SUMMER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 17301**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling error should not be given more importance (Not applicable for English and Communication Skill subjects).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Solve any TEN of the following:**	**20**
	a)	At what point on the curve $y=e^x$, the slope is 1?	**02**
	Ans	$y=e^x$	
		$\therefore \frac{dy}{dx}=e^x$	½
		$\because$ Slope $=1$	
		$\therefore e^x=1$	½
		$\therefore x=0$	
		$\therefore y=e^0=1$	
		$\therefore$ Point is $(0,1)$	1
		--	
	b)	Find the radius of curvature of the curve $y=x^3$ at $(2,8)$.	**02**
	Ans	$y=x^3$	
		$\therefore \frac{dy}{dx}=3x^2$	
		$\therefore \frac{d^2y}{dx^2}=6x$	
		$\therefore$ at $(2,8)$	
		$\frac{dy}{dx}=12$	½
		$\frac{d^2y}{dx^2}=12$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	$\therefore$ Radius of curvature is $\rho=\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$	
		$\therefore \rho=\dfrac{\left[1+12^2\right]^{\frac{3}{2}}}{12}$	½
		$\therefore \rho=145.5$	½
		$\therefore$ Radius of curvature is 145.5	
	c)	Evaluate $\int \dfrac{\cos(\log x)}{x}\ dx$	**02**
	Ans	$\int \dfrac{\cos(\log x)}{x}\ dx$	
		Let $\log x=t$	½
		$\therefore \dfrac{1}{x}dx=dt$	½
		$=\int \cos t dt$	
		$=\sin t+c$	½
		$=\sin(\log x)+c$	½
	d)	Evaluate $\int \cos ec^2\left(e^x\right)\times e^x\ dx$	**02**
	Ans	$\int \cos ec^2\left(e^x\right)\times e^x\ dx$	
		Let $e^x=t$	½
		$\therefore e^x dx=dt$	½
		$=\int \cos ec^2 t dt$	
		$=-\cot t+c$	½
		$=-\cot\left(e^x\right)+c$	½
	e)	Evaluate $\int x\times a^x\ dx$	**02**
	Ans	$\int x\times a^x\ dx$	
		$=x\int a^x\ dx-\int\left[\int a^x\ dx\dfrac{d}{dx}(x)\right]dx$	1
		$=x\dfrac{a^x}{\log a}-\int \dfrac{a^x}{\log a}dx$	

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1.	e)	$= x\frac{a^x}{\log a} - \frac{a^x}{(\log a)^2} + c$	1
	f)	Evaluate $\int \frac{1}{(x+3)(x-2)} dx$	**02**
	Ans	$\int \frac{1}{(x+3)(x-2)} dx$	
		Consider $\frac{1}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$	
		$\therefore 1 = A(x-2) + B(x+3)$	
		Put $x = 2 \quad \Rightarrow B = \frac{1}{5}$	½
		Put $x = -3 \Rightarrow A = \frac{-1}{5}$	½
		$\therefore \frac{1}{(x+3)(x-2)} = \frac{-1/5}{x+3} + \frac{1/5}{x-2}$	
		$\therefore \int \frac{1}{(x+3)(x-2)} dx = \int \left(\frac{-1/5}{x+3} + \frac{1/5}{x-2} \right) dx$	
		$= -\frac{1}{5}\log(x+3) + \frac{1}{5}\log(x-2) + c$	½ +½
	g)	Evaluate $\int_1^2 \frac{dx}{4x-1}$	**02**
	Ans	$\int_1^2 \frac{dx}{4x-1}$	
		$= \frac{1}{4}\left[\log(4x-1)\right]_1^2$	1
		$= \frac{1}{4}\left[\log(4(2)-1) - \log(4(1)-1)\right]$	½
		$= \frac{1}{4}[\log 7 - \log 3]$ or $\frac{1}{4}\log\left(\frac{7}{3}\right)$	½
	h)	Find the area enclosed by the curve $y = 3x^2$ and the lines $x = 1$, $x = 3$, and x – axis.	**02**
	Ans	Area $A = \int_a^b y dx$	

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1.	h)	$\therefore A=\int_{1}^{3}3x^2dx$	½
