SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any TEN of the following:**	**20**
	a)	Find the gradient of the curve $y=\sqrt{x^3}$ at $x=4$	**02**
	Ans	$y=\sqrt{x^3}$	
		$\therefore \frac{dy}{dx}=\frac{1}{2\sqrt{x^3}}.3x^2$	1
		at $x=4$	
		$\therefore \frac{dy}{dx}=\frac{1}{2\sqrt{(4)^3}}.3(4)^2=3$	1
		OR	
		$y=\sqrt{x^3}$	
		$\therefore y=x^{3/2}$	
		$\therefore \frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}$	1
		at $x=4$	
		$\therefore \frac{dy}{dx}=\frac{3}{2}(4)^{\frac{1}{2}}=3$	1
		---	
	b)	Divide 100 into two parts such that their product is maximum.	**02**
	Ans	Let x and y be two parts of 100	
		$\therefore x+y=100$	
		$\therefore y=100-x$	
		Product $P=xy$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	$\therefore P = x(100-x)$	
		$\therefore P = 100x - x^2$	½
		$\frac{dP}{dx} = 100 - 2x$	½
		$\therefore \frac{d^2P}{dx^2} = -2 \quad \therefore$ Product is maximum.	½
		Let $\frac{dP}{dx} = 0$	
		$\therefore 100 - 2x = 0$	
		$\therefore x = 50$ and $y = 50$	½
		--	
	c)	Evaluate: $\int \frac{x^2}{4+x^2}\ dx$	**02**
	Ans	$\int \frac{x^2}{4+x^2}\ dx$	
		$= \int \frac{4+x^2-4}{4+x^2}\ dx$	
		$= \int \left(1 - \frac{4}{4+x^2}\right)\ dx$	1
		$= \int \left(1 - \frac{4}{2^2+x^2}\right)\ dx$	
		$= x - \frac{4}{2}\tan^{-1}\frac{x}{2} + c$	½+½
		$= x - 2\tan^{-1}\frac{x}{2} + c$	
	d)	Evaluate: $\int \log x\ dx$	**02**
	Ans	$\int \log x\ dx$	
		$= \int \log x.1\ dx$	½
		$= \log x \int 1dx - \int \left(\int 1dx. \frac{d(\log x)}{dx} \right) dx$	½
		$= x.\log x - \int x.\frac{1}{x} dx$	½
		$= x.\log x - \int 1dx$	
		$= x.\log x - x + c$	½

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	e)	Evaluate: $\int \tan^3 xdx$	**02**
	Ans	Let $\int \tan^3 xdx$	
		$=\int \tan^2 x.\tan xdx$	
		$=\int \left(\sec^2 x-1\right).\tan xdx$	½
		$=\int \left(\sec^2 x.\tan x-\tan x\right)dx$	
		$=\int \sec^2 x.\tan xdx-\int \tan xdx$	
		$=\int \sec^2 x.\tan xdx-\log\left(\sec x\right)+c$	½
		Put $\tan x=t$	
		$\therefore \sec^2 xdx=dt$	½
		$=\int tdt-\log\left(\sec x\right)+c$	
		$=\dfrac{t^2}{2}-\log\left(\sec x\right)+c$	½
		$=\dfrac{\tan^2 x}{2}-\log\left(\sec x\right)+c$	
		--	
	f)	Evaluate: $\int \dfrac{dx}{(x+1)(x+2)}$	**02**
	Ans	$\dfrac{1}{(x+1)(x+2)}=\dfrac{A}{x+1}+\dfrac{B}{x+2}$	
		$\therefore 1=A(x+2)+B(x+1)$	
		Put $x=-1$	
		$\therefore A=1$	½
		Put $x=-2$	
		$\therefore B=-1$	½
		$\therefore \dfrac{1}{(x+1)(x+2)}=\dfrac{1}{x+1}+\dfrac{-1}{x+2}$	
		$\therefore \int \dfrac{1}{(x+1)(x+2)}dx=\int \dfrac{1}{x+1}+\dfrac{-1}{x+2}dx$	
		$=\log(x+1)-\log(x+2)+c$	½+½
		$=\dfrac{\log(x+1)}{\log(x+2)}+c$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	*OR*	
		$\int \frac{dx}{(x+1)(x+2)} = \int \frac{dx}{x^2+3x+2}$	
		$T.T. = \frac{9}{4}$	
		$\therefore \int \frac{dx}{x^2+3x+\frac{9}{4}-\frac{9}{4}+2}$	½
		$= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2-\frac{1}{4}}$	
		$= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2-\frac{1}{2^2}}$	½
		$= \frac{1}{2\times\frac{1}{2}} \log\left\|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right\|+c$	1
		$= \log\left\|\frac{x+1}{x+2}\right\|+c$	
	g)	Evaluate: $\int \frac{\sin x}{\sin 2x} dx$	**02**
	Ans	$\int \frac{\sin x}{\sin 2x} dx$	
		$= \int \frac{\sin x}{2\sin x \cos x} dx$	½
		$= \frac{1}{2}\int \frac{1}{\cos x} dx$	½
		$= \frac{1}{2}\int \sec x dx$	½
		$= \frac{1}{2}\log(\sec x+\tan x)+c$	½
	h)	If $\int_0^a 3x^2 dx = 8$ find the value of 'a'.	**02**
	Ans	$\int_0^a 3x^2 dx = 8$	

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	h)	$\left[3\frac{x^3}{3}\right]_0^a = 8$	1
		$\left[x^3\right]_0^a = 8$	
		$\therefore a^3 - 0 = 8$	½
		$\therefore a^3 = 8$	
		$\therefore a = 2$	½
		--	
	i)	Find the area bounded $y = x^2 - 9$, $x = 0$ to $x = 3$ and the X-axis	**02**
	Ans	Area $A = \int_a^b y dx$	
		$= \int_0^3 \left(x^2 - 9\right) dx$	½
		$= \left[\frac{x^3}{3} - 9x\right]_0^3$	½
		$= \left[\frac{3^3}{3} - 9(3)\right] - \left[\frac{0^3}{3} - 9(0)\right]$	½
		$= -18$ i.e. 18	½
		--	
	j)	Find order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} = \sqrt{\left(y + \frac{dy}{dx}\right)}$	**02**
	Ans	$\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} = \sqrt{\left(y + \frac{dy}{dx}\right)}$	
		$\therefore \left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} = \left(y + \frac{dy}{dx}\right)^{\frac{1}{2}}$	
		Taking 6^{th} power on both sides,	
		$\left(\frac{d^2y}{dx^2}\right)^4 = \left(y + \frac{dy}{dx}\right)^3$	
		$\therefore Order = 2$	1
		$Degree = 4$	1
		--	

