WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 17301**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling error should not be given more importance (Not applicable for English and Communication Skill subjects).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Solve any TEN of the following:**	**20**
	a)	Find the gradient of the curve $xy=6$ at pt $(1,6)$	**02**
	Ans	$xy=6$	
		$\therefore x\dfrac{dy}{dx}+y=0$	½
		$\therefore \dfrac{dy}{dx}=\dfrac{-y}{x}$	½
		at pt $(1,6)$	
		$\therefore \dfrac{dy}{dx}=\dfrac{-6}{1}=-6$	1
		OR	
		$xy=6$	
		$\therefore y=\dfrac{6}{x}$	
		$\therefore \dfrac{dy}{dx}=6.\left(\dfrac{-1}{x^2}\right)$	1
		at pt $(1,6)$	
		$\therefore \dfrac{dy}{dx}=6.\left(\dfrac{-1}{1^2}\right)=-6$	1
	b)	Divide 50 into two parts such that product is maximum.	**02**
	Ans	Let x and y be two parts of 50	
		$\therefore x+y=50$	
		$\therefore y=50-x$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	$P = xy$	½
		$\therefore P = x(50-x)$	
		$\therefore P = 50x - x^2$	½
		$\frac{dP}{dx} = 50 - 2x$	
		$\therefore \frac{d^2P}{dx^2} = -2 \quad \therefore$ Product is maximum.	½
		Let $\frac{dP}{dx} = 0$	
		$\therefore 50 - 2x = 0$	
		$\therefore x = 25$ and $y = 25$	½
			02
	c)	Evaluate: $\int \frac{x}{x+1}\, dx$	
	Ans	$\int \frac{x}{x+1}\, dx$	
		$= \int \frac{x+1-1}{x+1}\, dx$	
		$= \int \left(1 - \frac{1}{x+1}\right) dx$	1
		$= x - \log(x+1) + c$	1
			02
	d)	Evaluate: $\int \frac{\cos 2x}{\sin^2 x.\cos^2 x}\, dx$	
	Ans	$\int \frac{\cos 2x}{\sin^2 x.\cos^2 x}\, dx$	
		$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x.\cos^2 x}\, dx$	½
		$= \int \left(\frac{\cos^2 x}{\sin^2 x.\cos^2 x} - \frac{\sin^2 x}{\sin^2 x.\cos^2 x} \right) dx$	
		$= \int \operatorname{cosec}^2 x\, dx - \int \sec^2 x\, dx$	½
		$= -\cot x - \tan x + c$	1
	e)	Evaluate: $\int x.\sin x\, dx$	**02**
	Ans	$\int x.\sin x\, dx$	
		$= x\int \sin x\, dx - \int \left[\int \sin x\, dx \frac{d}{dx}(x) \right] dx$	½

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1.	e)	$= x(-\cos x)-\int(-\cos x).1\,dx$	½
		$=-x.\cos x+\int \cos x\,dx$	½
		$=-x.\cos x+\sin x+c$	½
			02
	f)	Evaluate $\int_2^4 \frac{1}{2x+3}dx$	
	Ans	$\int_2^4 \frac{1}{2x+3}dx$	
		$=\frac{1}{2}\left[\log(2x+3)\right]_2^4$	1
		$=\frac{1}{2}\left[\log(2(4)+3)-\log(2(2)+3)\right]$	½
		$=\frac{1}{2}\left[\log 11-\log 7\right]$ or $\frac{1}{2}\log\left(\frac{11}{7}\right)$	½
	g)	Find area between the line $y=2x$, x-axis and ordinates $x=0$ and $x=2$	**02**
	Ans	Area $A=\int_a^b y\,dx$	
		$=\int_0^2 2x\,dx$	
		$=2\left[\frac{x^2}{2}\right]_0^2$	1
		$=2\left[\frac{2^2}{2}-0\right]$	
		$=4$	1
	h)	Find order and degree of $\frac{d^2y}{dx^2}=\sqrt[3]{1+\frac{dy}{dx}}$	**02**
	Ans	$\frac{d^2y}{dx^2}=\sqrt[3]{1+\frac{dy}{dx}}$	
		$\therefore \frac{d^2y}{dx^2}=\left(1+\frac{dy}{dx}\right)^{\frac{1}{3}}$	
		Taking cube on both sides,	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	h)	$\therefore \left(\frac{d^2y}{dx^2}\right)^3 = \left(1+\frac{dy}{dx}\right)$	
		$\therefore Order = 2$	1
		$Degree = 3$	1
		--	
	i)	Form a differential eq^n of $y = ax^2 + b$	**02**
	Ans	$y = ax^2 + b$	
		$\frac{dy}{dx} = 2ax$	½
		$\frac{d^2y}{dx^2} = 2a$	½
		$\frac{d^2y}{dx^2} = \frac{1}{x}\frac{dy}{dx}$	½
		$\therefore \frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} = 0$	½
		OR	
		$y = ax^2 + b$	½
		$\therefore \frac{dy}{dx} = 2ax$	½
		$\therefore \frac{d^2y}{dx^2} = 2a$	
		$\therefore \frac{dy}{dx} = \frac{d^2y}{dx^2}x$	½
