LECTURE NOTES
ON

REINFORCED CONCRETE STRUCTURES DESIGN AND DRAWING
(ACE009)

III B. Tech I semester
(Regulation- R16)

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UNIT-I

DESIGN CONCEPTS

Introduction

In the design and analysis of reinforced concrete members, you are presented with a problem unfamiliar to most of you: "The mechanics of members consisting of two materials." To compound this problem, one of the materials (concrete) behaves differently in tension than in compression, and may be considered to be either elastic or inelastic, if it is not neglected entirely.

Although we will encounter some peculiar aspects of behavior of concrete members, we will usually be close to a solution for most problems if we can apply the following three basic ideas:

- Geometry of deformation of sections will be consistent under given types of loading; i.e., moment will always cause strain to vary linearly with distance from neutral axis, etc.
- Mechanics of materials will allow us to relate stresses to strains.
- Sections will be in equilibrium: external moments will be resisted by internal moment, external axial load will be equal to the sum of internal axial forces. (Many new engineers overly impressed speed and apparent accuracy of modern structural analysis computational procedures think less about equilibrium and details).

The overall goal is to be able to design reinforced concrete structures that are:

- Safe
- Economical
- Efficient

Reinforced concrete is one of the principal building materials used in engineered structures because:

- Low cost
- Weathering and fire resistance
- Good compressive strength
- Formability

Loads

Loads that act on structures can be divided into three general categories:

Dead Loads

Dead loads are those that are constant in magnitude and fixed in location throughout the lifetime of the structure such as: floor fill, finish floor, and plastered ceiling for buildings and wearing surface, sidewalks, and curbing for bridges

Live Loads

Live loads are those that are either fully or partially in place or not present at all, may also change in location; the minimum live loads for which the floors and roof of a building should be designed are usually specified in building code that governs at the site of construction (see Table1 - "Minimum Design Loads for Buildings and Other Structure.")

Environmental Loads

Environmental Loads consist of wind, earthquake, and snow loads. such as wind, earthquake, and snow loads.

Serviceability

Serviceability requires that

• Deflections be adequately small;
• Cracks if any be kept to a tolerable limits;
• Vibrations be minimized

Safety

A structure must be safe against collapse; strength of the structure must be dequate for all loads that might act on it. If we could build buildings as designed, and if the loads and their internal effects can be predicted accurately, we do not have to worry about safety. But there are uncertainties in:

• Actual loads;
• Forces/loads might be distributed in a manner different from what we assumed;
• The assumptions in analysis might not be exactly correct;
• Actual behavior might be different from that assumed;
• etc.

Finally, we would like to have the structure safe against

Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions.

Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones.

Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

Properties of Concrete

The important properties of concrete, which govern the design of concrete mix are as follows

(i) Weight

The unit weights of plain concrete and reinforced concrete made with sand, gravel of crushed natural stone aggregate may be taken as 24 KN/m3 and 25 KN/m3 respectively.

(ii) Compressive Strength

With given properties of aggregate the compressive strength of concrete depends primarily on age, cement content and the water cement ratio are given Table 2 of IS 456:2000. Characteristic strength are based on the strength at 28 days. The strength at 7 days is about two-thirds of that at 28 days with ordinary portland cement and generally good indicator of strength likely to be obtained.

(iii) Increase in strength with age

There is normally gain of strength beyond 28 days. The quantum of increase depends upon the grade and type of cement curing and environmental conditions etc.

(iv) Tensile strength of concrete

The flexure and split tensile strengths of various concrete are given in IS 516:1959 and IS 5816:1970 respectively when the designer wishes to use an estimate of the tensile strength from compressive strength, the following formula can be used

Flexural strength, $f_{cr}=0.7\sqrt{f_{ck}}$ N/mm2

(v) Elastic Deformation

The modulus of elasticity is primarily influenced by the elastic properties of the aggregate and to lesser extent on the conditions of curing and age of the concrete, the mix proportions and the type of cement. The modulus of elasticity is normally related to the compressive characteristic strength of concrete

$E_c=5000\sqrt{f_{ck}}$ N/mm2

Where E_c= the short-term static modulus of elasticity in N/mm2

f_{ck}=characteristic cube strength of concrete in N/mm2

(vi) Shrinkage of concrete

Shrinkage is the time dependent deformation, generally compressive in nature. The constituents of concrete, size of the member and environmental conditions are the factors on which the total shrinkage of concrete depends. However, the total shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing for a given humidity and temperature. The cement content, however, influences the total shrinkage of concrete to a lesser extent. The approximate value of the total shrinkage strain for design is taken as 0.0003 in the absence of test data (cl. 6.2.4.1).

(vii) Creep of concrete

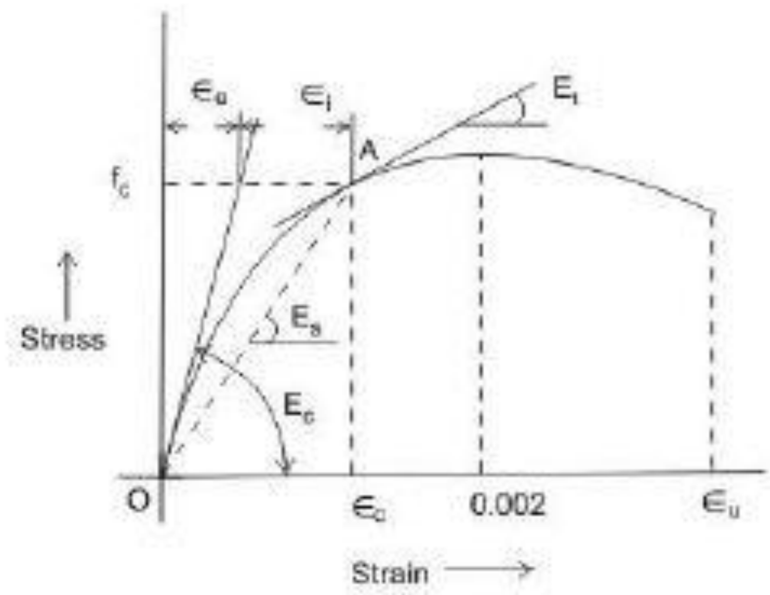


Figure1.1: Stress-strain curve of concrete

The effective modulus of *Ece* of concrete is used only in the calculation of creep deflection. It is seen that the value of creep coefficient θ is reducing with the age of concrete at loading. It may also be noted that the ultimate creep strain does not include short term strain.