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	(k)	Form a D.E. if $y = a\cos(x+b)$	**02**
	Ans	$y = a\cos(x+b)$	
		$\frac{dy}{dx} = -a\sin(x+b)$	½
		$\frac{d^2y}{dx^2} = -a\cos(x+b)$	½
		$\frac{d^2y}{dx^2} = -y$	1
		$\therefore \frac{d^2y}{dx^2} + y = 0$	
	l)	Verify that $y = 4\sin 3x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 9y = 0$	**02**
	Ans	$y = 4\sin 3x$	
		$\therefore \frac{dy}{dx} = 12\cos 3x$	½
		$\therefore \frac{d^2y}{dx^2} = -36\sin 3x$	½
		$\frac{d^2y}{dx^2} = -9(4\sin 3x)$	½
		$\frac{d^2y}{dx^2} = -9y$	
		$\therefore \frac{d^2y}{dx^2} + 9y = 0$	½
	m) Ans	Find the probability of occurrence of the digit 3 when an unbiased dice is thrown.	**02**
		$S=\{1,2,3,4,5,6\}$	
		$\therefore n(S)=6$	½
		$A=\{3\}$	
		$\therefore n(A)=1$	½
		$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6} = 0.1667$	1
	n)	A coin is tossed 3 times.What is the probability that appears an odd number of times?	**02**
		$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$	

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	n) Ans	$\therefore n(S)=8$	½
		A: head appears odd number of times	
		$A=\{HHH,,TTH,THT,HTT\} \quad \therefore n(A)=4$	½
		$\therefore p(A)=\frac{n(A)}{n(S)}=\frac{4}{8}=0.5$	1
		OR	
		$\therefore n(S)=8$	½
		A: tail appears odd number of times	
		$A=\{HHT,HTH,THH,TTT\} \quad \therefore n(A)=4$	½
		$\therefore p(A)=\frac{n(A)}{n(S)}=\frac{4}{8}=0.5$	1
		(Note: If student has considered either head or tail and attempted to solve give appropriate marks)	
		--	
2.		**Attempt any FOUR of the following:**	**16**
	a)	Determine a and b such that slope of curve $2y^3=ax^2+b$ at $(1,-1)$ is same as the slope of $x+y=0$	**04**
	Ans	$2y^3=ax^2+b$	
		$\therefore 6y^2\frac{dy}{dx}=2ax$	
		$\therefore \frac{dy}{dx}=\frac{2ax}{6y^2}=\frac{ax}{3y^2}$	1
		at $(1,-1)$	
		$\therefore \frac{dy}{dx}=\frac{a(1)}{3(-1)^2}=\frac{a}{3}$	1
		$\because x+y=0$	
		$\therefore 1+\frac{dy}{dx}=0$	
		$\therefore$ Slope is $\frac{dy}{dx}=-1$	1
		$\because$ slopes are equal.	
		$\therefore -1=\frac{a}{3}$	
		$\therefore a=-3$ and $b=1$	½+½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	b)	Find the maximum and minimum value of the function $y=2x^3-21x^2+36x-20$	**04**
	Ans	Let $y=2x^3-21x^2+36x-20$	
		$\therefore \frac{dy}{dx}=6x^2-42x+36$	½
		$\therefore \frac{d^2y}{dx^2}=12x-42$	½
		Consider $\frac{dy}{dx}=0$	
		$6x^2-42x+36=0$	½
		$\therefore x=6$ or $x=1$	½
		at $x=6$	
		$\frac{d^2y}{dx^2}=12(6)-42=30>0$	½
		$\therefore y$ is minimum at $x=6$	
		$y_{\min}=2(6)^3-21(6)^2+36(6)-20$	
		$=-128$	½
		at $x=1$	
		$\frac{d^2y}{dx^2}=12(1)-42=-30<0$	½
		$\therefore y$ is maximum at $x=1$	
		$y_{\max}=2(1)^3-21(1)^2+36(1)-20$	
		$=-3$	½
		--	
	c)	Find radius of curvature of $y=\log(\sin x)$ at $x=\pi/2$	**04**
	Ans	$y=\log(\sin x)$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sin x}.\cos x=\cot x$	½
		$\therefore \frac{d^2y}{dx^2}=-\cos ec^2 x$	½
		at $x=\pi/2$	
		$\frac{dy}{dx}=\cot\frac{\pi}{2}=0$	½
		$\frac{d^2y}{dx^2}=-\cos ec^2\frac{\pi}{2}=-1$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	c)	$\therefore$ Radius of curvature is, $\rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$	
		$\therefore \rho = \dfrac{\left[1+(0)^2\right]^{\frac{3}{2}}}{-1}$	1
		$\therefore \rho = -1$ i.e. 1	1
	d)	Evaluate $\int \sin^{-1} x dx$	**04**
	Ans	$\int \sin^{-1} x.1dx$	½
		$= \sin^{-1} x \int 1dx - \int\left(\int 1dx \dfrac{d}{dx}\sin^{-1} x\right)dx$	1
		$= \sin^{-1} x\ .x - \int \dfrac{x}{\sqrt{1-x^2}}dx$	1
		$= x\sin^{-1} x + \dfrac{1}{2}\int \dfrac{-2x}{\sqrt{1-x^2}}dx$	½
		$= x\sin^{-1} x + \dfrac{1}{2}2\sqrt{1-x^2} + c$	1
		$= x\sin^{-1} x + \sqrt{1-x^2} + c$	
	e)	Evaluate: $\int \dfrac{\cos xdx}{(1+\sin x)(2+\sin x)(3+\sin x)}$	**04**
	Ans	Put $\sin x = t$	
		$\therefore \cos xdx = dt$	½
		$= \int \dfrac{1}{(1+t)(2+t)(3+t)}dt$	
		consider $\dfrac{1}{(1+t)(2+t)(3+t)} = \dfrac{A}{1+t} + \dfrac{B}{2+t} + \dfrac{C}{3+t}$	
		$1 = (2+t)(3+t)A + (1+t)(3+t)B + (1+t)(2+t)C$	
		Put $t = -1$	
		$\therefore A = \dfrac{1}{2}$	½
		Put $t = -2$	
		$\therefore B = -1$	½
		Put $t = -3$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	e)	$\therefore C=\frac{1}{2}$	½
		$\frac{1}{(1+t)(2+t)(3+t)}=\frac{\frac{1}{2}}{1+t}+\frac{-1}{2+t}+\frac{\frac{1}{2}}{3+t}$	
		$\int\frac{1}{(1+t)(2+t)(3+t)}dt=\int\left(\frac{\frac{1}{2}}{1+t}+\frac{-1}{2+t}+\frac{\frac{1}{2}}{3+t}\right)dt$	
		$=\frac{1}{2}\log(1+t)-\log(2+t)+\frac{1}{2}\log(3+t)+c$	½+½+½
		$=\frac{1}{2}\log(1+\sin x)-\log(2+\sin x)+\frac{1}{2}\log(3+\sin x)+c$	½
		--	
	f)	Evaluate: $\int\frac{dx}{1+2(x+2)^2}$	**04**
	Ans	$\int\frac{dx}{1+2(x+2)^2}$	
		$=\int\frac{dx}{1+2(x^2+4x+4)}$	½
		$=\int\frac{dx}{1+2x^2+8x+8}$	
		$=\int\frac{dx}{2x^2+8x+9}$	
		$=\frac{1}{2}\int\frac{dx}{x^2+4x+\frac{9}{2}}$	½
		$T.T=\left(\frac{1}{2}\times4\right)^2=4$	½
		$=\frac{1}{2}\int\frac{dx}{x^2+4x+4+\frac{9}{2}-4}$	1
		$=\frac{1}{2}\int\frac{dx}{(x+2)^2+\frac{1}{2}}$	
		$=\frac{1}{2}\int\frac{dx}{(x+2)^2+\left(\frac{1}{\sqrt{2}}\right)^2}$	½

