SUMMER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	If $f(x)=3x^2-5x+7$ show that $f(-1)=3f(1)$	**02**
	Ans	$f(-1)=15$	1
		$f(1)=5$	
		$\therefore 3f(1)=15$	1
		$\therefore f(-1)=3f(1)$	
	b)	State whether the function $f(x)=3x^4+x^2+5-3\cos x+2\sin^2 x$ is even or odd	**02**
	Ans	$f(x)=3x^4+x^2+5-3\cos x+2\sin^2 x$	
		$\therefore f(-x)=3(-x)^4+(-x)^2+5-3\cos(-x)+2\sin^2(-x)$	½
		$\therefore f(-x)=3x^4+x^2+5-3\cos x+2\sin^2 x$	½
		$\therefore f(-x)=f(x)$	½
		$\therefore$ function is an even function	½
	c)	Find $\frac{dy}{dx}$ if $y=e^x.\sin^{-1}x$	**02**
	Ans	$y=e^x.\sin^{-1}x$	
		$\therefore \frac{dy}{dx}=e^x.\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x.e^x$	1+1

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	$\therefore \frac{dy}{dx}=e^{x}\left(\frac{1}{\sqrt{1-x^{2}}}+\sin^{-1}x\right)$	
	d)	Evaluate $e^{\int 2\log x\,dx}$	**02**
	Ans	$e^{\int 2\log x\,dx}$	
		$=e^{2\int \log x\,dx}$	
		$=e^{2\int \log x.1\,dx}$	½
		$=e^{2\left(\log x.x-\int x.\frac{1}{x}\,dx\right)}$	½
		$=e^{2\left(x\log x-\int 1\,dx\right)}$	½
		$=e^{2\left(x\log x-x\right)+c}$	½
		$=e^{2x\left(\log x-1\right)+c}$	
	e)	Evaluate $\int \sin^{2}x\,dx$	**02**
	Ans	$\int \sin^{2}x\,dx$	
		$=\frac{1}{2}\int 2\sin^{2}x\,dx$	
		$=\frac{1}{2}\int\left(1-\cos 2x\right)dx$	1
		$=\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c$	1
	f)	Find the area under the curve $y=x^{2}$ from $x=0$ to $x=3$ with $x-$axis.	**02**
	Ans	Area $A=\int_{a}^{b} y\,dx$	
		$=\int_{0}^{3} x^{2}dx$	½
		$=\left[\frac{x^{3}}{3}\right]_{0}^{3}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	f)	$=\left(\frac{3^3}{3}-0\right)$	½
		$=9$	½
	g)	Express $z=1-i$ in Polar form.	**02**
	Ans	$z=1-i$	
		$\therefore x=1, y=-1$	
		$\therefore r=\lvert z\rvert=\sqrt{x^2+y^2}=\sqrt{(1)^2+(-1)^2}=\sqrt{2}$	½
		$\theta=2\pi-\tan^{-1}\left(\left\lvert\frac{y}{x}\right\rvert\right)=2\pi-\tan^{-1}\left(\left\lvert\frac{-1}{1}\right\rvert\right)=2\pi-\tan^{-1}(1)$	
		$\theta=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$	½
		$\therefore$ Polar form is $z=r(\cos\theta+i\sin\theta)$	
		$\therefore\ 1-i=\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)$	1
2.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $x^2+y^2=4xy$	**04**
	Ans	$x^2+y^2=4xy$	
		$\therefore 2x+2y\frac{dy}{dx}=4\left(x\frac{dy}{dx}+y.1\right)$	2
		$\therefore 2x+2y\frac{dy}{dx}=4x\frac{dy}{dx}+4y$	
		$\therefore (2y-4x)\frac{dy}{dx}=4y-2x$	1
		$\therefore \frac{dy}{dx}=\frac{4y-2x}{2y-4x}$	1
		$\therefore \frac{dy}{dx}=\frac{2y-x}{y-2x}$	

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	b)	If $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$, find dy/dx at $\theta=\frac{\pi}{2}$	**04**
	Ans	$x=a(\theta+\sin\theta)$ $\quad$ $y=a(1-\cos\theta)$	