- Properties of concrete
- Water/cement ratio
- Humidity and temperature of curing
- Humidity during the period of use
- Age of concrete at first loading
- Magnitude of stress and its duration
- Surface-volume ratio of the member

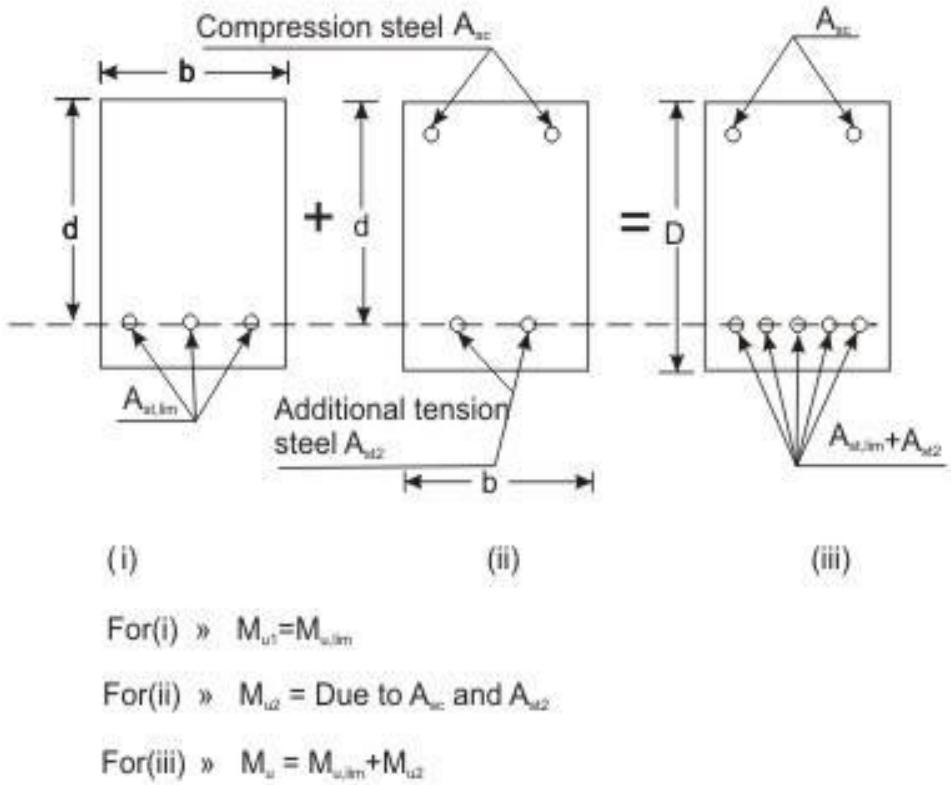


Fig. 4.8.1: Doubly reinforced beam

Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension. However, these beams have their respective limiting moments of resistance with specified width, depth and grades of concrete and steel. The amount of steel reinforcement needed is known as $A_{st,lim}$. Problem will arise, therefore, if such a section is subjected to bending moment greater than its limiting moment of resistance as a singly reinforced section.

There are two ways to solve the problem. First, we may increase the depth of the beam, which may not be feasible in many situations. In those cases, it is possible to increase both the compressive and tensile forces of the beam by providing steel reinforcement in compression face and additional reinforcement in tension face of the beam without increasing the depth (Fig. 4.8.1). The total compressive force of such beams comprises (i) force due to concrete in compression and (ii) force due to steel in compression. The tensile force also has two components: (i) the first provided by $A_{st,lim}$ which is equal to the compressive force of concrete in compression. The second part is due to the additional steel in tension - its force will be equal to the compressive force of steel in compression.

Such reinforced concrete beams having steel reinforcement both on tensile and compressive faces are known as doubly reinforced beams.

Doubly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams. However, other than in doubly reinforced beams compression steel reinforcement is provided when:

(i) some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.

(ii) the ductility requirement has to be followed.

(iii) the reduction of long term deflection is needed.

It may be noted that even in so called singly reinforced beams there would be longitudinal hanger bars in compression zone for locating and fixing stirrups.

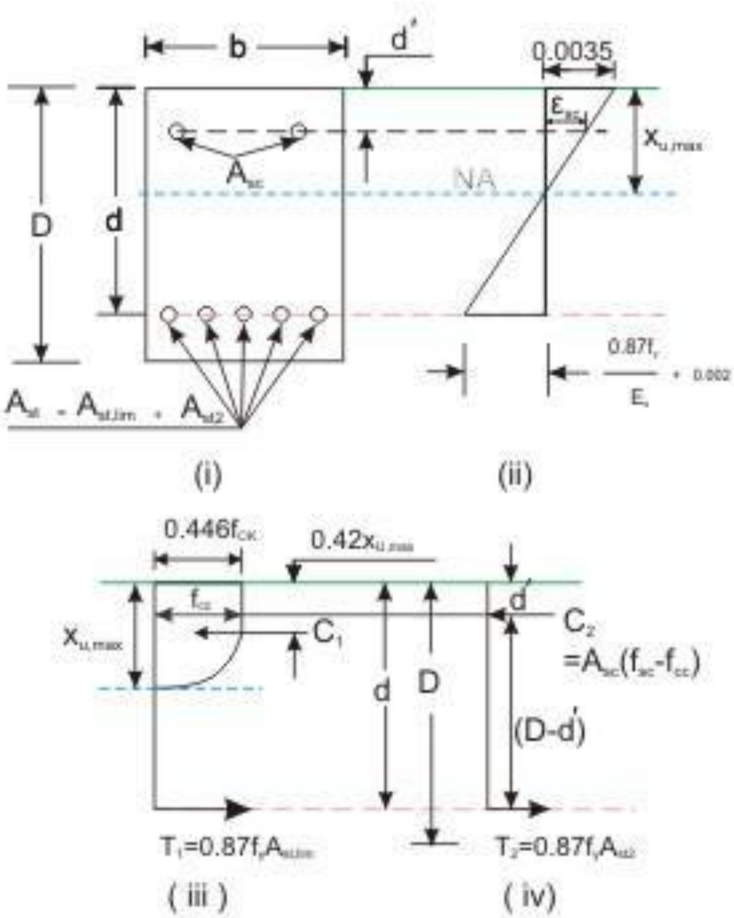
Assumptions

(i) The assumptions of sec. 3.4.2 of Lesson 4 are also applicable here.