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	f)	$=\frac{1}{2}\frac{1}{\frac{1}{\sqrt{2}}}\tan^{-1}\left(\frac{x+2}{\frac{1}{\sqrt{2}}}\right)+c$	1
		$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\left(x+2\right)\right)+c$	
		OR	
		$\int\frac{dx}{1+2\left(x+2\right)^2}$	
		Put $x+2=t$ $\therefore dx=dt$	1
		$=\int\frac{dt}{1+2t^2}$	½
		$=\frac{1}{2}\int\frac{dt}{\frac{1}{2}+t^2}$	
		$=\frac{1}{2}\int\frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2+t^2}$	1
		$=\frac{1}{2}\frac{1}{\frac{1}{\sqrt{2}}}\tan^{-1}\left(\frac{t}{\frac{1}{\sqrt{2}}}\right)+c$	1
		$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}t\right)+c$	
		$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\left(x+2\right)\right)+c$	½
		OR	
		$\int\frac{dx}{1+2\left(x+2\right)^2}$	
		$=\frac{1}{2}\int\frac{dx}{\frac{1}{2}+\left(x+2\right)^2}$	1
		$=\frac{1}{2}\int\frac{dx}{\left(\frac{1}{\sqrt{2}}\right)^2+\left(x+2\right)^2}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		$=\frac{1}{2}\frac{1}{\frac{1}{\sqrt{2}}}\tan^{-1}\left(\frac{x+2}{\frac{1}{\sqrt{2}}}\right)+c$	2
		$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\left(x+2\right)\right)+c$	
		OR	
		$\int\frac{dx}{1+2\left(x+2\right)^2}=\int\frac{dx}{1^2+\left(\sqrt{2}\left(x+2\right)\right)^2}$	2
		$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\left(x+2\right)\right)+c$	2

3.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate: $\int_0^1\frac{dx}{1-x+x^2}$	**04**
	Ans	$\int_0^1\frac{dx}{1-x+x^2}=\int_0^1\frac{dx}{x^2-x+1}$	
		$T.T.=\left(\frac{1}{2}\times\left(-1\right)\right)^2=\frac{1}{4}$	
		$\int_0^1\frac{dx}{x^2-x+\frac{1}{4}+1-\frac{1}{4}}$	1
		$=\int_0^1\frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}$	
		$=\int_0^1\frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$	½
		$=\left[\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right]_0^1=\left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right]_0^1$	1
		$=\left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\left(1\right)-1}{\sqrt{3}}\right)\right]-\left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\left(0\right)-1}{\sqrt{3}}\right)\right]$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	$=\dfrac{2\pi}{3\sqrt{3}}$	1
		--	
	b)	Evaluate: $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{9+16\cos^2 x}$	**04**
	Ans	$\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{9+16\cos^2 x}$	
		$\displaystyle=\int_0^{\frac{\pi}{2}} \frac{dx/\cos^2 x}{\frac{9+16\cos^2 x}{\cos^2 x}}$	½
		$\displaystyle=\int_0^{\frac{\pi}{2}} \frac{dx/\cos^2 x}{\frac{9}{\cos^2 x}+\frac{16\cos^2 x}{\cos^2 x}}$	
		$\displaystyle=\int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9\sec^2 x+16}$	½
		$\displaystyle=\int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9\left(1+\tan^2 x\right)+16}$	
		$\displaystyle=\int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9+9\tan^2 x+16}$	
		$\displaystyle=\int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{9\tan^2 x+25}$	½
		Put $\tan x = t$	
		$\sec^2 x dx = dt$	½
		when $x \to 0$ to $\frac{\pi}{2}$ $t \to 0$ to ∞	
		$\displaystyle=\int_0^{\infty} \frac{dt}{9t^2+25}$ OR $\displaystyle\int_0^{\infty} \frac{dt}{9t^2+25}=\int_0^{\infty} \frac{dt}{(3t)^2+5^2}$	½
		$\displaystyle=\frac{1}{9}\int_0^{\infty} \frac{dt}{t^2+\frac{25}{9}}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$=\frac{1}{9}\int_0^\infty \frac{dt}{t^2+\left(\frac{5}{3}\right)^2}$	
		$=\left[\frac{1}{9}\times\frac{1}{\frac{5}{3}}\tan^{-1}\left(\frac{t}{\frac{5}{3}}\right)\right]_0^\infty$ OR $\left[\frac{1}{5\times 3}\tan^{-1}\left(\frac{3t}{5}\right)\right]_0^\infty$	½
		$=\left[\frac{1}{15}\tan^{-1}\left(\frac{3t}{5}\right)\right]_0^\infty$	
		$=\left[\frac{1}{15}\tan^{-1}\left(\frac{3(\infty)}{5}\right)\right]-\left[\frac{1}{15}\tan^{-1}\left(\frac{3(0)}{5}\right)\right]$	
		$=\left[\frac{1}{15}\tan^{-1}(\infty)\right]-\left[\frac{1}{15}\tan^{-1}(0)\right]$	
		$=\frac{1}{15}\times\frac{\pi}{2}$	
		$=\frac{\pi}{30}$	1
		---	
	c)	Find the area included between the curves $y^2=4ax$ and $x^2=4ay$	**04**
	Ans	$y^2=4ax$ ---------(1)	
		$x^2=4ay$	
		$\therefore y=\frac{x^2}{4a}$	
		$\therefore \text{eq}^n.(1)\Rightarrow\left(\frac{x^2}{4a}\right)^2=4ax$	
		$\frac{x^4}{16a^2}=4ax$	
		$\therefore x^4=64a^3x$	
		$\therefore x^4-64a^3x=0$	
		$\therefore x\left(x^3-64a^3\right)=0$	
		$\therefore x=0,4a$	1
		Area $A=\int_a^b (y_1-y_2)\,dx$	
		$\therefore A=\int_0^{4a}\left(2\sqrt{a}\sqrt{x}-\frac{x^2}{4a}\right)dx$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$\therefore A=\int_0^{4a}\left(2\sqrt{a}x^{\frac{1}{2}}-\frac{x^2}{4a}\right)dx$	
		$\therefore A=\left(\frac{2\sqrt{a}x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12a}\right)_0^{4a}$	1
		$\therefore A=\left(\frac{2\sqrt{a}\left(4a\right)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{\left(4a\right)^3}{12a}\right)-0$	½
		$\therefore A=\frac{16a^2}{3}$ or $5.333a^2$	½
		---	
	d)	Solve $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$	**04**
	Ans	$\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$	
		$\sec^2 x\tan y dx=-\sec^2 y\tan x dy$	
		$\frac{\sec^2 x}{\tan x}dx=-\frac{\sec^2 y}{\tan y}dy$	1
		$\int\frac{\sec^2 x}{\tan x}dx=-\int\frac{\sec^2 y}{\tan y}dy$	1
		$\log\left(\tan x\right)=-\log\left(\tan y\right)+c$	1+1
		---	
	e)	Solve: $\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$	**04**
	Ans	$\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$	
		$\frac{dy}{dx}=\sin\left(x+y\right)$ ---------(1)	
		Put $x+y=v$	
		$\therefore 1+\frac{dy}{dx}=\frac{dv}{dx}$	
		$\therefore \frac{dy}{dx}=\frac{dv}{dx}-1$	1
		From (1)	
		$\frac{dv}{dx}-1=\sin v$	½