		$\frac{dx}{d\theta}=a(1+\cos\theta)$ $\quad$ $\frac{dy}{d\theta}=a(0+\sin\theta)=a\sin\theta$	1+1
		$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\sin\theta}{a(1+\cos\theta)}$	
		$\frac{dy}{dx}=\frac{\sin\theta}{1+\cos\theta}$	1
		at $\theta=\frac{\pi}{2}$	
		$\therefore\frac{dy}{dx}=\frac{\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}}=1$	1
		--	
	c)	Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=1$ at $\left(\frac{1}{4},\frac{1}{4}\right)$	**04**
	Ans	$\sqrt{x}+\sqrt{y}=1$	
		$\therefore\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$	½
		$\therefore\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$	½
		$\therefore\frac{d^2y}{dx^2}=-\frac{\left(\sqrt{x}\frac{1}{2\sqrt{y}}\frac{dy}{dx}-\sqrt{y}\frac{1}{2\sqrt{x}}\right)}{x}$	½
		$\therefore\frac{d^2y}{dx^2}=-\frac{\left(\frac{\sqrt{x}}{2\sqrt{y}}\cdot\frac{-\sqrt{y}}{\sqrt{x}}-\frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}=-\frac{\left(\frac{-1}{2}-\frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}$	½
		at $\left(\frac{1}{4},\frac{1}{4}\right)$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$\therefore \frac{dy}{dx}=-\frac{\sqrt{1/4}}{\sqrt{1/4}}=-1$	½
		$\therefore \frac{d^2y}{dx^2}=-\frac{\left(\frac{-1}{2}-\frac{\sqrt{1/4}}{2\sqrt{1/4}}\right)}{1/4}=4$	½
		$\therefore$ Radius of curvature $=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}=\frac{\left[1+(-1)^2\right]^{3/2}}{4}$	½
		$=0.707$	½
	d) Ans	Find the maximum and minimum value of x^3-9x^2+24y	**04**
		Let $y=x^3-9x^2+24x$	
		$\therefore \frac{dy}{dx}=3x^2-18x+24$	½
		$\therefore \frac{d^2y}{dx^2}=6x-18$	½
		Consider $\frac{dy}{dx}=0$	
		$3x^2-18x+24=0$	1
		$\therefore x=2$ or $x=4$	
		at $x=2$ $\therefore \frac{d^2y}{dx^2}=6(2)-18=-6<0$	½
		$\therefore y$ is maximum at $x=2$	
		$y_{\max}=(2)^3-9(2)^2+24(2)=20$	½
		at $x=4$ $\therefore \frac{d^2y}{dx^2}=6(4)-18=6>0$	½
		$\therefore y$ is minimum at $x=4$	
		$y_{\min}=(4)^3-9(4)^2+24(4)=16$	½
		***Note*: If students attempted to solve the question give appropriate marks.**	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.		**Attempt any THREE of the following:**	**12**
	a)	Find equation of tangent and normal to the curve $2x^2 - xy + 3y^2 = 18$ at $(3,1)$	**04**
	Ans	$2x^2 - xy + 3y^2 = 18$	
		$\therefore 4x - \left(x\frac{dy}{dx} + y.1\right) + 6y\frac{dy}{dx} = 0$	½
		$\therefore 4x - x\frac{dy}{dx} - y + 6y\frac{dy}{dx} = 0$	
		$\therefore (6y - x)\frac{dy}{dx} = y - 4x$	
		$\therefore \frac{dy}{dx} = \frac{y-4x}{6y-x}$	½
		at $(3,1)$	
		$\therefore \frac{dy}{dx} = \frac{1-4(3)}{6(1)-3}$	
		$\therefore \frac{dy}{dx} = \frac{-11}{3}$	
		$\therefore$ slope of tangent , $m = \frac{-11}{3}$	½
		Equation of tangent at $(3,1)$ is	
		$y - 1 = \frac{-11}{3}(x-3)$	