		$A=3\left[\frac{x^3}{3}\right]_1^3$ or $A=\left[x^3\right]_1^3$	½
		$A=3\left[\frac{3^3}{3}-\frac{1^3}{3}\right]$ or $A=\left[3^3-1^3\right]$	½
		$A=26$	½
		---	**02**
	i)	Find the order and degree of the differential equation	
	Ans	$\frac{d^3y}{dx^3}+\sqrt{1+\frac{dy}{dx}}=0$	
		$\frac{d^3y}{dx^3}=-\sqrt{1+\frac{dy}{dx}}$	
		Squaring both sides, we get	
		$\left(\frac{d^3y}{dx^3}\right)^2=1+\frac{dy}{dx}$	
		$\therefore$ Order $=3$	1
		Degree $=2$	1
		---	
	j)	Find integrating factor of $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}}x$	**02**
	Ans	$\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$	
		$\therefore \frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}$	½
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{1}{1+x^2}$	½
		$\therefore$ Integrating factor $=e^{\int Pdx}$	
		$=e^{\int\frac{1}{1+x^2}dx}$	
		$=e^{\tan^{-1}x}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	k)	From a pack of 52 cards,one card is drawn at random.Find the probability of getting a face card.	**02**
	Ans	$n(S)={}^{52}C_1$	½
		$n(A)={}^{12}C_1$	½
		$\therefore P(A)=\dfrac{n(A)}{n(S)}$	
		$\therefore P(A)=\dfrac{{}^{12}C_1}{{}^{52}C_1}=\dfrac{12}{52}$	1
		$\therefore P(A)=\dfrac{3}{13}$ or 0.2308	
		--	
	l)	A unbiased coin is tossed 5 times. Find the probability of getting 2 tails.	**02**
	Ans	Here $n=5,\ p=0.5,\ q=0.5,\ r=2$	
		$P(x=r)={}^{n}C_r\,p^r q^{n-r}$	
		$\therefore P(2)={}^{5}C_2\,(0.5)^2\,(0.5)^3$	1
		$\therefore P(2)=\dfrac{5}{16}$ or 0.3125	1
		--	
2.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Find the equation of the tangent and the normal to the parabola $y^2=4x$ at the point $(1,2)$	**04**
	Ans	$y^2=4x$	
		$\therefore 2y\dfrac{dy}{dx}=4$	1
		$\therefore \dfrac{dy}{dx}=\dfrac{4}{2y}$	
		at $(1,2)$	
		$\therefore \dfrac{dy}{dx}=\dfrac{4}{2(2)}$	
		$\therefore$ slope $m=1$	½
		$\therefore$ Equation of tgt is	
		$(y-2)=1(x-1)$	½
		$\therefore x-y+1=0$	½
		$\therefore$ slope of normal $=\dfrac{-1}{m}=\dfrac{-1}{1}=-1$	½

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2.	a)	$\therefore$ Equation of normal is	
		$(y-2)=-1(x-1)$	½
		$\therefore x+y-3=0$	½
	b)	Find the radius of curvature for $y=x^3+3x^2+2$ at $(1,2)$	**04**
	Ans	$y=x^3+3x^2+2$	
		$\therefore \frac{dy}{dx}=3x^2+6x$	½
		$\frac{d^2y}{dx^2}=6x+6$	½
		$\therefore$ at $(1,2)$	
		$\frac{dy}{dx}=3(1)^2+6(1)=9$	½
		$\frac{d^2y}{dx^2}=6(1)+6=12$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(9)^2\right]^{\frac{3}{2}}}{12}$	1
		$\therefore \rho=61.878$	1
		$\therefore$ Radius of curvature is 61.878	
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length of a rectangle $=x$, breadth of a rectangle $=y$	
		$\therefore 2x+2y=36$	½
		$\therefore x+y=18$	
		$\therefore y=18-x$	½
		$\therefore$ Area $A=xy$	
		$=x(18-x)$	
		$=18x-x^2$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c) Ans	$\therefore \frac{dA}{dx}=18-2x$	½
		Let $\frac{dA}{dx}=0$	
		$\therefore 18-2x=0$	½
		$\therefore x=9$	½