		$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$	½
		--	
	j)	A fair dice is rolled. What is the probability that no. appear on the dice is greater than 2.	**02**
	Ans	$S = \{1,2,3,4,5,6\}$	
		$\therefore n(s) = 6$	½
		$A = \{3,4,5,6\}$	
		$\therefore n(A) = 4$	½
		$P(A) = \frac{n(A)}{n(s)} = \frac{4}{6}$	
		$P(A) = \frac{2}{3}$ or 0.67	1
		--	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	k)	If two cards drawn from a pack of 52 cards. What is probability that both are king.	**02**
	Ans	$n(S)={}^{52}C_2$	½
		$n(A)={}^{4}C_2$	½
		$P(A)=\dfrac{n(A)}{n(S)}$	
		$P(A)=\dfrac{{}^{4}C_2}{{}^{52}C_2}=\dfrac{6}{1326}$	1
		$P(A)=\dfrac{1}{221}$ or 0.0045	
	l)	An unbiased coin is tossed 6 times. Find the probability of getting 2 Heads.	**02**
	Ans	Here $n=6,\ p=0.5,\ q=0.5,\ r=2$	
		$P(x=r)={}^{n}C_r\,p^r q^{n-r}$	
		$\therefore P(2)={}^{6}C_2\,(0.5)^2\,(0.5)^4$	1
		$\therefore P(2)=\dfrac{15}{64}$ or 0.2344	1
2.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Find eq^n of tgt and normal to the curve $y=x(2-x)$ at pt. $(2,0)$	**04**
	Ans	$y=x(2-x)$	
		$\therefore \dfrac{dy}{dx}=2-2x$	1
		at $(2,0)$	
		$\therefore \dfrac{dy}{dx}=2-2(2)=-2$	
		$\therefore$ slope $m=-2$	½
		Equation of tgt is	
		$(y-0)=-2(x-2)$	½
		$\therefore y=-2x+4$	
		$\therefore 2x+y-4=0$	½
		$\therefore$ slope of normal $=\dfrac{-1}{m}=\dfrac{1}{2}$	½
		Equation of normal is	
		$(y-0)=\dfrac{1}{2}(x-2)$	½
		$\therefore 2y=x-2$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	$\therefore x-2y-2=0$	½
		--	
	b)	Show that radius of curvature of the curve $x^2+y^2=25$ at $(3,4)$ is 5 units.	**04**
	Ans	$x^2+y^2=25$	
		$\therefore 2x+2y\frac{dy}{dx}=0$	½
		$\therefore \frac{dy}{dx}=-\frac{x}{y}$	
		$\frac{d^2y}{dx^2}=-\left(\frac{y(1)-x\frac{dy}{dx}}{y^2}\right)$	
		$\frac{d^2y}{dx^2}=-\left(\frac{y-x\left(-\frac{x}{y}\right)}{y^2}\right)$	½
		$\therefore$ at $(3,4)$	
		$\frac{dy}{dx}=-\frac{3}{4}$	½
		$\frac{d^2y}{dx^2}=-\left(\frac{4-3\left(-\frac{3}{4}\right)}{(4)^2}\right)=-\frac{25}{64}$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+\left(-\frac{3}{4}\right)^2\right]^{\frac{3}{2}}}{-\frac{25}{64}}$	1
		$\therefore \rho=-5$	1
		$\therefore$ Radius of curvature is 5	
		--	
	c)	Find maximum and minimum values of $y=x^3-9x^2+24x$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c) Ans	$y = x^3 - 9x^2 + 24x$	
		$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24$	½
		$\therefore \frac{d^2y}{dx^2} = 6x - 18$	½
		Consider $\frac{dy}{dx} = 0$	
		$3x^2 - 18x + 24 = 0$	
		$\therefore x = 2$ or $x = 4$	1
		at $x = 2$ $\quad \therefore \frac{d^2y}{dx^2} = 6(2) - 18 = -6 < 0$	½
		$\therefore y$ is maximum at $x = 2$	
		$y_{\max} = (2)^3 - 9(2)^2 + 24(2)$	½
		$= 20$	
		at $x = 4$ $\quad \therefore \frac{d^2y}{dx^2} = 6(4) - 18 = 6 > 0$	½
		$\therefore y$ is minimum at $x = 4$	
		$y_{\min} = (4)^3 - 9(4)^2 + 24(4)$	½
		$= 16$	
		---	**04**
	d)	Evaluate $\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx$	
	Ans	$\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx$	
		Put $xe^x = t$	
		$\therefore e^x(x+1)dx = dt$	1
		$= \int \frac{1}{\cos^2 t} dt$	½
		$= \int \sec^2 t\, dt$	1
		$= \tan t + c$	1
		$= \tan(xe^x) + c$	½
		---	**04**
	e)	Evaluate $\int \frac{3\tan^{-1} x}{1+x^2} dx$	
	Ans	$\int \frac{3\tan^{-1} x}{1+x^2} dx$	

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2.	e)	Put $\tan^{-1} x = t$	
		$\therefore \frac{1}{1+x^2}dx = dt$	1
		$= \int 3t\ dt$	1
		$= 3\left(\frac{t^2}{2}\right) + c$	1