		$\therefore 3y - 3 = -11x + 33$	
		$\therefore 11x + 3y - 36 = 0$	1
		$\therefore$ slope of normal , $m' = \frac{-1}{m} = \frac{3}{11}$	½
		Equation of normal at $(3,1)$ is	
		$y - 1 = \frac{3}{11}(x-3)$	
		$\therefore 11y - 11 = 3x - 9$	
		$\therefore 3x - 11y + 2 = 0$	1
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	Find $\frac{dy}{dx}$ if $y = x^x + (\sin x)^x$	**04**
	Ans	Let $u = x^x$	
		Taking log on both sides,	
		$\therefore \log u = \log x^x$	½
		$\therefore \log u = x \log x$	
		$\therefore \frac{1}{u}\frac{du}{dx} = x\frac{1}{x} + \log x.1$	½
		$\therefore \frac{1}{u}\frac{du}{dx} = 1 + \log x$	
		$\therefore \frac{du}{dx} = u(1 + \log x)$	½
		Let $v = (\sin x)^x$	
		taking log on both sides,	
		$\therefore \log v = \log(\sin x)^x$	½
		$\therefore \log v = x \log \sin x$	
		$\therefore \frac{1}{v}\frac{dv}{dx} = x\frac{1}{\sin x}.\cos x + \log \sin x.1$	½
		$\therefore \frac{1}{v}\frac{dv}{dx} = x \cot x + \log \sin x$	
		$\therefore \frac{dv}{dx} = v(x \cot x + \log \sin x)$	½
		$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	
		$\therefore \frac{dy}{dx} = x^x(1 + \log x) + (\sin x)^x(x \cot x + \log \sin x)$	1
		--	
	c)	If $y = e^{3\sec x + 4\tan x}$ find $\frac{dy}{dx}$	**04**
	Ans	$y = e^{3\sec x + 4\tan x}$	
		$\therefore \frac{dy}{dx} = e^{3\sec x + 4\tan x}\left(3\sec x \cdot \tan x + 4\sec^2 x\right)$	4
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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$	**04**
	Ans	$\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$	
		Put $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	1
		$\therefore \int \frac{1}{(1+t)(3+t)} dt$	
		$\frac{1}{(1+t)(3+t)} = \frac{A}{1+t} + \frac{B}{3+t}$	½
		$\therefore 1 = A(3+t) + B(1+t)$	
		$\therefore$ Put $t=-1$, $A = \frac{1}{2}$	½
		Put $t=-3$, $B = \frac{-1}{2}$	½
		$\therefore \frac{1}{(1+t)(3+t)} = \frac{1/2}{1+t} - \frac{1/2}{3+t}$	
		$\therefore \int \frac{1}{(1+t)(3+t)} dt = \int \left(\frac{1/2}{1+t} - \frac{1/2}{3+t} \right) dt$	
		$= \frac{1}{2}\log(1+t) - \frac{1}{2}\log(3+t) + c$	1
		$= \frac{1}{2}\log(1+\tan x) - \frac{1}{2}\log(3+\tan x) + c$	½
		OR	
		$\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$	
		Put $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	1
		$\int \frac{1}{(1+t)(3+t)} dt$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$=\int \frac{1}{t^2+4t+3}\,dt$	
		Third Term $=\frac{4^2}{4}=4$	½
		$=\int \frac{1}{t^2+4t+4-4+3}\,dt$	1
		$=\int \frac{1}{(t+2)^2-1}\,dt$	
		$=\frac{1}{2}\log\left\|\frac{t+2-1}{t+2+1}\right\|+c$	1
		$=\frac{1}{2}\log\left\|\frac{t+1}{t+3}\right\|+c$	
		$=\frac{1}{2}\log\left\|\frac{\tan x+1}{\tan x+3}\right\|+c$	½
		--	
4.		**Attempt any <u>THREE</u> of the following:**	**12**