(ii) Provision of compression steel ensures ductile failure and hence, the limitations of x/d ratios need not be strictly followed here.

(iii) The stress-strain relationship of steel in compression is the same as that in tension. So, the yield stress of steel in compression is 0.87 *fy*.

Basic Principle



(I) Beam cross section
(ii) Strain diagram
(iii) Force diagram of beam of $M_{u,lim}$
(iv) Force diagram of beam of M_{u2}

Fig. 4.8.2: Stress, strain and force diagrams of doubly reinforced beam

As mentioned in sec. 4.8.1, the moment of resistance M_u of the doubly reinforced beam consists of (i) $M_{u,lim}$ of singly reinforced beam and (ii) M_{u2} because of equal and opposite compression and tension forces (C_2 and T_2) due to additional steel reinforcement on compression and tension faces of the beam

(Figs. 4.8.1 and 2). Thus, the moment of resistance M_u of a doubly reinforced beam is

$$M_u = M_{u,lim} + M_{u2} \quad (4.1)$$

The $M_{u,lim}$ is as given in Eq. 3.24 of Lesson 5, i.e.,

$$M_{u,lim} = 0.36 \left(\frac{x_{u,max}}{d}\right)\left(1 - 0.42 \frac{x_{u,max}}{d}\right) b\, d^2 f_{ck} \quad (4.2)$$

Also, $M_{u,lim}$ can be written from Eq. 3.22 of Lesson 5, using $x_u = x_{u,max}$, i.e.,

$$M_{u,lim} = 0.87\, A_{st,lim}\, f_y\, (d - 0.42\, x_{u,max})$$

$$= 0.87 p_{t,lim} \left(1 - 0.42 \frac{x_{u,max}}{d}\right) b\, d^2 f_y \quad (4.3)$$

The additional moment M_{u2} can be expressed in two ways (Fig. 4.8.2): considering (i) the compressive force C_2 due to compression steel and (ii) the tensile force T_2 due to additional steel on tension face. In both the equations, the lever arm is $(d - d')$. Thus, we have

$$M_{u2} = A_{sc}\,(f_{sc} - f_{cc})\,(d - d') \quad (4.4)$$

$$M_{u2} = A_{st2}\,(0.87 f_y)\,(d - d') \quad (4.5)$$

where A_{sc} = area of compression steel reinforcement

f_{sc} = stress in compression steel reinforcement

f_{cc} = compressive stress in concrete at the level of centroid of compression steel reinforcement

A_{st2} = area of additional steel reinforcement

Since the additional compressive force C_2 is equal to the additional tensile force T_2, we have

$$A_{sc}\,(f_{sc} - f_{cc}) = A_{st2}\,(0.87 f_y) \quad (4.6)$$

Any two of the three equations (Eqs. 4.4 - 4.6) can be employed to determine A_{sc} and A_{st2}.

The total tensile reinforcement A_{st} is then obtained from:

$$A_{st} = A_{st1} + A_{st2} \quad (4.7)$$

where $A_{st1} = \frac{p_{t,\lim}\, b\, d}{100} = \frac{M_{u,\lim}}{0.87 f_y (d - 0.42\, x_{u,\max})}$ (4.8)

Determination of f_{sc} and f_{cc}

It is seen that the values of f_{sc} and f_{cc} should be known before calculating A_{sc}. The following procedure may be followed to determine the value of f_{sc} and f_{cc} for the design type of problems (and not for analysing a given section). For

the design problem the depth of the neutral axis may be taken as $x_{u,max}$ as shown in Fig. 4.8.2. From Fig. 4.8.2, the strain at the level of compression steel reinforcement ε_{sc} may be written as

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{u,\max}}\right) \tag{4.9}$$

The stress in compression steel f_{sc} is corresponding to the strain ε_{sc} of Eq. 4.9 and is determined for (a) mild steel and (b) cold worked bars Fe 415 and 500 as given below:

(a) Mild steel Fe 250

The strain at the design yield stress of 217.39 N/mm^2 (f_d = 0.87 f_y) is

0.0010869 (= 217.39/E_s). The f_{sc} is determined from the idealized stress-strain diagram of mild steel (Fig. 1.2.3 of Lesson 2 or Fig. 23B of IS 456) after computing the value of ε_{sc} from Eq. 4.9 as follows:

(i) If the computed value of $\varepsilon_{sc} \le 0.0010869$, $f_{sc} = \varepsilon_{sc} E_s = 2\,(10^5)\,\varepsilon_{sc}$

(ii) If the computed value of $\varepsilon_{sc} > 0.0010869$, f_{sc} = 217.39 N/mm^2.

(b) Cold worked bars Fe 415 and Fe 500

The stress-strain diagram of these bars is given in Fig. 1.2.4 of Lesson 2 and in Fig. 23A of IS 456. It shows that stress is proportional to strain up to a stress of 0.8 f_y. The stress-strain curve for the design purpose is obtained by substituting f_{yd} for f_y in the figure up to 0.8 f_{yd}. Thereafter, from 0.8 f_{yd} to f_{yd}, Table A of SP-16 gives the values of total strains and design stresses for Fe 415

and Fe 500. Table 4.1 presents these values as a ready reference here.

Table 4.1Values off_{sc} andε_{sc}

Stress level	Fe 415		Fe 500	
	Strain εsc	Stressf_{sc} (N/mm^2)	Strainε_{sc}	Stressf_{sc} (N/mm^2)
0.80 *fyd*	0.00144	288.7	0.00174	347.8
0.85 *fyd*	0.00163	306.7	0.00195	369.6
0.90 *fyd*	0.00192	324.8	0.00226	391.3
0.95 *fyd*	0.00241	342.8	0.00277	413.0
0.975 *fyd*	0.00276	351.8	0.00312	423.9
1.0 *fyd*	0.00380	360.9	0.00417	434.8

Linear interpolation may be done for intermediate values.

The above procedure has been much simplified for the cold worked bars by presenting the values of f_{sc} of compression steel in doubly reinforced beams for different values of d'/d only taking the practical aspects into consideration. In most of the doubly reinforced beams, d'/d has been found to be between 0.05 and 0.2. Accordingly, values of f_{sc} can be computed from Table 4.1 after determining the value of ε_{sc} from Eq. 4.9 for known values of d'/d as 0.05, 0.10, 0.15 and 0.2. Table F of SP-16 presents these values of f_{sc} for four values of d'/d (0.05, 0.10, 0.15 and 0.2) of Fe 415 and Fe 500. Table 4.2 below, however, includes Fe 250 also whose f_{sc} values are computed as laid down in sec.