		$\therefore \frac{d^2A}{dx^2}=-2$	½
		$\therefore A$ is maximum when $x=9$	
		$\therefore$ length $=9$, breadth $=9$	½
	d)	Evaluate $\int \frac{x^2+1}{(x+1)(x+2)(x+3)}dx$	**04**
	Ans	Consider	½
		$\frac{x^2+1}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}$	
		$\therefore x^2+1=A(x+2)(x+3)+B(x+1)(x+3)+C(x+1)(x+2)$	
		Put $x=-1$	½
		$\therefore A=1$	
		Put $x=-2$	½
		$\therefore B=-5$	
		Put $x=-3$	½
		$\therefore C=5$	½
		$\therefore \frac{x^2+1}{(x+1)(x+2)(x+3)}=\frac{1}{x+1}+\frac{-5}{x+2}+\frac{5}{x+3}$	
		$\therefore \int \frac{x^2+1}{(x+1)(x+2)(x+3)}dx=\int\left(\frac{1}{x+1}+\frac{-5}{x+2}+\frac{5}{x+3}\right)dx$	
		$=\log(x+1)-5\log(x+2)+5\log(x+3)+c$	½+½+½
	e)	Evaluate $\int \frac{\cos x}{\sin^2 x+10\sin x+26}dx$	**04**
	Ans	$\int \frac{\cos x}{\sin^2 x+10\sin x+26}dx$	
		Let $\sin x=t$	½
		$\therefore \cos x dx=dt$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	e)Ans	$= \int \frac{1}{t^2+10t+26}dt$	½
		Third term $= \frac{(10t)^2}{4t^2} = 25$	½
		$= \int \frac{1}{t^2+10t+25-25+26}dt$	
		$= \int \frac{1}{(t+5)^2+1}dt$	1
		$= \tan^{-1}(t+5)+c$	½
		$= \tan^{-1}(\sin x+5)+c$	½
		--	
	f)	Evaluate $\int \frac{\cos ec^2 x}{(1+\cot x)(3+\cot x)}dx$	**04**
	Ans	$\int \frac{\cos ec^2 x}{(1+\cot x)(3+\cot x)}\ dx$	
		Put $\cot x = t$	½
		$\therefore -\cos ec^2 x dx = dt$	½
		$\therefore \int \frac{-1}{(1+t)(3+t)}dt$	
		$\frac{-1}{(1+t)(3+t)} = \frac{A}{1+t}+\frac{B}{3+t}$	½
		$\therefore -1 = A(3+t)+B(1+t)$	
		$\therefore$ Put $t=-1$, $A = \frac{-1}{2}$	½
		Put $t=-3$, $B = \frac{1}{2}$	½
		$\therefore \frac{-1}{(1+t)(3+t)} = \frac{-1/2}{1+t}+\frac{1/2}{3+t}$	
		$\therefore \int \frac{-1}{(1+t)(3+t)}dt = \int\left(\frac{-1/2}{1+t}+\frac{1/2}{3+t}\right)dt$	
		$= -\frac{1}{2}\log(1+t)+\frac{1}{2}\log(3+t)+c$	½+½
		$= -\frac{1}{2}\log(1+\cot x)+\frac{1}{2}\log(3+\cot x)+c$	½
		OR	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	f) Ans	$\int \frac{\cos ec^2 x}{(1+\cot x)(3+\cot x)}\, dx$	
		Put $\cot x = t$	½
		$\therefore -\cos ec^2 x dx = dt$	½
		$\therefore \int \frac{-1}{(1+t)(3+t)} dt$	
		$= \int \frac{-1}{t^2+4t+3}\, dt$	
		Third Term $= \frac{4^2}{4} = 4$	1
		$= \int \frac{-1}{t^2+4t+4-4+3}\, dt$	½
		$= \int \frac{-1}{(t+2)^2-1}\, dt$	½
		$= -\frac{1}{2}\log\left\|\frac{t+2-1}{t+2+1}\right\| + c$	½
		$= -\frac{1}{2}\log\left\|\frac{t+1}{t+3}\right\| + c$	
		$= -\frac{1}{2}\log\left\|\frac{\cot x+1}{\cot x+3}\right\| + c$	½
3.		**Solve any FOUR of the following :**	**16**
	a)	Evaluate $\int_0^{\pi} \cos^3 x.\sin x.dx$	**04**
	Ans	$\int_0^{\pi} \cos^3 x.\sin x.dx$	
		Put $\cos x = t$	½
		$-\sin x dx = dt \quad \Rightarrow \sin x dx = -dt$	½
		when $x \to 0$ to π, $t \to 1$ to -1	
		$= -\int_1^{-1} t^3 dt$	1
		$= \left[-\frac{t^4}{4}\right]_1^{-1}$	1

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3.	a)	$=\left[-\frac{(-1)^4}{4}\right]-\left[-\frac{(1)^4}{4}\right]$	½
		$=0$	½
		--	
	b)	Evaluate $\int_0^{\pi/2}\frac{1}{1+\cot x}dx$	**04**