	a)	Evaluate $\int x\tan^{-1}x\,dx$	**04**
	Ans	$\int \tan^{-1}x\cdot x dx$	
		$=\tan^{-1}x\int x dx-\int\left(\int x dx\frac{d}{dx}\left(\tan^{-1}x\right)\right)dx$	1
		$=\tan^{-1}x\;\frac{x^2}{2}-\int\frac{x^2}{2}\frac{1}{1+x^2}dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$	
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2-1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)+c$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	Evaluate $\int \frac{dx}{4+5\cos x}$	**04**
	Ans	$\int \frac{dx}{4+5\cos x}$	
		Put $\tan\frac{x}{2}=t$, $dx=\frac{2dt}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$	1
		$=\int \frac{\frac{2dt}{1+t^2}}{4+5\left(\frac{1-t^2}{1+t^2}\right)}$	
		$=\int \frac{2dt}{4\left(1+t^2\right)+5\left(1-t^2\right)}$	½
		$=2\int \frac{dt}{4+4t^2+5-5t^2}$	
		$=2\int \frac{dt}{9-t^2}$	½
		$=2\int \frac{dt}{(3)^2-t^2}$	½
		$=2\frac{1}{2(3)}\log\left\|\frac{3+t}{3-t}\right\|+c$	1
		$=\frac{1}{3}\log\left\|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right\|+c$	½
	c)	Evaluate $\int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	**04**
	Ans	$\int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	
		Consider $\frac{2x^2+5}{(x-1)(x+2)(x+3)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3}$	½
		$\therefore 2x^2+5=A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2)$	
		Put $x=1$	
		$\therefore 7=12A$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	$\therefore A=\frac{7}{12}$	½
		Put $x=-2 \quad \Rightarrow 13=-3B$	
		$\therefore B=\frac{-13}{3}$	½
		Put $x=-3 \quad \Rightarrow 23=4C$	
		$\therefore C=\frac{23}{4}$	½
		$\therefore \int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx=\int\left(\frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3}\right)dx$	½
		$=\frac{7}{12}\log(x-1)-\frac{13}{3}\log(x+2)+\frac{23}{4}\log(x+3)+c$	½+½+½
	d)	Evaluate $\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$	**04**
	Ans	$\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$	
		Third Term $=\frac{(6)^2}{4}=9$	1
		$=\int \frac{1}{\sqrt{16+9-9-6x-x^2}}\,dx$	1
		$=\int \frac{1}{\sqrt{25-(9+6x+x^2)}}\,dx$	
		$=\int \frac{1}{\sqrt{(5)^2-(x+3)^2}}\,dx$	1
		$=\sin^{-1}\left(\frac{x+3}{5}\right)+c$	1
		OR	
		$\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	$=\int \frac{1}{\sqrt{-\left(x^2+6x-16\right)}}\,dx$	½
		Third Term $=\frac{(6)^2}{4}=9$	1
		$=\int \frac{1}{\sqrt{-\left(x^2+6x+9-9-16\right)}}\,dx$	½
		$=\int \frac{1}{\sqrt{-\left(x^2+6x+9-25\right)}}\,dx$	
		$=\int \frac{1}{\sqrt{-\left[(x+3)^2-(5)^2\right]}}\,dx$	
		$=\int \frac{1}{\sqrt{(5)^2-(x+3)^2}}\,dx$	1
		$=\sin^{-1}\left(\frac{x+3}{5}\right)+c$	1
		--	
	e)	Evaluate $\int_0^{\pi/2} \frac{dx}{1+\cot x}$	**04**
	Ans	$\int_0^{\pi/2} \frac{dx}{1+\cot x}$	