4.8.4(a) (i) and (ii) along with those of Fe 415 and Fe 500. This table is very useful and easy to determine the f_{sc} from the given value of d'/d. The table also includes strain values at yield which are explained below:

(i)The strain at yield of Fe 250=

$$\frac{\text{Design YieldStress}}{E_s} = \frac{250}{1.15\ (200000)} = 0.0010869$$

Here, there is only elastic component of the strain without any inelastic strain.

(ii)The strain at yield of Fe 415 = Inelastic Strain+ $\dfrac{\text{Design YieldStress}}{E_s}$

$$= 0.002 + \frac{415}{1.15\ (200000)} = 0.0038043$$

(iii) The strain at yield of Fe 500 = $0.002 + \dfrac{500}{1.15\ (200000)} = 0.0041739$

Table 4.2 Values off_{sc} for different values ofd'/d

f_y (N/mm^2)	d'/d				Strain at yield
	0.05	0.10	0.15	0.20	
250	217.4	217.4	217.4	217.4	0.0010869
415	355	353	342	329	0.0038043
500	412	412	395	370	0.0041739

Minimum and maximum steel

Minimum and maximum steel in compression

There is no stipulation in IS 456 regarding the minimum compression steel in doubly reinforced beams. However, hangers and other bars provided up to 0.2% of the whole area of cross section may be necessary for creep and shrinkage of concrete. Accordingly, these bars are not considered as compression reinforcement. From the practical aspects of consideration, therefore, the minimum steel as compression reinforcement should be at least 0.4% of the area of concrete in compression or 0.2% of the whole cross -sectional area of the beam so that the doubly reinforced beam can take care of the extra loads in addition to resisting the effects of creep and shrinkage of concrete.

The maximum compression steel shall not exceed 4 per cent of the whole area of cross-section of the beam as given in cl. 26.5.1.2 of IS 456.

Minimum and maximum steel in tension

As stipulated in cl. 26.5.1.1(a) and (b) of IS 456, the minimum amount of tensile reinforcement shall be at least (0.85 bd/f_y) and the maximum area of tension reinforcement shall not exceed (0.04 bD).

It has been discussed in sec. 3.6.2.3 of Lesson 6 that the singly reinforced

beams shall have A_{st} normally not exceeding 75 to 80% of $A_{st,lim}$ so that x_u remains less than $x_{u,max}$ with a view to ensuring ductile failure. However, in the

case of doubly reinforced beams, the ductile failure is ensured with the presence of compression steel. Thus, the depth of the neutral axis may be taken as $x_{u,\ max}$ if the beam is over-reinforced. Accordingly, the A_{st1} part of tension steel can go

up to $A_{st,\ lim}$ and the additional tension steel A_{st2} is provided for the additional moment M_u - $M_{u,\ lim}$. The quantities of A_{st1} and A_{st2} together form the total A_{st}, which shall not exceed 0.04 bD.

Types of problems and steps of solution

Similar to the singly reinforced beams, the doubly reinforced beams have two types of problems: (i) design type and (ii) analysis type. The different steps of solutions of these problems are taken up separately.

Design type of problems

In the design type of problems, the given data are b, d, D, grades of concrete and steel. The designer has to determine A_{sc} and A_{st} of the beam from the given factored moment. These problems can be solved by two ways: (i) use of the equations developed for the doubly reinforced beams, named here as direct computation method, (ii) use of charts and tables of SP-16.

(a) Direct computation method

Step 1: To determine $M_{u,\ lim}$ and $A_{st,\ lim}$ from Eqs. 4.2 and 4.8, respectively.

Step 2: To determine M_{u2}, A_{sc}, A_{st2} and A_{st} from Eqs. 4.1, 4.4, 4.6 and 4.7, respectively.

Step 3: To check for minimum and maximum reinforcement in compression and tension as explained in sec. 4.8.5.

Step 4: To select the number and diameter of bars from known values of A_{sc} and A_{st}.

(b) Use of SP table

Tables 45 to 56 present the p_t and p_c of doubly reinforced sections for d'/d = 0.05, 0.10, 0.15 and 0.2 for different f_{ck} and f_y values against $M_u\ /bd^2$. The values of p_t and p_c are obtained directly selecting the proper table with known values of M_u/bd^2 and d'/d.

Analysis type of problems

In the analysis type of problems, the data given are b, d, d', D, f_{ck}, f_y, A_{sc} and A_{st} . It is required to determine the moment of resistance M_u of such beams.

These problems can be solved: (i) by direct computation method and (ii) by using tables of SP-16.

(a) Direct computation method

Step 1: To check if the beam is under-reinforced or over-reinforced.

First, $x_{u,max}$ is determined assuming it has reached limiting stage using $\underline{x_{u,ma}}$ coefficients as given in cl. 38.1, Note of IS 456. The strain of tensile steel

$$\frac{x}{d}$$

ε_{st} is computed from $\varepsilon_{st} = \dfrac{\varepsilon_c \, (d - x_{u,\max})}{x_{u,\max}}$ and is checked if ε_{st} has reached the yield strain of steel:

$$\varepsilon_{st\,at\,yield} = \frac{f_y}{1.15\,(E)} + 0.002$$

The beam is under-reinforced or over-reinforced if ε_{st} is less than or more than the yield strain.

Step 2: To determine $M_{u,lim}$ from Eq. 4.2 and $A_{st,lim}$ from the $p_{t,\ lim}$ given in Table 3.1 of Lesson 5.

Step 3: To determine A_{st2} and A_{sc} from Eqs. 4.7 and 4.6, respectively.

Step 4: To determine M_{u2} and M_u from Eqs. 4.4 and 4.1, respectively.

(b) Use of tables of SP-16

As mentioned earlier Tables 45 to 56 are needed for the doubly reinforced beams. First, the needed parameters d'/d, p_t and p_c are calculated. Thereafter, M_u/bd^2 is computed in two stages: first, using d'/d and p_t and then using d'/d and p_c . The lower value of M_u is the moment of resistance of the beam.