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	e)	$\therefore \dfrac{dv}{dx}=\sin v+1$	
		$\therefore \dfrac{dv}{\sin v+1}=dx$	
		$\therefore \int \dfrac{dv}{1+\sin v}=\int dx$	1
		$\int \dfrac{1}{1+\sin v}\times\dfrac{1-\sin v}{1-\sin v}dv=\int dx$	
		$\int \dfrac{1-\sin v}{1-\sin^2 v}dv=x+c$	
		$\int \dfrac{1-\sin v}{\cos^2 v}dv=x+c$	
		$\int \sec^2 v-\tan v\sec v dv=x+c$	
		$\tan v-\sec v=x+c$	1
		$\tan(x+y)-\sec(x+y)=x+c$	½
		OR	
		$\therefore \int \dfrac{dv}{1+\sin v}=\int dx$	1
		Put $\tan\dfrac{v}{2}=t,\ dv=\dfrac{2dt}{1+t^2},\quad \sin v=\dfrac{2t}{1+t^2}$	
		$\therefore \int \dfrac{\frac{2dt}{1+t^2}}{1+\left(\frac{2t}{1+t^2}\right)}=x+c$	
		$\therefore \int \dfrac{2dt}{1+t^2+2t}=x+c$	
		$\therefore \int \dfrac{2dt}{(t+1)^2}=x+c$	
		$\therefore \int 2(t+1)^{-2}dt=x+c$	
		$\therefore 2\dfrac{(t+1)^{-1}}{-1}=x+c \qquad \therefore \dfrac{-2}{(t+1)}=x+c$	1
		$\dfrac{-2}{\left(\tan\dfrac{(x+y)}{2}+1\right)}=x+c$	½
		OR	
		$\therefore \int \dfrac{dv}{1+\sin v}=\int dx$	1
		$\therefore \int \dfrac{dv}{1+\cos\left(\dfrac{\pi}{2}-v\right)}=x+c$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	e)	$\therefore \int \frac{dv}{2\cos^2\left(\frac{\pi}{4}-\frac{v}{2}\right)} = x + c$	
		$\therefore \frac{1}{2}\int \sec^2\left(\frac{\pi}{4}-\frac{v}{2}\right)dv = x + c$	
		$\therefore \frac{1}{2}\frac{\tan\left(\frac{\pi}{4}-\frac{v}{2}\right)}{-\frac{1}{2}} = x + c$	1
		$\therefore -\tan\left(\frac{\pi}{4}-\frac{x+y}{2}\right) = x + c$	½
		--	
	f)	Solve : $\left(y^2 - x^2\right)dx - 2xydy = 0$	**04**
	Ans	$\therefore \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$	
		Put $y = vx$	
		$\frac{dy}{dx} = v + x\frac{dv}{dx}$	½
		$\therefore v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x(vx)}$	½
		$\therefore v + x\frac{dv}{dx} = \frac{x^2\left(v^2 - 1\right)}{2vx^2}$	
		$\therefore v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$	
		$\therefore x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$	
		$\therefore x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$	
		$\therefore x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$	
		$\therefore \frac{-2v}{1+v^2}dv = \frac{1}{x}dx$	1
		$\therefore \int \frac{-2v}{1+v^2}dv = \int \frac{1}{x}dx$	½
		$\therefore -\log\left(1+v^2\right) = \log x + c$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	f)	$\therefore -\log\left(1+\frac{y^2}{x^2}\right)=\log x+c$	½
		--	
4.		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$	**04**
	Ans	Let $I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx ------(1)$	
		$\therefore I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$	
		$\therefore I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx -------(2)$	1
		add (1) and (2)	
		$\therefore I+I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx+\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$	1
		$\therefore 2I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$	
		$\therefore 2I=\int_0^{\frac{\pi}{2}} 1\,dx$	
		$\therefore 2I=\left[x\right]_0^{\pi/2}$	½
		$\therefore 2I=\frac{\pi}{2}-0$	1
		$\therefore I=\frac{\pi}{4}$	½
		--	
	b)	Evaluate: $\int_0^{\pi} \frac{dx}{5+4\cos x}$	**04**
	Ans	Put $\tan\frac{x}{2}=t$	