		$\therefore I=\int_0^{\pi/2} \frac{1}{1+\frac{\cos x}{\sin x}}\,dx$	½
		$\therefore I=\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$ --------------------(1)	
		$\therefore I=\int_0^{\pi/2} \frac{\sin\left(\pi/2-x\right)}{\sin\left(\pi/2-x\right)+\cos\left(\pi/2-x\right)}\,dx$	1
		$\therefore I=\int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\,dx$ --------------------(2)	
		add (1) and (2)	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$\therefore I+I=\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}\,dx+\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}\,dx$	½
		$2I=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sin x+\cos x}\,dx$	
		$2I=\int_0^{\pi/2}1\,dx$	
		$2I=[x]_0^{\pi/2}$	1
		$2I=\frac{\pi}{2}-0$	
		$\therefore I=\frac{\pi}{4}$	1
		--	
5.		**Attempt any <u>TWO</u> of the following:**	**04**
	a)	Find the area between the curves $y=x$ and $y=x^2$	
	Ans	$y=x$	
		$y=x^2$	
		$\therefore x-x^2=0$	
		$\therefore x(1-x)=0$	2
		$\therefore x=0,1$	1
		$\therefore A=\int_0^1(x-x^2)$	
		$A=\left(\frac{x^2}{2}-\frac{x^3}{3}\right)_0^1$	1
		$A=\left(\frac{1}{2}-\frac{1}{3}\right)$	1
		$\therefore A=\frac{1}{6}$ or 0.167	1
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5.		Attempt the following	**12**
	b)(i)	Find the order and degree of the differential equation	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b) (i) Ans	$\frac{d^2y}{dx^2}=\sqrt{1+\frac{dy}{dx}}$ $\frac{d^2y}{dx^2}=\sqrt{1+\frac{dy}{dx}}$ Squaring both sides, we get $\left(\frac{d^2y}{dx^2}\right)^2=1+\frac{dy}{dx}$ $\therefore$ Order $=2$ Degree $=2$	**03** 1 1 1
	b)(ii) Ans	Solve $\frac{dy}{dx}+y\cot x=\cos ecx$ $\frac{dy}{dx}+y\cot x=\cos ecx$ Comparing with $\frac{dy}{dx}+Py=Q$ $P=\cot x$, $Q=\cos ecx$ Integrating factor $IF=e^{\int \cot x dx}=e^{\log(\sin x)}=\sin x$ $y.IF=\int Q.IFdx+c$ $\therefore y\sin x=\int \cos ecx.\sin x\ dx$ $\therefore y\sin x=\int 1\ dx$ $\therefore y\sin x=x+c$	**03** 1 1 ½ ½
	c) Ans	If $L\frac{di}{dt}=30.\sin(10\pi t)$, find i in terms of t, given that $L=2$ and $i=0$ at $t=0$ $Ldi=30\sin(10\pi t)dt$ $\int Ldi=\int 30\sin(10\pi t)dt$ $Li=30\left(\frac{-\cos(10\pi t)}{10\pi}\right)+c$	**06** 1 2

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	$Li=\frac{-3\cos(10\pi t)}{\pi}+c$ at $t=0, i=0$ $L(0)=\frac{-3\cos(0)}{\pi}+c$ $0=\frac{-3}{\pi}+c$ $\therefore c=\frac{3}{\pi}$ $\therefore Li=\frac{-3\cos(10\pi t)}{\pi}+\frac{3}{\pi}$ at $L=2$ $\therefore 2i=\frac{-3\cos(10\pi t)}{\pi}+\frac{3}{\pi}$ $\therefore i=\frac{3}{2\pi}(-\cos(10\pi t)+1)$	½ 1 ½ 1
		--	
6.		**Attempt any TWO of the following:**	**04**
	a)	Attempt the following	
	(i)	Express $\frac{2-\sqrt{3}i}{1+i}$ in $x+iy$ form.	**03**