	Ans	$\int_0^{\pi/2}\frac{1}{1+\cot x}dx$	
		$I=\int_0^{\pi/2}\frac{1}{1+\frac{\cos x}{\sin x}}\,dx$	
		$\therefore I=\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}\,dx$ --------------------(1)	½
		$I=\int_0^{\pi/2}\frac{\sin\left(\pi/2-x\right)}{\sin\left(\pi/2-x\right)+\cos\left(\pi/2-x\right)}\,dx$	1
		$\therefore I=\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}\,dx$ --------------------(2)	½
		add (1) and (2)	
		$I+I=\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}\,dx+\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}\,dx$	½
		$2I=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sin x+\cos x}\,dx$	
		$2I=\int_0^{\pi/2}1\,dx$	
		$2I=[x]_0^{\pi/2}$	½
		$2I=\frac{\pi}{2}-0$	
		$I=\frac{\pi}{4}$	1

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3.	c)	Find the area bounded by the curve $y = x^2$ and the line $y = x$	**04**
	Ans	$y = x$ and $y = x^2$	
		$\therefore x - x^2 = 0$	
		$\therefore x(1-x) = 0$	
		$\therefore x = 0,1$	1
		$\therefore A = \int_a^b (y_1 - y_2)dx$	
		$= \int_0^1 (x - x^2)dx$	1
		$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1$	1
		$= \left(\frac{1}{2} - \frac{1}{3}\right)$	½
		$\therefore A = \frac{1}{6}$ or 0.167	½
		---	
	d)	Solve $\frac{dy}{dx} = \sin(x+y)$	**04**
	Ans	Put $x + y = v$	
		$\therefore 1 + \frac{dy}{dx} = \frac{dv}{dx}$	
		$\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1$	1
		$\therefore \frac{dv}{dx} - 1 = \sin v$	
		$\therefore \frac{dv}{dx} = \sin v + 1$	
		$\therefore \frac{dv}{\sin v + 1} = dx$	
		$\therefore$ solution is	
		$\therefore \int \frac{dv}{\sin v + 1} = \int dx$	1
		$\therefore \int \frac{1}{1+\sin v} \cdot \frac{1-\sin v}{1-\sin v} dv = \int dx$	½
		$\therefore \int \frac{1-\sin v}{1-\sin^2 v} dv = \int dx$	
		$\therefore \int \frac{1-\sin v}{\cos^2 v} dv = \int dx$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	$\therefore \int\left(\frac{1}{\cos^2 v}-\frac{\sin v}{\cos^2 v}\right)dv=\int dx$	
		$\therefore \int\left(\sec^2 v-\sec v\tan v\right)dv=\int dx$	
		$\therefore \tan v-\sec v=x+c$	1
		$\therefore \tan\left(x+y\right)-\sec\left(x+y\right)=x+c$	½
		--	
	e)	Solve $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$	**04**
	Ans	Put $y=vx$	½
		$\frac{dy}{dx}=v+x\frac{dv}{dx}$	
		$\therefore v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2x\left(vx\right)}$	½
		$\therefore v+x\frac{dv}{dx}=\frac{x^2\left(1+v^2\right)}{2vx^2}$	
		$\therefore v+x\frac{dv}{dx}=\frac{1+v^2}{2v}$	
		$\therefore x\frac{dv}{dx}=\frac{1+v^2}{2v}-v$	
		$\therefore x\frac{dv}{dx}=\frac{1+v^2-2v^2}{2v}$	
		$\therefore x\frac{dv}{dx}=\frac{1-v^2}{2v}$	
		$\therefore \frac{2v}{1-v^2}dv=\frac{1}{x}dx$	1
		$\therefore -\int\frac{-2v}{1-v^2}dv=\int\frac{1}{x}dx$	½
		$\therefore -\log\left(1-v^2\right)=\log x+c$	1
		$-\log\left(1-\left(\frac{y}{x}\right)^2\right)=\log x+c$	½
		--	
	f)	Solve: $\frac{dy}{dx}+y\tan x=\sec x$	**04**