UNIT-II

LIMIT STATE DESIGN

Introduction:

- identify the regions where the beam shall be designed as a flanged and where it will be rectangular in normal slab beam construction,
- define the effective and actual widths of flanged beams,
- state the requirements so that the slab part is effectively coupled with the flanged beam,
- write the expressions of effective widths of *T* and *L*-beams both for continuous and isolated cases,
- derive the expressions of *C*, *T* and M_u for four different cases depending on the location of the neutral axis and depth of the flange.

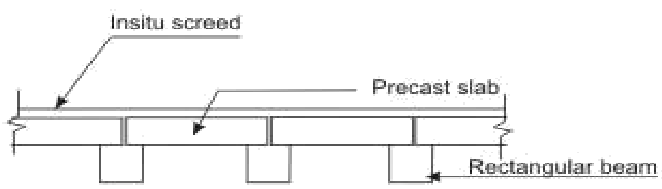


Fig. 5.10.1(c) Precast slab on rectangular beams

Reinforced concrete slabs used in floors, roofs and decks are mostly cast monolithic from the bottom of the beam to the top of the slab. Such rectangular beams having slab on top are different from others having either no slab (bracings of elevated tanks, lintels etc.) or having disconnected slabs as in some pre-cast systems (Figs. 5.10.1 a, b and c). Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e., between the supports of a continuous beam, the slab, up to a certain width greater than the width of the beam, forms the top part of the beam. Such beams having slab on top of the rectangular rib are designated as the flanged beams - either *T* or *L* type depending on whether the slab is on both sides or on one side of the beam (Figs. 5.10.2 a to e) . Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression. The continuous beam at support is thus equivalent to a rectangular beam (Figs. 5.10.2 a, c, f and g).