		$= \frac{3}{2}\left(\tan^{-1} x\right)^2 + c$	1
	f)	Evaluate $\int \frac{1}{4\sin^2 x + 5\cos^2 x} dx$	**04**
	Ans	$\int \frac{1}{4\sin^2 x + 5\cos^2 x} dx$	
		$= \int \frac{\frac{1}{\cos^2 x}}{4\frac{\sin^2 x}{\cos^2 x} + 5\frac{\cos^2 x}{\cos^2 x}} dx$	½
		$= \int \frac{\sec^2 x}{4\tan^2 x + 5} dx$	½
		Put $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	1
		$\therefore \int \frac{1}{4t^2 + 5} dt$	½
		$= \frac{1}{4}\int \frac{1}{t^2 + \frac{5}{4}}\ dt$	
		$= \frac{1}{4}\int \frac{1}{t^2 + \left(\frac{\sqrt{5}}{2}\right)^2}\ dt$	
		$= \frac{1}{4} \times \frac{1}{\frac{\sqrt{5}}{2}} \tan^{-1}\left(\frac{t}{\frac{\sqrt{5}}{2}}\right) + c$	1
		$= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{2t}{\sqrt{5}}\right) + c$	
		$= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c$	½
		OR	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	f)	$\int \frac{1}{4\sin^2 x+5\cos^2 x}dx$	
		$=\int \frac{\frac{1}{\cos^2 x}}{4\frac{\sin^2 x}{\cos^2 x}+5\frac{\cos^2 x}{\cos^2 x}}dx$	½
		$=\int \frac{\sec^2 x}{4\tan^2 x+5}dx$	½
		Put $\tan x=t$	
		$\therefore \sec^2 xdx=dt$	1
		$\therefore \int \frac{1}{4t^2+5}dt$	½
		$=\int \frac{1}{(2\text{t})^2+\left(\sqrt{5}\right)}dt$	
		$=\frac{1}{\sqrt{5}}\frac{\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right)}{2}+c$	1
		$=\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$	½
3.		**Solve any FOUR of the following :**	**16**
	a)	Evaluate $\int_0^{\pi/2}\frac{1}{3+4\cos x}dx$	**04**
	Ans	$\int_0^{\pi/2}\frac{1}{3+4\cos x}dx$	
		Put $\tan\frac{x}{2}=t$, $dx=\frac{2dt}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$	1
		when $x\to 0$ to $\pi/2$ $t\to 0$ to 1	
		$\int_0^{\pi/2}\frac{1}{3+4\cos x}dx=\int_0^1\frac{\frac{2dt}{1+t^2}}{3+4\left(\frac{1-t^2}{1+t^2}\right)}$	
		$=\int_0^1\frac{2dt}{3\left(1+t^2\right)+4\left(1-t^2\right)}$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	$=2\int_0^1 \frac{dt}{7-t^2}=2\int_0^1 \frac{dt}{\left(\sqrt{7}\right)^2-t^2}$	½
		$=2\left[\frac{1}{2\sqrt{7}}\log\left(\frac{\sqrt{7}+t}{\sqrt{7}-t}\right)\right]_0^1$	1
		$=\frac{1}{\sqrt{7}}\left[\log\left(\frac{\sqrt{7}+1}{\sqrt{7}-1}\right)-\log\left(\frac{\sqrt{7}}{\sqrt{7}}\right)\right]$	1
		$=\frac{1}{\sqrt{7}}\log\left(\frac{\sqrt{7}+1}{\sqrt{7}-1}\right)$	
		---	**04**
	b)	Evaluate $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$	
	Ans	$\int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$	
		$I=\int_0^{\pi/2} \frac{1}{1+\sqrt{\frac{\cos x}{\sin x}}}\ dx$	½
		$=\int_0^{\pi/2} \frac{1}{1+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}}\ dx$	
		$\therefore I=\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\ dx$ --------------------(1)	½
		$I=\int_0^{\pi/2} \frac{\sqrt{\sin\left(\pi/2-x\right)}}{\sqrt{\sin\left(\pi/2-x\right)}+\sqrt{\cos\left(\pi/2-x\right)}}\ dx$	1
		$\therefore I=\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\ dx$ --------------------(2)	½
		add (1) and (2)	
		$I+I=\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\ dx+\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\ dx$	
		$2I=\int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\ dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$2I = \int_0^{\pi/2} 1\,dx$	
		$2I = [x]_0^{\pi/2}$	½
		$2I = \frac{\pi}{2} - 0$	
		$I = \frac{\pi}{4}$	½
	c)	Find area enclosed by parabolas $y^2 = 9x$ and $x^2 = 9y$	**04**
	Ans	$y^2 = 9x$ ---------(1)	
		$x^2 = 9y$	
		$\therefore y = \frac{x^2}{9}$	
		$\therefore \text{eq}^{\text{n}}.(1) \Rightarrow \left(\frac{x^2}{9}\right)^2 = 9x$	