	Ans	$\frac{dy}{dx}+y\tan x=\sec x$	

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3.	f)	Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\tan x,\ Q=\sec x$	1
		$\therefore I.F.=e^{\int Pdx}=e^{\int \tan x\ dx}$	
		$\therefore I.F.=e^{\log(\sec x)}$	
		$\therefore I.F.=\sec x$	1
		$\therefore$ Solution is	
		$y.I.F.=\int QI.F.dx+c$	
		$y.\sec x=\int \sec x.\sec x\ dx+c$	1
		$y.\sec x=\int \sec^2 x\ dx+c$	
		$y.\sec x=\tan x+c$	1
4.		**Solve any FOUR of the following :**	**16**
	a)	Evaluate $\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx$	**04**
	Ans	Let $I=\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx$ ---------------------(1)	
		$I=\int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{7-(7-x)}}\,dx$	
		$\therefore I=\int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{x}}\,dx$---------------------(2)	1
		add (1) and (2)	
		$\therefore I+I=\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx+\int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{x}}\,dx$	1
		$\therefore 2I=\int_0^7 \frac{\sqrt[3]{x}+\sqrt[3]{7-x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx$	
		$\therefore 2I=\int_0^7 1\,dx$	½
		$\therefore 2I=[x]_0^7$	1
		$\therefore 2I=7-0$	
		$\therefore I=\frac{7}{2}$	½

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4.	b)	Evaluate $\int_0^1 x\cdot\tan^{-1} x\,dx$	**04**
	Ans	$\int_0^1 x\cdot\tan^{-1} x\,dx$	
		$=\left[\tan^{-1}x\int x\,dx-\int\left(\int x\,dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx\right]_0^1$	1
		$=\left[\frac{x^2}{2}\tan^{-1}x-\int\frac{x^2}{2}\cdot\frac{1}{x^2+1}\cdot dx\right]_0^1$	½
		$=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{x^2+1}\cdot dx\right]_0^1$	
		$=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{\left(x^2+1\right)-1}{x^2+1}\cdot dx\right]_0^1$	½
		$=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{x^2+1}\right)\cdot dx\right]_0^1$	½
		$=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)\right]_0^1$	½
		$=\left[\frac{1^2}{2}\tan^{-1}1-\frac{1}{2}\left(1-\tan^{-1}1\right)\right]-[0]$	½
		$=\frac{\pi}{8}-\frac{1}{2}\left(1-\frac{\pi}{4}\right)=\frac{\pi}{4}-\frac{1}{2}$ or 0.2854	½
	c)	Find by integration the area of the circle $x^2+y^2=25$	**04**
	Ans	$x^2+y^2=25$	
		$\therefore y^2=25-x^2$	
		$\therefore y=\sqrt{25-x^2}$	
		$At\ \ y=0,\quad 25-x^2=0$	
		$\therefore x=5$	
		$\therefore A=4\int_a^b y\,dx$	
		$=4\int_0^5\sqrt{25-x^2}\,dx$	1
		$=4\int_0^5\sqrt{5^2-x^2}\,dx$	
		$=4\left[\frac{x}{2}\sqrt{5^2-x^2}+\frac{5^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$	1

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4.	c)	$=4\left[0+\frac{25}{2}\sin^{-1}(1)\right]-\left[0+\frac{25}{2}\sin^{-1}(0)\right]$	1
		$=4\left[\frac{25}{2}\cdot\frac{\pi}{2}\right]$	
		$=25\pi$	1
		--	
	d)	Solve $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$	**04**
	Ans	$\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$	
		$\sec^2 x\tan y dx=-\sec^2 y\tan x dy$	