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$\cos x=\frac{1-t^2}{1+t^2}$, $dx=\frac{2dt}{1+t^2}$	
		when $x\to 0$ to π $t\to 0$ to ∞	1
		$\therefore I=\int_0^\infty \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}\frac{2dt}{1+t^2}$	½
		$\therefore I=2\int_0^\infty \frac{1}{5\left(1+t^2\right)+4\left(1-t^2\right)}dt$	
		$\therefore I=2\int_0^\infty \frac{1}{5+5t^2+4-4t^2}dt$	
		$\therefore I=2\int_0^\infty \frac{1}{t^2+9}dt$	½
		$\therefore I=2\int_0^\infty \frac{1}{t^2+\left(3^2\right)}dt$	
		$\therefore I=\frac{2}{3}\left[\tan^{-1}\left(\frac{t}{3}\right)\right]_0^\infty$	1
		$\therefore I=\frac{2}{3}\left[\tan^{-1}\left(\frac{\infty}{3}\right)-\tan^{-1}\left(\frac{0}{3}\right)\right]$	½
		$\therefore I=\frac{2}{3}\left[\tan^{-1}\left(\infty\right)-\tan^{-1}\left(0\right)\right]$	
		$\therefore I=\frac{2}{3}\frac{\pi}{2}=\frac{\pi}{3}$ or 60	½
		--	
	c)	Find the area of the circle $x^2+y^2=9$ using integration.	**04**
	Ans	$x^2+y^2=9$ $\therefore y^2=9-x^2$ $\therefore y=\sqrt{9-x^2}=\sqrt{3^2-x^2}$ $\therefore A=4\int_a^b ydx$	
		$=4\int_0^3\sqrt{3^2-x^2}dx$	1
		$=4\left[\frac{x}{2}\sqrt{3^2-x^2}+\frac{3^2}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$	1

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	$=4\left[0+\frac{3^2}{2}\sin^{-1}(1)\right]-\left[0+\frac{3^2}{2}\sin^{-1}(0)\right]$	1
		$=4\left[\frac{3^2}{2}\cdot\frac{\pi}{2}\right]$	
		$=9\pi$	1
		--	
	d)	Solve: $x\frac{dy}{dx}+y=x^3$	**04**
	Ans	$x\frac{dy}{dx}+y=x^3$	
		Divide by x	
		$\therefore \frac{dy}{dx}+\frac{y}{x}=x^2$	½
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{1}{x}, Q=x^2$	½
		$\therefore I.F.=e^{\int Pdx}=e^{\int\frac{1}{x}dx}$	
		$=e^{\log x}=x$	1
		Solution is	
		$y.I.F.=\int QI.F.dx+c$	
		$y.x=\int x^2.xdx+c$	1
		$\therefore xy=\int x^3dx+c$	
		$\therefore xy=\frac{x^4}{4}+c$	1
		--	
	e)	Solve: $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+\left[x+\log x-x\sin y\right]dy=0$	**04**
	Ans	Let $M=y+\frac{y}{x}+\cos y$, $N=x+\log x-x\sin y$	
		$\therefore \frac{\partial M}{\partial y}=1+\frac{1}{x}-\sin y$, $\frac{\partial N}{\partial x}=1+\frac{1}{x}-\sin y$	1
		$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	$\therefore$ D.E. is exact	
		Solution is, $\int_{y-constant} M\ dx + \int_{terms\ not\ containing\ 'x'} N dy = c$	
		$\therefore \int_{y-constant} \left(y + \frac{y}{x} + \cos y \right) dx + \int 0\ dy = c$	1
		$\therefore yx + y\log x + x\cos y = c$	1
		--	
	f)	Verify that $y = e^{m\sin^{+} x}$ is the solution of the differential equation	**04**
		$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$	
	Ans	Consider $y = e^{m\sin^{-1} x}$	
		$\therefore \frac{dy}{dx} = e^{m\sin^{-1} x} \cdot \frac{d}{dx}\left(m\sin^{-1} x\right)$	
		$\therefore \frac{dy}{dx} = e^{m\sin^{-1} x} \cdot m \cdot \frac{1}{\sqrt{1-x^2}}$	½
		$\therefore \sqrt{1-x^2}\frac{dy}{dx} = my$	
		$\therefore \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = m^2 y^2$	½
		$\therefore \left(1-x^2\right) \cdot 2\frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \cdot (-2x) = m^2 \cdot 2y\frac{dy}{dx}$	1
		$\therefore 2\frac{dy}{dx}\left[\left(1-x^2\right) \cdot \frac{d^2y}{dx^2} - x\frac{dy}{dx}\right] = 2\frac{dy}{dx}\left[m^2 y\right]$	1
		$\therefore \left(1-x^2\right) \cdot \frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2 y$	
		$\therefore \left(1-x^2\right) \cdot \frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$	1
		OR	
		Consider $y = e^{m\sin^{-1} x}$	
		$\therefore \frac{dy}{dx} = e^{m\sin^{-1} x} \cdot \frac{d}{dx}\left(m\sin^{-1} x\right)$	
		$\therefore \frac{dy}{dx} = e^{m\sin^{-1} x} \cdot m \cdot \frac{1}{\sqrt{1-x^2}}$	½
		$\therefore \sqrt{1-x^2}\frac{dy}{dx} = my$ ---------(1)	

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	$\sqrt{1-x^2}\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}\dfrac{1}{2\sqrt{1-x^2}}(-2x)=m\dfrac{dy}{dx}$	1
		Multiply by $\sqrt{1-x^2}$	
		$(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=m\dfrac{dy}{dx}\sqrt{1-x^2}$	1
		$(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=m^2y \quad \text{from}(1)$	1
		$\therefore (1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-m^2y=0$	½
		(Note: If student has considered -1 at index place and attempted to solve give appropriate marks)	
5.		**Attempt any FOUR of the following:**	**16**
	a)	Two unbiased dice are thrown.Find the probability that the sum of the numbers obtained on two dice is neither a multiple of 2 nor a multiple of 3.	**04**
	Ans	$S=\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$ $(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$ $(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$ $(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$ $(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$ $(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$	
		$\therefore n(S)=36$	1
		$A=$ multiple of 2 or multiple of 3	
		$A=\{(1,1)(1,2)(1,3)(1,5)(2,1)(2,2)(2,4)(2,6)$ $(3,1)(3,3)(3,5)(3,6)(4,2)(4,4)(4,5)(4,6)$ $(5,1)(5,3)(5,4)(5,5)(6,2)(6,3)(6,4)(6,6)\}$	
		$\therefore n(A)=24$	1
		$\therefore P(A)=\dfrac{n(A)}{n(S)}=\dfrac{24}{36}=0.6667$	1
		$A'=$ neither a multiple of 2 nor a multiple of 3	
		$\therefore P(A')=1-P(A)=1-0.667=0.3333$	1
		OR	