		$\frac{x^4}{81} = 9x$	
		$\therefore x^4 = 729x$	
		$\therefore x^4 - 729x = 0$	
		$\therefore x\left(x^3 - 9^3\right) = 0$	
		$\therefore x = 0, 9$	1
		Area $A = \int_a^b (y_1 - y_2)\,dx$	
		$\therefore A = \int_0^9 \left(3\sqrt{x} - \frac{x^2}{9}\right)dx$	1
		$\therefore A = \left(\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{27}\right)_0^9$	1
		$\therefore A = \left(\frac{3(9)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(9)^3}{27}\right) - 0$	
		$\therefore A = 27$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	Solve : $y^2dx-\left(xy-x^2\right)dy=0$	**04**
	Ans	$y^2dx-\left(xy-x^2\right)dy=0$	
		$\frac{dy}{dx}=\frac{y^2}{xy-x^2}$	
		Put $y=vx \quad \therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$	1
		$v+x\frac{dv}{dx}=\frac{(vx)^2}{vx^2-x^2}$	
		$v+x\frac{dv}{dx}=\frac{x^2\left(v^2\right)}{x^2(v-1)}$	
		$v+x\frac{dv}{dx}=\frac{v^2}{v-1}$	1
		$x\frac{dv}{dx}=\frac{v^2}{v-1}-v$	
		$x\frac{dv}{dx}=\frac{v}{v-1}$	½
		$\left(\frac{v-1}{v}\right)dv=\frac{1}{x}dx$	
		$\therefore$ solution is	
		$\int\left(1-\frac{1}{v}\right)dv=\int\frac{1}{x}dx$	
		$v-\log v=\log x+c$	1
		$\frac{y}{x}-\log\left(\frac{y}{x}\right)=\log x+c$	½
		--	
	e)	Solve : $\frac{dy}{dx}=\left(\frac{y}{x}\right)+\tan\left(\frac{y}{x}\right)$	**04**
	Ans	$\frac{dy}{dx}=\left(\frac{y}{x}\right)+\tan\left(\frac{y}{x}\right)$	
		$\therefore \frac{dy}{dx}-\left(\frac{y}{x}\right)=\tan\left(\frac{y}{x}\right)$	
		Put $\frac{y}{x}=v$	
		$\Rightarrow y=vx$	
		$\therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	e)	$\therefore v+x\dfrac{dv}{dx}-v=\tan v$	
		$\therefore x\dfrac{dv}{dx}=\tan v$	1
		$\therefore \dfrac{dv}{\tan v}=\dfrac{dx}{x}$	
		$\therefore$ solution is	
		$\therefore \int\dfrac{dv}{\tan v}=\int\dfrac{dx}{x}$	½
		$\therefore \int\cot v\ dv=\int\dfrac{dx}{x}$	1
		$\therefore \log(\sin v)=\log x+c$	
		$\therefore \log\left(\sin\left(\dfrac{y}{x}\right)\right)=\log x+c$	½
		---	
	f)	Solve: $\dfrac{dy}{dx}+2\tan x.y=2\sin x$	**04**
	Ans	$\dfrac{dy}{dx}+2\tan x.y=2\sin x$	
		Comparing with $\dfrac{dy}{dx}+Py=Q$	
		$\therefore P=2\tan x,\ Q=2\sin x$	
		$\therefore I.F.=e^{\int Pdx}=e^{\int 2\tan x\ dx}$	
		$\therefore I.F.=e^{2\log(\sec x)}=e^{\log(\sec x)^2}$	
		$\therefore I.F.=\sec^2 x$	1
		$\therefore$ Solution is	
		$y.I.F.=\int QI.F.dx+c$	1
		$y.\sec^2 x=\int 2\sin x.\sec^2 x\ dx+c$	
		$y.\sec^2 x=2\int \sin x.\dfrac{1}{\cos^2 x}\ dx+c$	
		$y.\sec^2 x=2\int \sec x.\tan x\ dx+c$	1
		$y.\sec^2 x=2\sec x+c$	1

4.		**Solve any FOUR of the following :**	**16**
	a)	Evaluate: $\int_2^5\dfrac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\ dx$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a) Ans	$I=\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$ --------------------(1)	
		$I=\int_2^5 \frac{\sqrt{7-(2+5-x)}}{\sqrt{(2+5-x)}+\sqrt{7-(2+5-x)}}\,dx$	1
		$\therefore I=\int_2^5 \frac{\sqrt{7-7+x}}{\sqrt{7-x}+\sqrt{7-7+x}}\,dx$	
		$\therefore I=\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$ --------------------(2)	½
		add (1) and (2)	
		$I+I=\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx+\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$	
		$\therefore 2I=\int_2^5 \frac{\sqrt{7-x}+\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}\,dx$	1
		$\therefore 2I=\int_2^5 1\,dx$	½
		$\therefore 2I=[x]_2^5$	½
		$\therefore 2I=5-2$	
		$\therefore 2I=3$	
		$I=\frac{3}{2}$	½
		--	
	b)	Evaluate $\int_5^{10} \frac{1}{(x-1)(x-2)}\,dx$	**04**