		$\frac{\sec^2 x}{\tan x}dx=-\frac{\sec^2 y}{\tan y}dy$	1
		$\therefore$ Solution is,	
		$\int\frac{\sec^2 x}{\tan x}dx=-\int\frac{\sec^2 y}{\tan y}dy$	1
		$\log(\tan x)=-\log(\tan y)+c$	1+1
		--	
	e)	Solve $\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^2\right)dy=0$	**04**
	Ans	$\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^2\right)dy=0$	
		Comparing with $Mdx+Ndy=0$	
		$\therefore M=3x^2+6xy^2, N=6x^2y+4y^2$	
		$\frac{\partial M}{\partial y}=12xy$, $\frac{\partial N}{\partial x}=12xy$	1
		$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ $\therefore$ D.E. is an exact	1
		$\therefore$ Solution is,	
		$\int\limits_{y-constant} Mdx+\int\limits_{terms\ free\ from\ 'x'} Ndy=c$	
		$\therefore \int\limits_{y-constant}\left(3x^2+6xy^2\right)dx+\int 4y^2dy=c$	1
		$\therefore x^3+3x^2y^2+\frac{4}{3}y^3=c$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	Verify that $y=\log x$ is a solution of $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	04
	Ans	$\because y=\log x$	
		$\therefore \frac{dy}{dx}=\frac{1}{x}$	1
		$\therefore x\frac{dy}{dx}=1$	1
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	2
5.		**Solve any <u>FOUR</u> of the following :**	04
	a)	A and B are two Independent events.From a sample space S,such that $P(A)=0.8$, $P(B)=0.6$ and $P(A\cup B)=0.9$. Find $(i)\ P(A\cap B)$ $(ii)\ P\left(A/B\right)$	
	Ans	$(i) P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
		$\therefore 0.9=0.8+0.6-P(A\cap B)$	1
		$\therefore P(A\cap B)=0.5$	1
		$(ii)\ P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}$	
		$=\frac{0.5}{0.6}$	1
		$=\frac{5}{6}$ or 0.8333	1
	b)	If 30% of the bulbs produced are defective,find the probability that out of 4 bulbs selected (i) One is defective (ii) at the most two are defective	04
	Ans	$p=30\%=\frac{30}{100}=0.3 \quad q=1-0.3=0.7$	1
		$n=4$	
		$(i)\because p(r)={}^nC_r(p)^r(q)^{n-r}$	1
		$\therefore p(1)={}^4C_1(0.3)^1(0.7)^{4-1}=0.4116$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)	$ii)\, p(\text{at most } 2) = p(0)+p(1)+p(2)$	
		$= {}^4C_0(0.3)^0(0.7)^{4-0} + {}^4C_1(0.3)^1(0.7)^{4-1} + {}^4C_2(0.3)^2(0.7)^{4-2}$	1
		$=0.9163$	1
	c)	Fit a Poisson distribution to the set of observations.	**04**
		x: 20, 30, 40, 50, 60, 70; f: 8, 12, 20, 10, 6, 4	
	Ans		
		(table below)	2
		$\therefore$ mean $m = \dfrac{\sum xy}{\sum y} = \dfrac{2460}{60} = 41$	1
		$\therefore P(r) = \dfrac{e^{-m}m^r}{r!}$	
		$\therefore P(r) = \dfrac{e^{-41}(41)^r}{r!}$	1
	d)	Evaluate $\displaystyle\int \frac{dx}{5+4\cos x}$	**04**
	Ans	$\displaystyle\int \frac{dx}{5+4\cos x}$	
		Put $\tan\dfrac{x}{2}=t$, $dx=\dfrac{2dt}{1+t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$	
		$= \displaystyle\int \frac{\frac{2dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)}$	1

x	20	30	40	50	60	70
f	8	12	20	10	6	4

x	y	xy
20	8	160
30	12	360
40	20	800
50	10	500
60	6	360
70	4	280
	$\sum y =$ **60**	$\sum xy =$ **2460**