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	a)	$\therefore n(S)=36$ $A=$ neither a multiple of 2 nor a multiple of 3 $A=\{(1,4)(1,6)(2,3)(2,5)(3,2)(3,4)(4,1)(4,3)(5,2)(5,6)(6,1)(6,5)\}$ $\therefore n(A)=12$ $\therefore P(A)=\frac{n(A)}{n(S)}=\frac{12}{36}=0.3333$ *OR* $n(S)=36$ $A=$ multiple of 2 $A=\{(1,1)(1,3)(1,5)(2,2)(2,4)(2,6)(3,1)(3,3)(3,5)$ $(4,2)(4,4)(4,6)(5,1)(5,3)(5,5)(6,2)(6,4)(6,6)\}$ $B=$ multiple of 3 $B=\{(1,2)(1,5)(2,1)(2,4)(3,3)(3,6)$ $(4,2)(4,5)(5,1)(5,4)(6,3)(6,6)\}$ $A\cup B=$ multiple of 2 or multiple of 3 $A\cup B=\{(1,1)(1,2)(1,3)(1,5)(2,1)(2,2)(2,4)(2,6)$ $(3,1)(3,3)(3,5)(3,6)(4,2)(4,4)(4,5)(4,6)$ $(5,1)(5,3)(5,4)(5,5)(6,2)(6,3)(6,4)(6,6)\}$ $\therefore n(A\cup B)=24$ $\therefore p(A\cup B)=\frac{n(A\cup B)}{n(S)}=\frac{24}{36}=0.6667$ $(A\cup B)'=$ neither a multiple of 2 nor a multiple of 3 $\therefore P(A\cup B)'=1-P(A)=1-0.6667=0.3333$ *OR* $n(S)=36$ $A=$ not multiple of 2 $A=\{(1,2)(1,4)(1,6)(2,1)(2,3)(2,5)(3,2)(3,4)(3,6)$ $(4,1)(4,3)(4,5)(5,2)(5,4)(5,6)(6,1)(6,3)(6,5)\}$ $B=$ not multiple of 3 $B=\{(1,1)(1,3)(1,4)(1,6)(2,2)(2,3)(2,5)(2,6)(3,1)(3,2)(3,4)(3,5)$ $(4,1)(4,3)(4,4)(4,6)(5,2)(5,3)(5,5)(5,6)(6,1)(6,2)(6,4)(6,5)\}$ $A\cap B=$ neither a multiple of 2 nor a multiple of 3 $A\cap B=\{(1,4)(1,6)(2,3)(2,5)(3,2)(3,4)(4,1)(4,3)(5,2)(5,6)(6,1)(6,5)\}$	1 1 2 1 2 1 1

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	a)	$\therefore n(A\cap B)=12$	2
		$\therefore P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{12}{36}=0.3333$	1
		OR	
		$n(S)=36$	
		$A=$ multiple of 2	
		$A=\{(1,1)(1,3)(1,5)(2,2)(2,4)(2,6)(3,1)(3,3)(3,5)$ $(4,2)(4,4)(4,6)(5,1)(5,3)(5,5)(6,2)(6,4)(6,6)\}$ $\therefore n(A)=18$	
		$\therefore p(A)=\frac{n(A)}{n(S)}=\frac{18}{36}=\frac{9}{4}$	1
		$B=$ multiple of 3	
		$B=\{(1,2)(1,5)(2,1)(2,4)(3,3)(3,6)$ $(4,2)(4,5)(5,1)(5,4)(6,3)(6,6)\}$ $\therefore n(B)=12$	
		$\therefore p(B)=\frac{n(B)}{n(S)}=\frac{12}{36}=\frac{2}{6}$	1
		$\therefore A\cap B=\{(1,5)(2,4)(3,3)(4,2)(5,1)(6,6)\}$ $\therefore n(A\cap B)=6$	
		$\therefore p(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{6}{36}=\frac{1}{6}$	1
		$\therefore p(A\cup B)=p(A)+p(B)-p(A\cap B)=\frac{18}{36}+\frac{12}{36}-\frac{6}{36}=\frac{24}{36}=0.6667$	½
		$\therefore p(A\cup B)'=1-p(A\cup B)=1-\frac{24}{36}=\frac{12}{36}=0.3333$	½
	b)	Probability that a bomb dropped from a plane hits a target is 0.4. Two bombs can destroy a bridge,if in all 6 bombs are dropped,find probability that the bridge will be destroyed.	**04**
	Ans	Given $p=0.4, n=6$ and $q=1-p=0.6$	1
		$p(r)={}^{n}C_{r}p^{r}q^{n-r}$	
		p(at least two bombs are required to destroy bridge)	
		$p(r)=1-\left[p(0)+p(1)\right]$	1
		$=1-\left[{}^{6}C_{0}(0.4)^{0}(0.6)^{6-0}+{}^{6}C_{1}(0.4)^{1}(0.6)^{6-1}\right]$	1
		$=0.7667$	1

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	If the probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get a bad reaction. (Given: $e=2.718$)	**04**
	Ans	$p=0.001, n=2000$	
		$\therefore m=np=0.001\times 2000=2$	1
		$p(\text{more than }2)=p(3)+p(4)+p(5)+...$	
		$=1-\left[p(0)+p(1)+p(2)\right]$	1
		$=1-\left[\frac{e^{-2}\cdot(2)^0}{0!}+\frac{e^{-2}\cdot(2)^1}{1!}+\frac{e^{-2}\cdot(2)^2}{2!}\right]$	1
		$=0.3233$	1
		---	
	d)	Evaluate: $\int \frac{\sin(x+a)}{\sin x}dx$	**04**
	Ans	$\int \frac{\sin(x+a)}{\sin x}dx$	
		$=\int \frac{\sin x.\cos a+\cos x.\sin a}{\sin x}\, dx$	1
		$=\int\left(\frac{\sin x.\cos a}{\sin x}+\frac{\cos x.\sin a}{\sin x}\right)dx$	1
		$=\int(\cos a+\cot x.\sin a)dx$	1
		$=\cos a.x+\sin a\log(\sin x)+c$	1
		---	
	e)	Evaluate: $\int_0^{\pi/2}\log(\sin x)dx$	**04**
	Ans	Let $I=\int_0^{\pi/2}\log(\sin x)dx$ --------------------(1)	
		By property $\int_0^{2a} f(x)dx=\int_0^a f(x)dx+\int_0^a f(2a-x)dx$	
		$I=\int_0^{\pi/4}\log(\sin x)dx+\int_0^{\pi/4}\log\left(\sin\left(\frac{\pi}{2}-x\right)\right)dx$	½
		$I=\int_0^{\pi/4}\log(\sin x)dx+\int_0^{\pi/4}\log(\cos x)dx$	