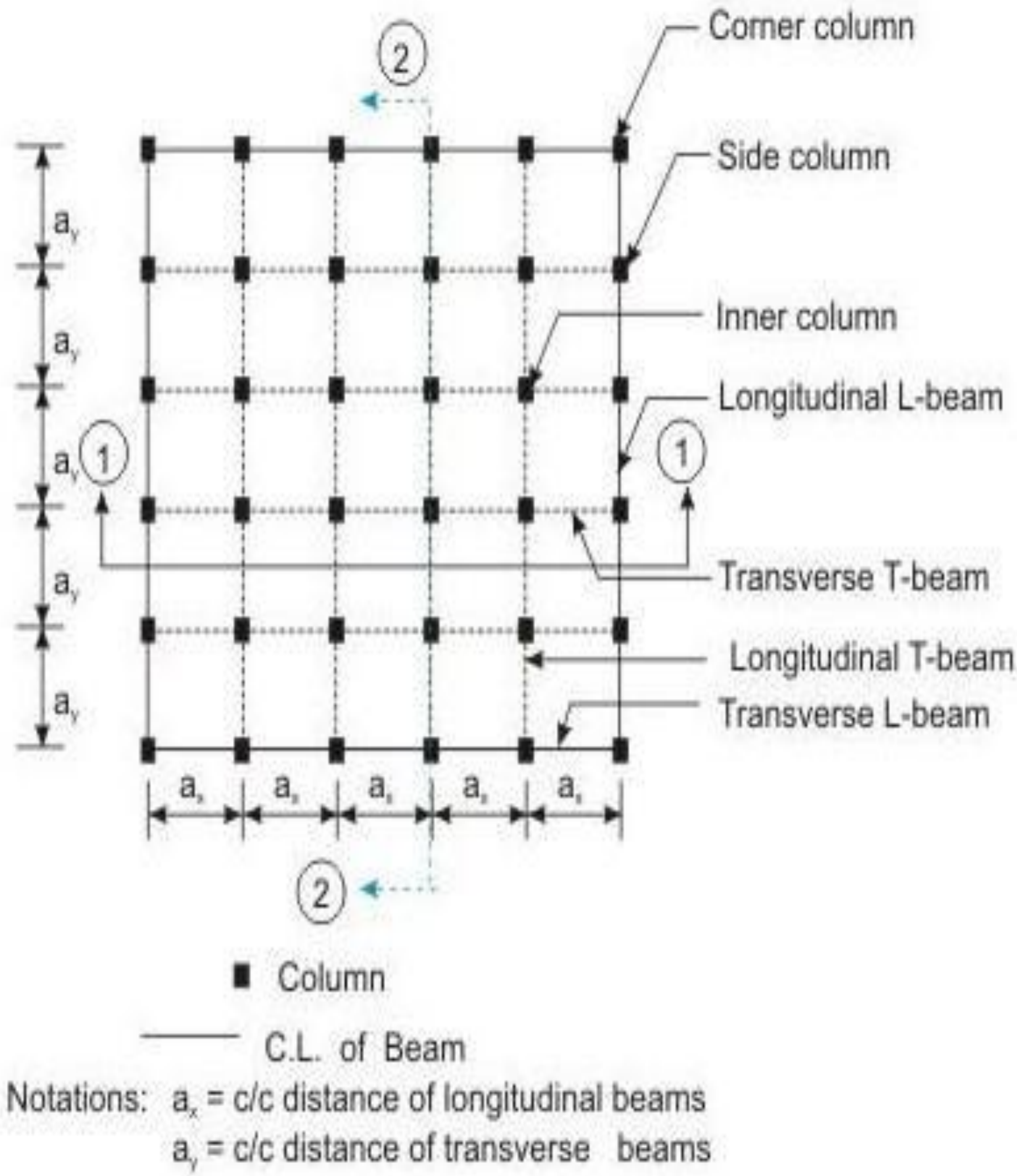


Fig. 5.10.2 (a): Key plan

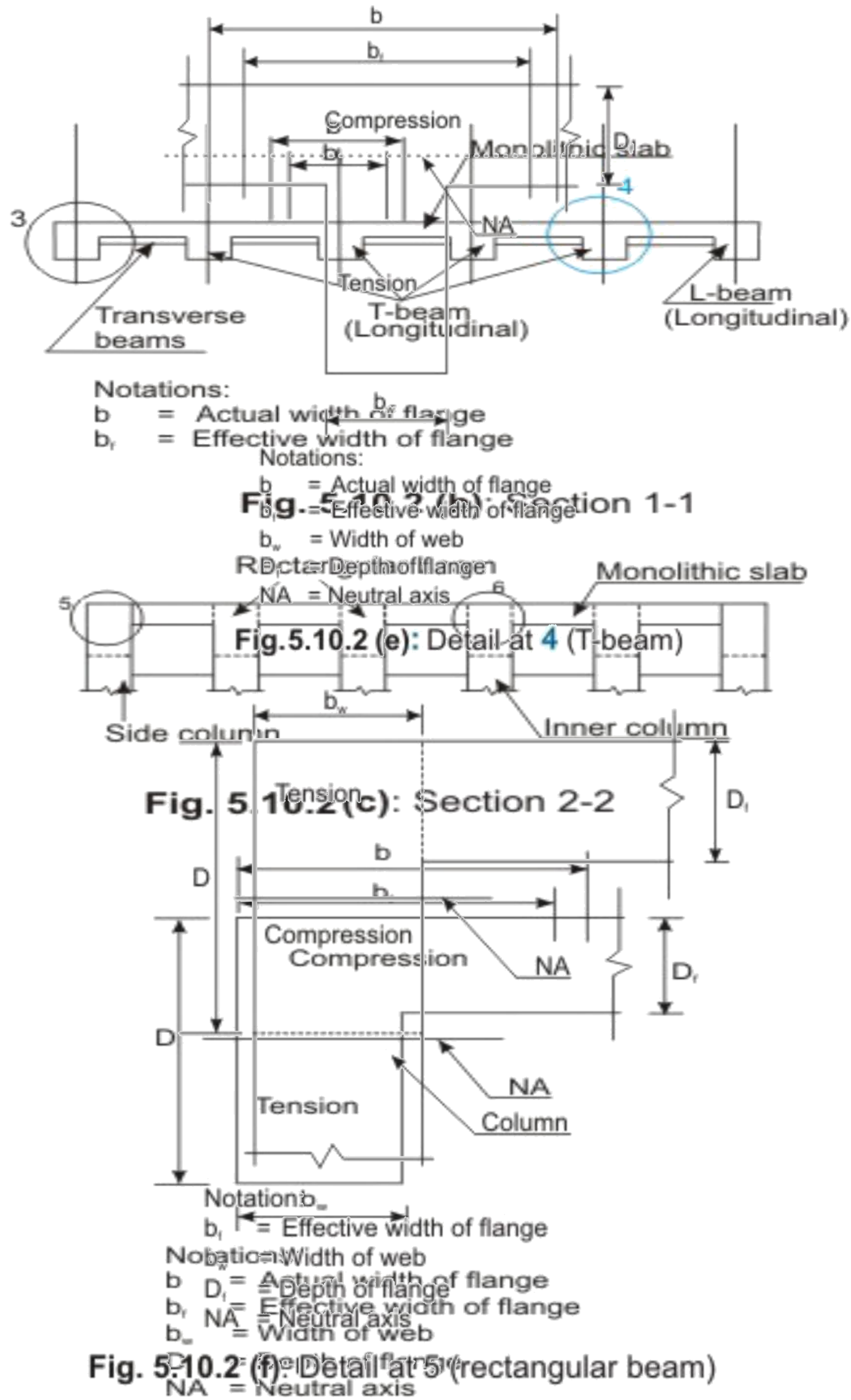


Fig. 5.10.2 (b): Section 1-1

Fig.5.10.2 (e): Detail at 4 (T-beam)

Fig. 5.10.2 (c): Section 2-2

Fig. 5.10.2 (f): Detail at 5 (rectangular beam)

Fig. 5.10.2 (d): Detail at 3 (L-beam)

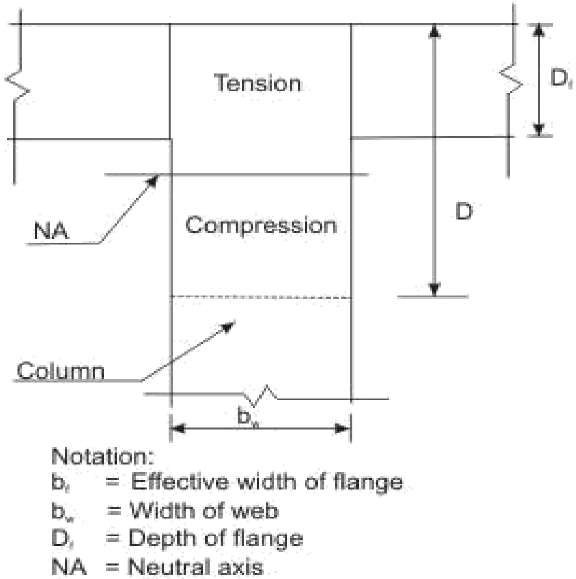


Fig. 5.10.2 (g): Detail at 6 (rectangular beam)

The actual width of the flange is the spacing of the beam, which is the same as the distance between the middle points of the adjacent spans of the slab, as shown in Fig. 5.10.2 b. However, in a flanged beam, a part of the width less than the actual width, is effective to be considered as a part of the beam. This width of the slab is designated as the effective width of the flange.

Effective Width

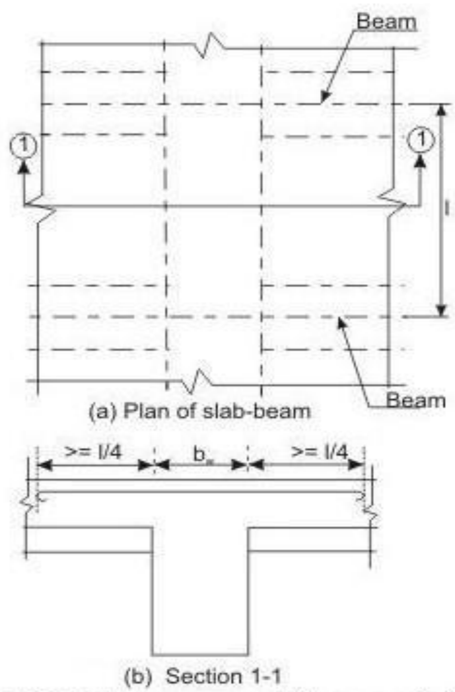


Fig. 5.10.3: Transverse reinforcement of flange of T-beam

IS code requirements

The following requirements (cl. 23.1.1 of IS 456) are to be satisfied to ensure the combined action of the part of the slab and the rib (rectangular part of the beam).

4.8.3 The slab and the rectangular beam shall be cast integrally or they shall be effectively bonded in any other manner.

4.8.4 Slabs must be provided with the transverse reinforcement of at least 60 per cent of the main reinforcement at the mid span of the slab if the main reinforcement of the slab is parallel to the transverse beam (Figs. 5.10.3 a and b).