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5.	d)	$=\int\frac{2dt}{5\left(1+t^2\right)+4\left(1-t^2\right)}$	
		$=2\int\frac{dt}{5+5t^2+4-4t^2}$	1
		$=2\int\frac{dt}{t^2+9}$	
		$=2\int\frac{dt}{t^2+3^2}$	½
		$=2\frac{1}{3}\tan^{-1}\frac{t}{3}+c$	1
		$=\frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$	½
		--	
	e)	Evaluate $\int\frac{2x}{\left(x^2-1\right)\left(x^2+3\right)}dx$	**04**
	Ans	$\int\frac{2x}{\left(x^2-1\right)\left(x^2+3\right)}dx$	
		Let $x^2=t$	
		$\therefore 2xdx=dt$	1
		$=\int\frac{1}{(t-1)(t+3)}dt$	
		Consider	
		$\frac{1}{(t-1)(t+3)}=\frac{A}{t-1}+\frac{B}{t+3}$	
		$\therefore 1=A(t+3)+B(t-1)$	
		Put $t=1$	
		$\therefore A=\frac{1}{4}$	½
		Put $t=-3$	
		$\therefore B=\frac{-1}{4}$	½
		$\therefore\frac{1}{(t-1)(t+3)}=\frac{1/4}{t-1}+\frac{-1/4}{t+3}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	e)	$\therefore \int \frac{1}{(t-1)(t+3)}dt = \int\left(\frac{1/4}{t-1}+\frac{-1/4}{t+3}\right)dt$	
		$= \frac{1}{4}\log(t-1) - \frac{1}{4}\log(t+3)+c$	1
		$= \frac{1}{4}\log(x^2-1) - \frac{1}{4}\log(x^2+3)+c$	½
	f)	Solve $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$	**04**
	Ans	$(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$	
		$\therefore \frac{dy}{dx} - \frac{y}{x+1} = e^x(x+1)$	½
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P = \frac{-1}{x+1},\ \ Q = e^x(x+1)$	½
		Integrating Factor $= e^{\int Pdx} = e^{\int \frac{-1}{x+1}dx}$	
		$= e^{-\log(x+1)}$	
		$= \frac{1}{x+1}$	1
		$\therefore$ Solution is,	
		$y\cdot I.F. = \int Q\cdot I.F.dx + c$	
		$y\cdot\frac{1}{x+1} = \int e^x(x+1)\cdot\frac{1}{x+1}dx + c$	1
		$\therefore \frac{y}{x+1} = \int e^x dx + c$	
		$\therefore \frac{y}{x+1} = e^x + c$	1
6.		**Solve any FOUR of the following :**	**16**
	a)	A bag contains 20 tickets numbered from 1 to 20. One ticket is drawn at random. Find the probability that it is numbered with multiple of 3 or 4	**04**

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)Ans	$n(S)=20$	1
		$A=\{3,4,6,8,9,12,15,16,18,20\}$	
		$\therefore n(A)=10$	1
		$\therefore p=\frac{n(A)}{n(S)}=\frac{10}{20}=0.5$	2
		--	
	b)	The chance of two students to win a competition are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If they participate in the same condition,what is the probability that at least one will win?	**04**
	Ans	Given $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$	
		The probability that at least one will win	
		$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
		since A and B are Independent,	
		$\therefore P(A\cup B)=P(A)+P(B)-P(A)\times P(B)$	
		$\therefore P(A\cup B)=\frac{1}{2}+\frac{1}{3}-\frac{1}{2}\times\frac{1}{3}$	2
		$\therefore P(A\cup B)=\frac{2}{3}$ or 0.667	2
		OR	
		Given $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$	
		$\therefore P(A')=1-\frac{1}{2}=\frac{1}{2}$	½
		$P(B')=1-\frac{1}{3}=\frac{2}{3}$	½
		The probability that no one will win	