	Ans	$\int_5^{10} \frac{1}{(x-1)(x-2)}\,dx$	
		Consider $\frac{1}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2}$	
		$\therefore\ 1=A(x-2)+B(x-1)$	
		Put $x=1$ $\therefore A=-1$	½
		Put $x=2$ $\therefore B=1$	½
		$\frac{1}{(x-1)(x-2)}=\frac{-1}{x-1}+\frac{1}{x-2}=-\frac{1}{x-1}+\frac{1}{x-2}$	
		$\therefore\int_5^{10} \frac{1}{(x-1)(x-2)}\,dx$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$=\int_5^{10}\left(\frac{1}{x-2}-\frac{1}{x-1}\right)dx$	½
		$=\int_5^{10}\frac{1}{x-2}dx-\int_5^{10}\frac{1}{x-1}dx$	
		$=\left[\log(x-2)-\log(x-1)\right]_5^{10}$	1
		$=\left[\log\left(\frac{x-2}{x-1}\right)\right]_5^{10}$	
		$=\log\left(\frac{8}{9}\right)-\log\left(\frac{3}{4}\right)$	1
		$=\log\left(\frac{32}{27}\right)$	½
		OR	
		$\int_5^{10}\frac{1}{(x-1)(x-2)}dx$	
		$=\int_5^{10}\frac{1}{x^2-3x+2}dx$	
		$\therefore \text{Third Term}=\frac{(-3)^2}{4}=\frac{9}{4}$	½
		$=\int_5^{10}\frac{1}{x^2-3x+\frac{9}{4}-\frac{9}{4}+2}dx$	½
		$=\int_5^{10}\frac{1}{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$	½
		$=\frac{1}{2\left(\frac{1}{2}\right)}\left[\log\left(\frac{x-\frac{3}{2}-\frac{1}{2}}{x-\frac{3}{2}+\frac{1}{2}}\right)\right]_5^{10}$	1
		$=\left[\log\left(\frac{x-2}{x-1}\right)\right]_5^{10}$	
		$=\log\left(\frac{8}{9}\right)-\log\left(\frac{3}{4}\right)$	1
		$=\log\left(\frac{32}{27}\right)$	½
		--	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	Using integration find area of the loop $y^2 = x^2(1-x)$	**04**
	Ans	$y^2 = x^2(1-x)$	
		$\because y = 0$	
		$\therefore x^2(1-x) = 0$	1
		$\therefore x = 0,1$	
		Area $A = \int_a^b y\,dx$	½
		$\therefore A = \int_0^1 x\sqrt{1-x}\,dx$	
		$\therefore A = \int_0^1 (1-x)\sqrt{x}\,dx$	
		$\therefore A = \int_0^1 (1-x)x^{\frac{1}{2}}\,dx$	
		$\therefore A = \int_0^1 \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx$	½
		$\therefore A = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1$	1
		$\therefore A = \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$ or 0.2667	½
		$\therefore$ Area of loop $= 2\times A = \frac{8}{15} = 0.533$	½
		OR	
		$y^2 = x^2(1-x)$	
		$\because y = 0$	
		$\therefore x^2(1-x) = 0 \qquad \therefore x = 0,1$	1
		$y = x\sqrt{1-x}$	
		Area $A = \int_a^b y\,dx$	
		$\therefore A = \int_0^1 x\sqrt{1-x}\,dx$	½
		Put $1-x = t \Rightarrow 1-t = x$ [when $x \to 0$ to 1, $t \to 1$ to 0]	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	$\therefore -dx = dt$	
		$\therefore dx = -dt$	½
		$\therefore A = \int_1^0 (1-t)\sqrt{t}\ (-dt)$	
		$\therefore A = -\int_1^0 (1-t)t^{\frac{1}{2}}\ dt$	½
		$\therefore A = -\int_1^0 \left(t^{\frac{1}{2}} - t^{\frac{3}{2}}\right)dt$	
		$\therefore A = -\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}}\right]_1^0$	½
		$\therefore A = 0 + \left(\frac{2}{3} - \frac{2}{5}\right) = \frac{4}{15}$ or 0.2667	
		$\therefore$ Area of loop $= 2\times A = \frac{8}{15} = 0.533$	½
	d)	Solve $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$	**04**
	Ans	$\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$	
		$\frac{dy}{dx} = e^{-2y}\left(e^{3x} + x^2\right)$	1
		$\int e^{2y}dy = \int \left(e^{3x} + x^2\right)dx$	1
		$\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$	2
	e)	Solve : $\left(3x^2 + 6xy^2\right)dx + \left(6x^2y + 4y^2\right)dy$	**04**
	Ans	$\left(3x^2 + 6xy^2\right)dx + \left(6x^2y + 4y^2\right)dy = 0$	
		Comparing with $Mdx + Ndy = 0$	
		$\therefore M = 3x^2 + 6xy^2, N = 6x^2y + 4y^2$	
		$\frac{\partial M}{\partial y} = 12xy$, $\frac{\partial N}{\partial x} = 12xy$	1+1