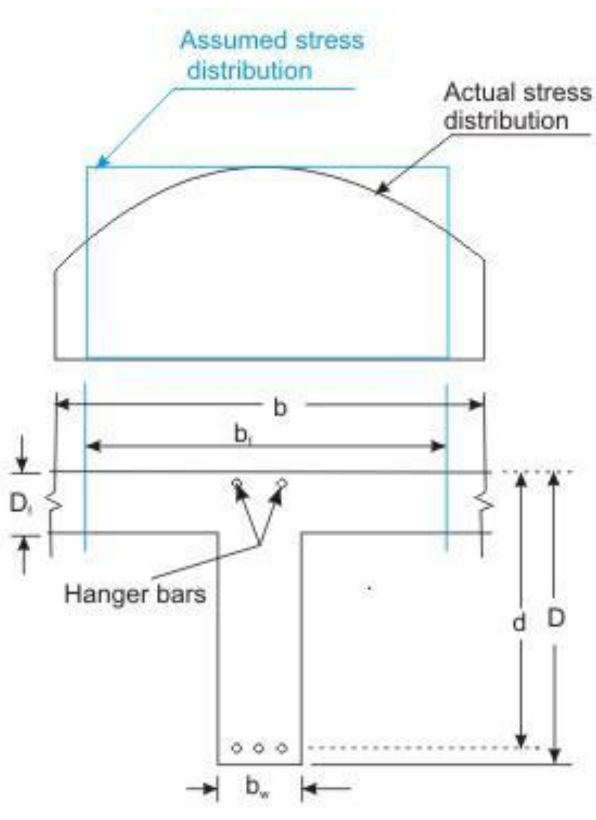


Fig. 5.10.4: Variation of compressive stress

The variation of compressive stress (Fig. 5.10.4) along the actual width of the flange shows that the compressive stress is more in the flange just above the rib than the same at some distance away from it. The nature of variation is complex and, therefore, the concept of effective width has been introduced. The effective width is a convenient hypothetical width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress.

5.10.2.2 IS code specifications

Clause 23.1.2 of IS 456 specifies the following effective widths of *T* and *L*-beams:

(a) For *T*-beams, the lesser of

(i) $b_f = l_o/6 + b_w + 6\,D_f$

(iv) b_f=Actual width of the flange

4.8.3 For isolated *T*-beams, the lesser of

$$\text{(i)}\, b_f = \frac{l_o}{(l_o/b) + 4} + b_w$$

b_f=Actual width of the flange

(ii) For*L*-beams, the lesser of

(i) $b_f = l_o/12 + b_w + 3\,D_f$

b_f=Actual width of the flange

(i) For isolated *L*-beams, the lesser of

$$\text{(i)}\, b_f = \frac{0.5\; l_o}{(l_o/b) + 4} + b_w$$

(ii) b_f = Actual width of the flange

where b_f = effective width of the flange,

l_o = distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

b_w = beadth of the web,

D_f = thickness of the flange,

and b = actual width of the flange.

Four Different Cases

The neutral axis of a flanged beam may be either in the flange or in the web depending on the physical dimensions of the effective width of flange b_f, effective width of web b_w, thickness of flange D_f and effective depth of flanged beam d (Fig. 5.10.4). The flanged beam may be considered as a rectangular beam of width b_f and effective depth d if the neutral axis is in the flange as the concrete in tension is ignored. However, if the neutral axis is in the web, the compression is taken by the flange and a part of the web.

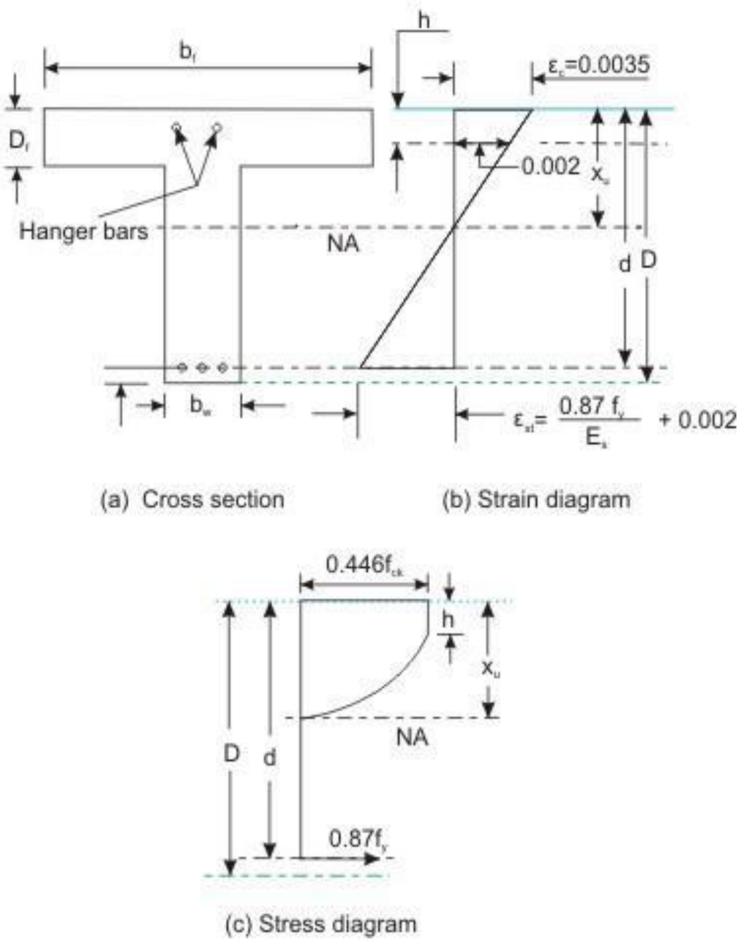


Fig. 5.10.5: A typical T-beam section

All the assumptions made in sec. 3.4.2 of Lesson 4 are also applicable for the flanged beams. As explained in Lesson 4, the compressive stress remains constant between the strains of 0.002 and 0.0035. It is important to find the depth h of the beam where the strain is 0.002 (Fig. 5.10.5 b). If it is located in the web, the whole of flange will be under the constant stress level of 0.446 f_{ck}. The following gives the relation of D_f and d to facilitate the determination of the depth h where the strain will be 0.002.