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	e)	$I=\int_0^{\pi/4}\log\left(\sin x.\cos x\right)dx$	½
		$\therefore I=\int_0^{\pi/4}\log\left(\frac{2\sin x.\cos x}{2}\right)dx$	
		$\therefore I=\int_0^{\pi/4}\log\left(\frac{\sin 2x}{2}\right)dx$	½
		$\therefore I=\int_0^{\pi/4}\log\left(\sin 2x\right)-\log\left(2\right)dx$	½
		$\therefore I=\int_0^{\pi/4}\log\left(\sin 2x\right)dx-\int_0^{\pi/4}\log\left(2\right)dx$	
		Put $2x=t$	
		$\therefore dx=\frac{dt}{2}$	½
		when $x\to 0$ to $\pi/4$ $t\to 0$ to $\pi/2$	
		$I=\int_0^{\pi/2}\log\left(\sin t\right)\frac{dt}{2}-\log 2\left[\frac{\pi}{4}-0\right]$	½
		$I=\frac{1}{2}I-\frac{\pi\log 2}{4}$	½
		$\therefore I-\frac{1}{2}I=-\frac{\pi\log 2}{4}$	
		$\frac{I}{2}=-\frac{\pi\log 2}{4}$	
		$\therefore I=-\frac{\pi\log 2}{2}$	½
		OR	
		$I=\int_0^{\pi/2}\log\left(\sin x\right)dx$ --------------------(1)	
		Using Property $\int_0^a f\left(x\right)dx=\int_0^a f\left(a-x\right)dx$	
		$\therefore I=\int_0^{\pi/2}\log\left(\sin\left(\frac{\pi}{2}-x\right)\right)dx$	½

SUMMER – 2018 EXAMINATION

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	e)	$I=\int_0^{\pi/2}\log(\cos x)\,dx$ ----------------(2)	
		Add (1) and (2)	
		$I+I=\int_0^{\pi/2}\log(\sin x)\,dx+\int_0^{\pi/2}\log(\cos x)\,dx$	½
		$2I=\int_0^{\pi/2}\log(\sin x.\cos x)\,dx$	½
		$\therefore 2I=\int_0^{\pi/2}\log\left(\frac{2\sin x.\cos x}{2}\right)dx$	
		$\therefore 2I=\int_0^{\pi/2}\log\left(\frac{\sin 2x}{2}\right)dx$	½
		$\therefore 2I=\int_0^{\pi/2}\left(\log(\sin 2x)-\log 2\right)dx$	½
		$\therefore 2I=\int_0^{\pi/2}\log(\sin 2x)\,dx-\log 2\int_0^{\pi/2}dx$	
		Put $2x=t$ $\therefore\ dx=\frac{dt}{2}$	½
		when $x\to 0$ to $\pi/2$ $t\to 0$ to π	
		$2I=\int_0^{\pi}\log(\sin t)\frac{dt}{2}-\log 2\left[\frac{\pi}{2}-0\right]$	½
		$2I=\frac{1}{2}\int_0^{\pi}\log(\sin t)dt-\frac{\pi\log 2}{2}$	
		$2I=\frac{1}{2}\int_0^{\pi}\log(\sin x)dx-\frac{\pi\log 2}{2}$	
		$2I=\frac{1}{2}\times 2\int_0^{\pi/2}\log(\sin x)dx-\frac{\pi\log 2}{2}$ Using Property	
		$2I=I-\frac{\pi\log 2}{2}$	
		$\therefore I=-\frac{\pi\log 2}{2}$	½

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	f)	Evaluate: $\int x\sin x\cos x dx$	**04**
	Ans	$\int x\sin x\cos x\ dx$	
		$=\frac{1}{2}\int x2\sin x\cos x\ dx$	½
		$=\frac{1}{2}\int x\sin 2x\ dx$	½
		$=\frac{1}{2}\left[x\int\sin 2xdx-\int\left(\int\sin 2xdx.\frac{d}{dx}(x)\right)dx\right]$	1
		$=\frac{1}{2}\left[x\left(\frac{-\cos 2x}{2}\right)-\int\left(\frac{-\cos 2x}{2}\right)dx\right]$	1
		$=\frac{1}{2}\left[x\left(\frac{-\cos 2x}{2}\right)+\frac{1}{2}\frac{\sin 2x}{2}\right]+c$	1
		$=\frac{1}{2}\left[x\left(\frac{-\cos 2x}{2}\right)+\frac{\sin 2x}{4}\right]+c$	
		OR	
		$\int x\sin x\cos x\ dx$	
		Put $\sin x=t\Rightarrow x=\sin^{-1}t$ $\therefore \cos xdx=dt$	½
		$\int\sin^{-1}t.t\ dt$	
		$=\sin^{-1}t\int t\ dt-\int\left(\int t\ dt\frac{d}{dt}\left(\sin^{-1}t\right)\right)dt$	½
		$=\sin^{-1}t\frac{t^2}{2}-\int\frac{t^2}{2}\frac{1}{\sqrt{1-t^2}}\ dt$	1
		$=\frac{t^2\sin^{-1}t}{2}-\frac{1}{2}\int\frac{t^2}{\sqrt{1-t^2}}\ dt$	
		$=\frac{t^2\sin^{-1}t}{2}+\frac{1}{2}\int\frac{-t^2}{\sqrt{1-t^2}}\ dt$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	f)	$=\frac{t^2\sin^{-1}t}{2}+\frac{1}{2}\int\frac{1-t^2-1}{\sqrt{1-t^2}}\,dt$	
		$=\frac{t^2\sin^{-1}t}{2}+\frac{1}{2}\int\left(\frac{1-t^2}{\sqrt{1-t^2}}-\frac{1}{\sqrt{1-t^2}}\right)dt$	
		$=\frac{t^2\sin^{-1}t}{2}+\frac{1}{2}\int\left(\sqrt{1-t^2}-\frac{1}{\sqrt{1-t^2}}\right)dt$	1
		$=\frac{t^2\sin^{-1}t}{2}+\frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t-\sin^{-1}t\right]+c$	½
		$=\frac{t^2\sin^{-1}t}{2}+\frac{1}{4}\left[t\sqrt{1-t^2}+\sin^{-1}t-2\sin^{-1}t\right]+c$	
		$=\frac{(\sin x)^2\sin^{-1}(\sin x)}{2}+\frac{1}{4}\left[(\sin x)\sqrt{1-(\sin x)^2}-\sin^{-1}(\sin x)\right]+c$	½
		--	
6.		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	It is given that mean and variance of a binomial distribution are 2 and 4/3 respectively what is the probability of obtaining (i) Exactly two successes. (ii) Less than two successes.	**04**
	Ans	Given mean $= np = 2$ variance $=\frac{4}{3}$	
		$npq=\frac{4}{3}$	
		$2q=\frac{4}{3}$	
		$\therefore q=\frac{2}{3}$	½
		$p=1-\frac{2}{3}=\frac{1}{3}$	½
		$\therefore 2=np$	
		$\therefore 2=n\frac{1}{3}$	
		$\therefore n=6$	½
		(i) Exactly two successes	
		$\therefore p(2)=6C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{6-2}$	
		$=0.3292$	1