		$\therefore$ D.E. is an exact	
		$\therefore$ Solution is,	
		$\int_{y-constant} Mdx + \int_{terms\ free\ from\ 'x'} Ndy = c$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	$\therefore \int_{y-\text{constant}} \left(3x^2+6xy^2\right)dx+\int 4y^2 dy = c$	1
		$\therefore x^3+3x^2y^2+\frac{4}{3}y^3=c$	1
		--	
	f)	Verify that $y.\sec^2 x=\sec x+c$ is solution of differential eq^n $\frac{dy}{dx}+2y\tan x=\sin x$.	**04**
		Also find its particular sol^n when $x=0$ and $y=0$	
	Ans	$y.\sec^2 x=\sec x+c$	
		$\therefore y\left(2\sec x.\sec x.\tan x\right)+\sec^2 x\frac{dy}{dx}=\sec x.\tan x+0$	1
		$y\left(2\sec x.\tan x\right)+\sec x\frac{dy}{dx}=\tan x$	
		$\therefore \sec x\frac{dy}{dx}=\tan x\left(1-2y\sec x\right)$	½
		$\therefore \frac{dy}{dx}=\sin x\left(1-2y\sec x\right)$	
		$\therefore \frac{dy}{dx}+2y\tan x=\sin x$	1
		when $x=0$ and $\text{y}=0$	
		$y.\sec^2 x=\sec x+c \quad \Rightarrow \quad 0=1+c \quad \Rightarrow c=-1$	1
		$y.\sec^2 x=\sec x-1$	½
		$\therefore y=\cos x-\cos^2 x$	
		OR	
		$\frac{dy}{dx}+2y\tan x=\sin x$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=2\tan x,\ Q=2\sin x$	
		$\therefore I.F.=e^{\int Pdx}=e^{\int 2\tan x\ dx}$	
		$=e^{2\log(\sec x)}=e^{\log(\sec x)^2}=\sec^2 x$	1
		$\therefore$ Solution is , $y.I.F.=\int QI.F.dx+c$	
		$y.\sec^2 x=\int \sin x.\sec^2 x\ dx+c$	½
		$y.\sec^2 x=\int \sin x.\frac{1}{\cos^2 x}\ dx+c$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	$y.\sec^2 x = \int \sec x.\tan x\, dx + c$	½
		$y.\sec^2 x = \sec x + c$	½
		when $x = 0$ and $y = 0$	
		$y.\sec^2 x = \sec x + c \quad \Rightarrow \quad 0 = 1 + c \quad \Rightarrow c = -1$	1
		$y.\sec^2 x = \sec x - 1$	
		$\therefore y = \cos x - \cos^2 x$	½
		--	
5.		**Solve any <u>FOUR</u> of the following :**	**16**
	a)	The probability of solving problem by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. What is probability that problem is solved.	**04**
	Ans	Given $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$	
		The problem is solved is given by ,	
		$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
		since A and B are Independent,	
		$\because P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$	
		$\therefore P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{2} \times \frac{1}{3}$	2
		$\therefore P(A \cup B) = \frac{2}{3}$ or 0.667	2
		<u>OR</u>	
		Given $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$	
		$\therefore P(A') = 1 - \frac{1}{2} = \frac{1}{2}$	½
		$P(B') = 1 - \frac{1}{3} = \frac{2}{3}$	½
		The problem is not solved is given by ,	
		$\because P(A' \cap B') = P(A') \times P(B')$, since A and B are Independent	
		$= \frac{1}{2} \times \frac{2}{3}$	
		$= \frac{1}{3}$	1
		The problem is solved is given by ,	
		$= 1 - P(A' \cap B')$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	a)	$=1-\frac{1}{3}$	½
		$=\frac{2}{3}$ or 0.667	1
		--	
	b)	The probability that a machine manufactured by a company will be defective $\frac{1}{10}$. If 5 such machines are manufactured find probability that (i) Exactly two will be defective. (ii) At least two will be defective.	**04**
	Ans	Given $p=\frac{1}{10}=0.1,\ \therefore q=1-p=\frac{9}{10}=0.9$, $n=5$	1
		Binomial distribution is , $P(x=r)={}^{n}C_{r}(p)^{r}(q)^{n-r}$	
		(i) Exactly two will be defective	
		$\therefore p(2)={}^{5}C_{2}(0.1)^{2}(0.9)^{5-2}$	½
		$=0.0729$	1
		(ii) At Least two will be defective	
		i.e $r=2$ *or* 3 *or* 4 *or* 5	