		$\because P(A'\cap B')=P(A')\times P(B')$, since A and B are Independent	
		$=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$	1
		The probability that at least one will win	
		$P(A\cup B)=1-P(A\cup B)'=1-P(A'\cap B')$	
		$=1-\frac{1}{3}$	
		$=\frac{2}{3}$ or 0.667	2

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	I.Q.'s are normally distributed with mean 100 and standard deviation 15. Find the probability that a randomly selected person has (i) An I.Q. more than 130, (ii) An I.Q. between 85 and 115. Given $\begin{bmatrix} Z=2,\ \text{Area}=0.4772 \\ Z=1,\ \text{Area}=0.3413 \end{bmatrix}$	**04**
	Ans	Given $\bar{x}=100 \quad \sigma=15$	
		$i)\quad z=\frac{130-100}{15}=2$	1
		$\therefore P(130\le x)=P(2\le z)$ $=0.5-P(0\le z\le 2)$ $=0.5-0.4772$	
		$=0.0228$	1
		2) $z=\frac{85-100}{15}=-1 \qquad z=\frac{115-100}{15}=1$	1
		$\therefore P(85\le x\le 115)=P(-1\le z\le 1)$ $=P(-1\le z\le 0)+P(0\le z\le 1)$ $=0.3413+0.3413$	
		$=0.6826$	1
	d)	The equation of the tangent at $(2, 3)$ on the curve $y=ax^3+b$, is $y=4x-5$. Find the values of a and b.	**04**
	Ans	$y=ax^3+b$	
		$\therefore \frac{dy}{dx}=3ax^2$	1
		$\therefore$ slope $m=\frac{dy}{dx}=3a(2)^2=12a$	½
		$\because$ the equation of tangent is $y=4x-5$	
		$\therefore$ slope $m=4$	½
		$\therefore 12a=4$	
		$\therefore a=\frac{4}{12}=\frac{1}{3}$	1
		$\because$ the point (2, 3) is on the curve $y=ax^3+b$	
		$\therefore 3=a(2)^3+b$	
		$\therefore b=3-8a=3-8\left(\frac{1}{3}\right)=\frac{1}{3}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	e)	Find the maximum and minimum values of $x^3 - 9x^2 + 24x$	**04**
	Ans	Let $y = x^3 - 9x^2 + 24x$	
		$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24$	½
		$\therefore \frac{d^2y}{dx^2} = 6x - 18$	½
		Let $\frac{dy}{dx} = 0$	
		$\therefore 3x^2 - 18x + 24 = 0$	½
		$\therefore x = 2,\ 4$	½
		At $x = 2,\ \frac{d^2y}{dx^2} = 6(2) - 18 = -6 < 0$	½
		$\therefore$ At $x = 2,\ y$ is maximum	
		$y_{max} = (2)^3 - 9(2)^2 + 24(2) = 20$	½
		At $x = 4,\ \frac{d^2y}{dx^2} = 6(4) - 18 = 6 > 0$	½
		$\therefore$ At $x = 8,\ y$ *is* minimum	
		$y_{min} = (4)^3 - 9(4)^2 + 24(4) = 16$	½
		--	
	f)	Find the area bounded by the parabola $y^2 = 9x$ and $x^2 = 9y$.	**04**
	Ans	$y^2 = 9x$ ---------(1)	
		$x^2 = 9y$	
		$\therefore y = \frac{x^2}{9}$	
		$\therefore \text{eq}^n.(1) \Rightarrow \left(\frac{x^2}{9}\right)^2 = 9x$	
		$\frac{x^4}{81} = 9x$	
		$\therefore x^4 = 729x$	
		$\therefore x^4 - 729x = 0$	
		$\therefore x(x^3 - 9^3) = 0$	1
		$\therefore x = 0, 9$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	f)	Area $A=\int_a^b (y_1-y_2)\,dx$	
		$\therefore A=\int_0^9\left(3\sqrt{x}-\frac{x^2}{9}\right)dx$	1
		$\therefore A=\left(\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{27}\right)_0^9$	1
		$\therefore A=\left(\frac{3(9)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(9)^3}{27}\right)-0$	
		$\therefore A=27$	1

		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	