	Ans	$\frac{2-\sqrt{3}i}{1+i}$ $=\frac{2-\sqrt{3}i}{1+i}\times\frac{1-i}{1-i}$ $=\frac{2-2i-\sqrt{3}i+\sqrt{3}i^2}{1-i^2}$ $=\frac{2-(2+\sqrt{3})i+\sqrt{3}(-1)}{1-i^2}$ $=\frac{2-(2+\sqrt{3})i-\sqrt{3}}{1+1}$	1 ½ ½

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(i)	$=\dfrac{\left(2-\sqrt{3}\right)-\left(2+\sqrt{3}\right)i}{2}$	
		$=\dfrac{\left(2-\sqrt{3}\right)}{2}-\dfrac{\left(2+\sqrt{3}\right)i}{2}$	1
		--	**03**
	a)(ii)	Find $\mathrm{L}\left\{e^{-4t}t^2\right\}$	
	Ans	$\mathrm{L}\left\{e^{-4t}t^2\right\}$	1
		$L\left\{t^2\right\}=\dfrac{2!}{s^3}=\dfrac{2}{s^3}$	
		$\therefore \mathrm{L}\left\{e^{-4t}t^2\right\}=\dfrac{2}{\left(s+4\right)^3}$	2
		--	
	b)	Find $L^{-1}\left\{\dfrac{2s^2-4}{\left(s+1\right)\left(s-2\right)\left(s-3\right)}\right\}$	**04**
	Ans	$L^{-1}\left\{\dfrac{2s^2-4}{\left(s+1\right)\left(s-2\right)\left(s-3\right)}\right\}$	
		Let	
		$\dfrac{2s^2-4}{\left(s+1\right)\left(s-2\right)\left(s-3\right)}=\dfrac{A}{s+1}+\dfrac{B}{s-2}+\dfrac{C}{s-3}$	
		$2s^2-4=\left(s-2\right)\left(s-3\right)A+\left(s+1\right)\left(s-3\right)B+\left(s+1\right)\left(s-2\right)C$	
		Put $s=-1$	
		$\therefore -2=12A$	
		$\therefore A=-\dfrac{1}{6}$	1
		Put $s=2$	
		$4=-3B$	
		$\therefore B=-\dfrac{4}{3}$	1
		Put $s=3$	
		$14=4C$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	$\therefore C=\frac{7}{2}$	1
		$\therefore \frac{2s^2-4}{(s+1)(s-2)(s-3)}=\frac{-\frac{1}{6}}{s+1}+\frac{-\frac{4}{3}}{s-2}+\frac{\frac{7}{2}}{s-3}$	
		$\therefore L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}=-\frac{1}{6}L^{-1}\left\{\frac{1}{s+1}\right\}-\frac{4}{3}L^{-1}\left\{\frac{1}{s-2}\right\}+\frac{7}{2}L^{-1}\left\{\frac{1}{s-3}\right\}$	
		$=-\frac{1}{6}e^{-t}-\frac{4}{3}e^{2t}+\frac{7}{2}e^{3t}$	1+1+1
	c)	Solve using Laplace transform	**06**
		$\frac{dx}{dt}+2x=e^{-t}$, given that $x(0)=2$	
	Ans	$\frac{dx}{dt}+2x=e^{-t}$	
		$\therefore L\left\{\frac{dx}{dt}+2x\right\}=L\left\{e^{-t}\right\}$	
		$\therefore sL(x)-x(0)+2L(x)=\frac{1}{s+1}$	1
		$\therefore sL(x)-2+2L(x)=\frac{1}{s+1}$	
		$\therefore (s+2)L(x)-2=\frac{1}{s+1}$	
		$\therefore (s+2)L(x)=\frac{1}{s+1}+2$	
		$\therefore (s+2)L(x)=\frac{2s+3}{s+1}$	
		$\therefore L(x)=\frac{2s+3}{(s+1)(s+2)}$	1
		$\therefore x=L^{-1}\left\{\frac{2s+3}{(s+1)(s+2)}\right\}$	½
		$\therefore \frac{2s+3}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$	½
		$\therefore 2s+3=A(s+2)+B(s+1)$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	Put $s=-1$ $\therefore A=1$ Put $s=-2$ $\therefore B=1$ $\therefore \frac{2s+3}{(s+1)(s+2)}=\frac{1}{s+1}+\frac{1}{s+2}$ $\therefore L^{-1}\left\{\frac{2s+3}{(s+1)(s+2)}\right\}=L^{-1}\left\{\frac{1}{s+1}+\frac{1}{s+2}\right\}$ $=L^{-1}\left\{\frac{1}{s+1}\right\}+L^{-1}\left\{\frac{1}{s+2}\right\}$ $=e^{-t}+e^{-2t}$	½ ½ 1+1

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.