		$\therefore p(r=2\ or\ 3\ or\ 4\ or\ 5)=1-p(r=0\ or\ 1)$	
		$=1-(p(0)+p(1))$	½
		$=1-\left[{}^{5}C_{0}(0.1)^{0}(0.9)^{5-0}+{}^{5}C_{1}(0.1)^{1}(0.9)^{4}\right]$	
		$=0.08146$	1
		--	
	c)	Fit a Poisson distribution's for the following observations x_i: 0, 1, 2, 3, 4 f_i: 21, 18, 7, 3, 1	**04**
	Ans	Mean $=m=\frac{\sum f_i x_i}{\sum f_i}$	
		$\therefore m=\frac{0(21)+1(18)+2(7)+3(3)+4(1)}{21+18+7+3+1}$	1
		$\therefore m=\frac{45}{50}=\frac{9}{10}=0.9$	1
		Poisson distribution is ,	

Table in 5 c):

x_i	0	1	2	3	4
f_i	21	18	7	3	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$P(x=r)=\frac{e^{-m}m^r}{r!}$ $\therefore P(r)=\frac{e^{-0.9}(0.9)^r}{r!}$	2
	d)	Evaluate $\int x^2\tan^{-1}x\,dx$	**04**
	Ans	$\int x^2\tan^{-1}x\,dx$	
		$=\tan^{-1}x\int x^2dx-\int\left(\int x^2dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx$	1
		$=\tan^{-1}x\cdot\frac{x^3}{3}-\int\frac{x^3}{3}\cdot\frac{1}{1+x^2}dx$ $=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\int\frac{x^3}{1+x^2}dx$	1
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\int\left[x-\frac{x}{1+x^2}\right]dx$	1
		$=\frac{x^3\tan^{-1}x}{3}-\frac{1}{3}\left[\frac{x^2}{2}-\frac{1}{2}\log\left(1+x^2\right)\right]+c$	1
	e)	Evaluate: $\int_0^{\frac{\pi}{2}}\sin5x\cos3x\,dx$	**04**
	Ans	$\int_0^{\frac{\pi}{2}}\sin5x\cos3x\,dx$ $=\frac{1}{2}\int_0^{\frac{\pi}{2}}2\sin5x\cos3x\,dx$	
		$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\sin(5x+3x)+\sin(5x-3x)\right)dx$	1
		$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\sin8x+\sin2x\right)dx$	
		$=\frac{1}{2}\left[\frac{-\cos8x}{8}-\frac{\cos2x}{2}\right]_0^{\frac{\pi}{2}}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	e)	$=-\frac{1}{2}\left[\frac{\cos 8x}{8}+\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$	
		$=-\frac{1}{2}\left[\frac{\cos 8\left(\frac{\pi}{2}\right)}{8}+\frac{\cos 2\left(\frac{\pi}{2}\right)}{2}-\frac{\cos 0}{8}-\frac{\cos 0}{2}\right]$	1
		$=-\frac{1}{2}\left[\frac{\cos 4\pi}{8}+\frac{\cos \pi}{2}-\frac{\cos 0}{8}-\frac{\cos 0}{2}\right]$	
		$=-\frac{1}{2}\left[\frac{1}{8}+\frac{(-1)}{2}-\frac{1}{8}-\frac{1}{2}\right]=\frac{1}{2}$	1
		--	
	f)	Solve : $(x+1)\frac{dy}{dx}-y=e^x(x+1)^2$	**04**
	Ans	$(x+1)\frac{dy}{dx}-y=e^x(x+1)^2$	
		Divide by $(x+1)$	
		$\frac{dy}{dx}-\frac{1}{(x+1)}y=e^x(x+1)$	½
		$\therefore$ Comparing with $\frac{dy}{dx}+Py=Q$	
		$P=\frac{-1}{x+1}$, $Q=e^x(x+1)$	
		Integrating factor $IF=e^{\int\frac{-1}{x+1}dx}=e^{-\log x+1}=\frac{1}{x+1}$	1
		$y.IF=\int Q.IFdx$	
		$y\frac{1}{x+1}=\int e^x(x+1).\frac{1}{x+1}\,dx$	½
		$\frac{y}{x+1}=\int e^x dx$	1
		$\frac{y}{x+1}=e^x+c$	1
		--	
6.		**Solve any FOUR of the following:**	**16**
	a)	If $P(A)=\frac{1}{2},P(B')=\frac{2}{3},$ find $P(A'\cap B')$ and $P(A/B)$	**04**

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a) Ans	$P(A)=\frac{1}{2}, P(B')=\frac{2}{3}$ and $P(A\cap B)=\frac{1}{4}$	
		$P(B)=1-P(B')=1-\frac{2}{3}=\frac{1}{3}$	½
		$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
		$\therefore P(A\cup B)=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$	
		$\therefore P(A\cup B)=\frac{7}{12}$	1
		$i)\ P(A'\cap B')=P(A\cup B)'$	
		$P(A'\cap B')=1-P(A\cup B)$	
		$\therefore P(A'\cap B')=1-\frac{7}{12}$	½
		$\therefore P(A'\cap B')=\frac{5}{12}$	½
		$ii) P(A/B)=\frac{P(A\cap B)}{P(B)}$	
		$\therefore P(A/B)=\frac{\frac{1}{4}}{\frac{1}{3}}$	1
		$\therefore P(A/B)=\frac{3}{4}$	½
		Note: "If Students attempted to solve the problem by assuming appropriate value. Consider and Give appropriate marks."	