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	a)	(ii) Less than two success	
		$p(0)+p(1)=6C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{6-0}+6C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{6-1}$	½
		$=0.3512$	1
	b)	A card is drawn from a pack of 100 cards numbered 1 to 100 find the probability of drawing a number which is a square.	**04**
	Ans	$n(S)=100$	1
		$A=$ No. which is square $=\{1,4,9,16,25,36,49,64,81,100\}$	
		$\therefore n(A)=10$	1
		$\therefore p(A)=\frac{n(A)}{n(S)}=\frac{10}{100}=\frac{1}{10}$ or 0.1	2
	c)	Divide 20 into two parts so that the product of the square of the one and the cube of the other may be the greatest possible.	**04**
	Ans	Let two parts of 20 be x and y	
		$\therefore x+y=20 \Rightarrow y=20-x$	
		$\therefore P=xy$	½
		$P=(20-x)^2x^3$	
		$\therefore P=\left(400-40x+x^2\right)x^3$	
		$\therefore P=400x^3-40x^4+x^5$	½
		$\therefore \frac{dP}{dx}=1200x^2-160x^3+5x^4$	½
		$\therefore \frac{d^2P}{dx^2}=2400x-480x^2+20x^3$	½
		Put $\frac{dP}{dx}=0$	
		$\therefore 1200x^2-160x^3+5x^4=0$	½
		$\therefore x^2\left(1200-160x+5x^2\right)=0$	
		$\therefore x=0, x=20, x=12$	½
		at $x=12$	
		$\frac{d^2P}{dx^2}=2400(12)-480(12)^2+20(12)^3=-5760<0$	½
		Product is greatest when 20 is divided into two parts 12 and 8	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	*OR*	
		$\therefore x+y=20 \Rightarrow y=20-x$	½
		$P=xy$	
		$P=(20-x)^3 x^2$	
		$\therefore P=(8000-1200x+60x^2-x^3)x^2$	
		$\therefore P=8000x^2-1200x^3+60x^4-x^5$	½
		$\therefore \frac{dP}{dx}=16000x-3600x^2+240x^3-5x^4$	½
		$\therefore \frac{d^2P}{dx^2}=16000-7200x+720x^2-20x^3$	½
		Let $\frac{dP}{dx}=0$	
		$16000x-3600x^2+240x^3-5x^4=0$	½
		$x(16000-3600x+240x^2-5x^3)=0$	
		$x=0, x=8, x=20$	½
		at $x=8$	
		$\frac{d^2P}{dx^2}=16000-7200(8)+720(8)^2-20(8)^3=-5760<0$	½
		Product is greatest when 20 is divided into two parts 8 and 12	½
		--	
	d)	Find the equation of tangent to the curve $x=\frac{1}{t},\ \ y=t-\frac{1}{t}$, when $t=2$	**04**
	Ans	$x=\frac{1}{t},\ \ y=t-\frac{1}{t}$	
		$\therefore \frac{dx}{dt}=-\frac{1}{t^2}$ and $\frac{dy}{dt}=1+\frac{1}{t^2}$	½+½
		$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{1+1/t^2}{-1/t^2}=-(t^2+1)$	1
		at $t=2$, $\frac{dy}{dx}=-5$	
		at $t=2, x=\frac{1}{2}$ and $y=\frac{3}{2}$	½
		and slope $m=-5$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	d)	$\therefore$ equation is, $y-y_1=m(x-x_1)$ $\therefore y-\frac{3}{2}=-5\left(x-\frac{1}{2}\right)$ $\therefore 2y-3=-10x+5$ $\therefore 10x+2y-8=0$ $\therefore 5x+y-4=0$	1
	e)	Given $p(A)=\frac{1}{4}$ $p(B)=\frac{1}{3}$ and $p(A\cup B)=\frac{1}{2}$ Evaluate : (i) $p(A/B)$ (ii) $p(B/A)$ (iii) $p(A\cap B')$ (iv) $p(A/B')$	**04**
	Ans	$p(A\cup B)=p(A)+p(B)-p(A\cap B)$ $\frac{1}{2}=\frac{1}{4}+\frac{1}{3}-p(A\cap B)$ $\therefore p(A\cap B)=\frac{1}{12}$	
		(i) $p(A/B)=\frac{p(A\cap B)}{p(B)}=\frac{\frac{1}{12}}{\frac{1}{3}}=\frac{1}{4}$	1
		(ii) $p(B/A)=\frac{p(A\cap B)}{p(A)}=\frac{\frac{1}{12}}{\frac{1}{4}}=\frac{1}{3}$	1
		(iii) $p(A\cap B')=p(A)-p(A\cap B)=\frac{1}{4}-\frac{1}{12}=\frac{1}{6}$	1
		(iv) $p(A/B')=\frac{p(A\cap B')}{p(B')}=\frac{\frac{1}{6}}{1-p(B)}=\frac{\frac{1}{6}}{1-\frac{1}{3}}=\frac{\frac{1}{6}}{\frac{2}{3}}=\frac{1}{4}$	1

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 17301

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	f)	In a certain examination 500 students appeared. Mean score is 68 with S. D. 8. Find the number of students scoring (i) less than 50, (ii) more than 60. Given Area between z = 0 to z = 2.25 is 0.4878 Area between z = 0 to z = 1 is 0.3413	**04**
	Ans	Given $\bar{x} = 68 \quad \sigma = 8 \quad N = 500$	
		$i)\ z = \frac{x - \bar{x}}{\sigma} = \frac{50 - 68}{8} = -2.25$	½
		$\therefore p(\text{Less than } 50) = A(\text{less than } -2.25)$	
		$= 0.5 - A(\text{z = 0 to z = 2.25})$	½
		$= 0.5 - 0.4878$	
		$= 0.0122$	½
		$\therefore \text{No. of students} = N \cdot p$	
		$= 500 \times 0.0122 = 6.1 \ i.e., \ 6$	½
		$ii)\ z = \frac{x - \bar{x}}{\sigma} = \frac{60 - 68}{8} = -1$	½
		$\therefore p(\text{More than } 60) = A(\text{more than } -1)$	
		$= A(\text{z = 0 to z = 1}) + 0.5$	½
		$= 0.3413 + 0.5$	
		$= 0.8413$	½
		$\therefore \text{No. of students} = N \cdot p = 500 \times 0.8413$	
		$= 420.65 \ i.e., \ 421$	½
		--- ***Important Note*** *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* ---	