		--	
	b)	If 5% of electric bulbs manufactured by a company are defective. Use Poisson's distribution to find probability that in a sample of 100 bulbs. (i) None is defective (ii) five are defective	**04**
	Ans	$p=5\%=0.05$, $n=100$	
		$\therefore$ mean $m=np$	
		$\therefore m=100\times 0.05=5$	1
		Poisson's distribution is,	
		$P(r)=\frac{e^{-m}m^{r}}{r!}$	
		(i) None is defective $\quad \therefore r=0$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	$\therefore P(0)=\dfrac{e^{-5}(5)^{0}}{0!}$	1
		$\therefore P(0)=0.0067$	½
		(i) five are defective $\therefore r=5$	
		$\therefore P(5)=\dfrac{e^{-5}(5)^{5}}{5!}$	1
		$\therefore P(0)=0.1755$	½
		--	
	c)	In a certain examination 500 students appeared. Mean score is 68 and standard deviation is 8. Assuming data normally distributed. Find no of students scoring	**04**
		(i) less than 50 and	
		(ii) more than 60.	
		Where $A(2.25)=0.4878$	
		$A(1)=0.3413$	
	Ans	Given $\bar{x}=68$ $\sigma=8$ $N=500$	
		$i)$ For $x=50$	
		$z=\dfrac{x-\bar{x}}{\sigma}=\dfrac{50-68}{8}=-2.25$	½
		$\therefore p(\text{less than }50)=A(\text{less than }-2.25)$	
		$=0.5-A(z=0\text{ to }z=2.25)$	
		$=0.5-0.4878$	
		$=0.0122$	1
		$\therefore$ No.of students $=N\cdot p$	
		$=500\times0.0122=6.1$ *i.e.*, 6	½
		$ii)$ For $x=60$	
		$z=\dfrac{x-\bar{x}}{\sigma}=\dfrac{60-68}{8}=-1$	½
		$\therefore p(\text{more than }60)=A(\text{more than }-1)$	
		$=A(z=0\text{ to }z=1)+0.5$	
		$=0.3413+0.5$	
		$=0.8413$	1
		$\therefore$ No.of students $=N\cdot p=500\times0.8413$	½
		$=420.65$ *i.e.*, 421	
		--	
	d)	A bullet is fired into a mud bank and penetrates $(120t-3600t^{2})$ meters in t seconds after	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	d)	impact.Calculate maximum depth of penetration.	**04**
	Ans	Let $s=120t-3600t^2$	
		$\therefore \frac{ds}{dt}=120-7200t$	1
		$\therefore \frac{d^2s}{dt^2}=-7200$	1
		Consider $\frac{ds}{dt}=0$	
		$\therefore 120-7200t=0$	
		$\therefore 120=7200t$	
		$\therefore t=\frac{1}{60}$	½
		at $t=\frac{1}{60}$	
		$\therefore \frac{d^2s}{dt^2}=-7200<0$	½
		$\therefore$ The maximum depth of penetration is,	
		$s=120\left(\frac{1}{60}\right)-3600\left(\frac{1}{60}\right)^2$	½
		$\therefore s=1$ meter	½
		--	
	e)	Find the equation of tangent to the curve $y=9x^2-12x+7$ which is parallel to $x-$axis.	**04**
	Ans	$y=9x^2-12x+7$	
		$\therefore \frac{dy}{dx}=18x-12$	1
		Slope of tangent which is parallel to $x-axis$ is $m=0$	
		$\therefore 18x-12=0$	
		$\therefore x=\frac{12}{18}=\frac{2}{3}$	
		$\therefore y=9\left(\frac{2}{3}\right)^2-12\left(\frac{2}{3}\right)+7=3$	
		$\therefore (x,y)=\left(\frac{2}{3},3\right)$	1
		at $\left(\frac{2}{3},3\right)$	
		Equation of tangent is,	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	e)	$\therefore y - y_1 = m(x - x_1)$	
		$y - 3 = 0\left(x - \frac{2}{3}\right)$	1
		$y = 3$	1
		--	
	f)	Find area bounded by parabola $y = 4x - x^2$ and $x-$axis.	**04**
	Ans	$y = 0$	
		$\therefore 4x - x^2 = 0$	
		$\therefore x = 0, 4$	1
		Area $A = \int_a^b y\,dx$	
		$\therefore A = \int_0^4 (4x - x^2)\,dx$	½
		$\therefore A = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4$	1
		$\therefore A = \left[\frac{4(4)^2}{2} - \frac{(4)^3}{3}\right]_0^4$	½
		$\therefore A = 32 - \frac{64}{3}$	
		$\therefore A = \frac{32}{3}$ or 10.667	1
		--	
		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
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		--	