From the strain diagram of Fig. 5.10.5 b:

$$\frac{0.002}{0.0035} = \frac{x_u - h}{x_u}$$

or $$\frac{h}{x_u} = \frac{3}{7} = 0.43 \qquad (5.1)$$

when $x_u = x_{u,\max}$, we get

$h = \frac{3}{7} x_{u,\max} = 0.227\,d$, $0.205\,d$ and $0.197\,d$, for Fe250,Fe415andFe 500, respectively. In general, we can adopt, say

$$h/d = 0.2 \qquad (5.2)$$

The same relation is obtained below from the values of strains of concrete and steel of Fig. 5.10.5 b.

$$\frac{\varepsilon_{st}}{\varepsilon_c} = \frac{d - x_u}{x_u}$$

or $$\frac{d}{x_u} = \frac{\varepsilon_{st} + \varepsilon_c}{\varepsilon_c} \qquad (5.3)$$

Dividing Eq. 5.1 by Eq. 5.3

$$\frac{h}{d} = \frac{0.0015}{\varepsilon_{st} + 0.0035} \qquad (5.4)$$

Using $\varepsilon_{st} = (0.87 f_y / E_s) + 0.002$ in Eq. 5.4, we get $h/d = 0.227$, 0.205 and 0.197 for Fe 250, Fe 415 and Fe 500 respectively, and we can adopt $h/d = 0.2$ (as in Eq. 5.2).

Thus, we get the same Eq. 5.2 from Eq. 5.4,

$h/d = 0.2$ (5.2)

following gives the relation of D_f and d to facilitate the determination of the depth h where the strain will be 0.002.

From the strain diagram of Fig. 5.10.5 b:

$$\frac{0.002}{0.0035} = \frac{x_u - h}{x_u}$$

or

$$\frac{h}{x_u} = \frac{3}{7} = 0.43$$

(5.1)

when $x_u = x_{u,\max}$, we get

$$h = \frac{3}{7} x_{u,\max} = 0.227\,d,\ 0.205\,d \text{ and } 0.197\,d, \text{ for}$$

Fe250,Fe415andFe 500, respectively. In general, we can adopt, say

$$h/d = 0.2 \qquad (5.2)$$

The same relation is obtained below from the values of strains of concrete and steel of Fig. 5.10.5 b.

$$\frac{\varepsilon_{st}}{\varepsilon_c} = \frac{d - x_u}{x_u}$$

or

$$\frac{d}{x_u} = \frac{\varepsilon_{st} + \varepsilon_c}{\varepsilon_c} \qquad (5.3)$$

Dividing Eq. 5.1 by Eq. 5.3

$$\frac{h}{d} = \frac{0.0015}{\varepsilon_{st} + 0.0035} \qquad (5.4)$$

Using $\varepsilon_{st} = (0.87 f_y / E_s) + 0.002$ in Eq. 5.4, we get h/d = 0.227, 0.205 and 0.197 for Fe 250, Fe 415 and Fe 500 respectively, and we can adopt $h/d = 0.2$ (as in Eq. 5.2).

Thus, we get the same Eq. 5.2 from Eq. 5.4,

$h/d = 0.2$ (5.2)

It is now clear that the three values of h are around 0.2 d for the three grades of steel. The maximum value of h may be D_f, at the bottom of the flange where the strain will be 0.002, if $D_f/d = 0.2$. This reveals that the thickness of the flange may be considered small if D_f/d does not exceed 0.2 and in that case, the position of the fibre of 0.002 strain will be in the web and the entire flange will be under a constant compressive stress of 0.446 f_{ck} .

On the other hand, if D_f is > 0.2 d, the position of the fibre of 0.002 strain will be in the flange. In that case, a part of the slab will have the constant stress of 0.446 f_{ck} where the strain will be more than 0.002.

Thus, in the balanced and over-reinforced flanged beams (when $x_u = x_{u,\max}$), the ratio of Df/d is important to determine if the rectangular stress block is for the full depth of the flange (when D_f/d does not exceed 0.2) of for a part of the flange (when $D_f/d > 0.2$). Similarly, for the under-reinforced flanged beams, the ratio of D_f/x_u is considered in place of D_f/d. If D_f/x_u does not exceed

0.43 (see Eq. 5.1), the constant stress block is for the full depth of the flange. If $D_f/x_u > 0.43$, the constant stress block is for a part of the depth of the flange.

Based on the above discussion, the four cases of flanged beams are as follows:

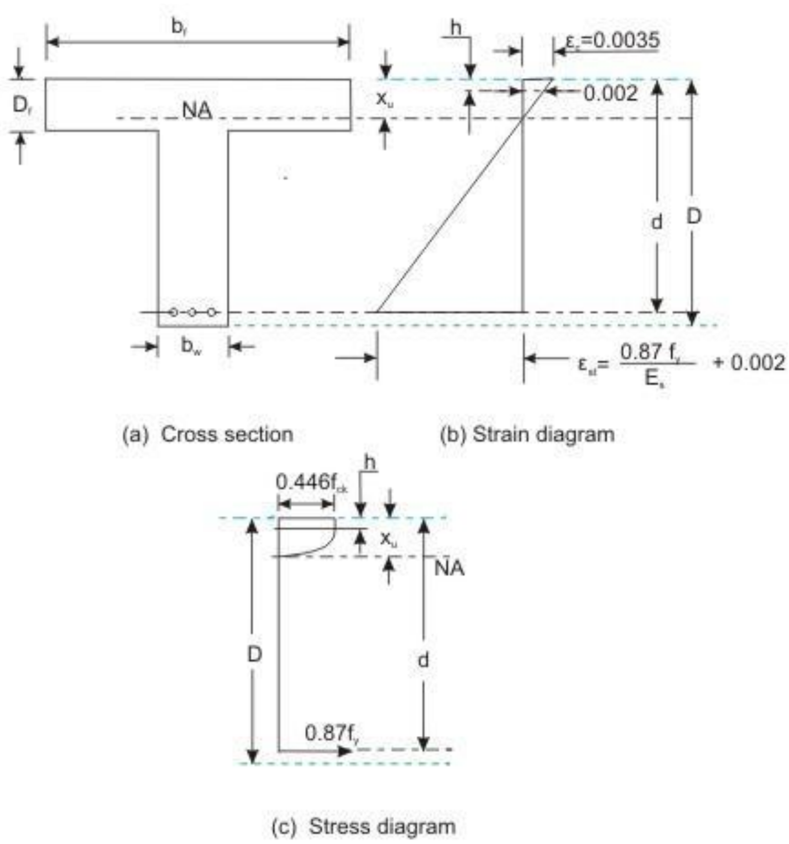
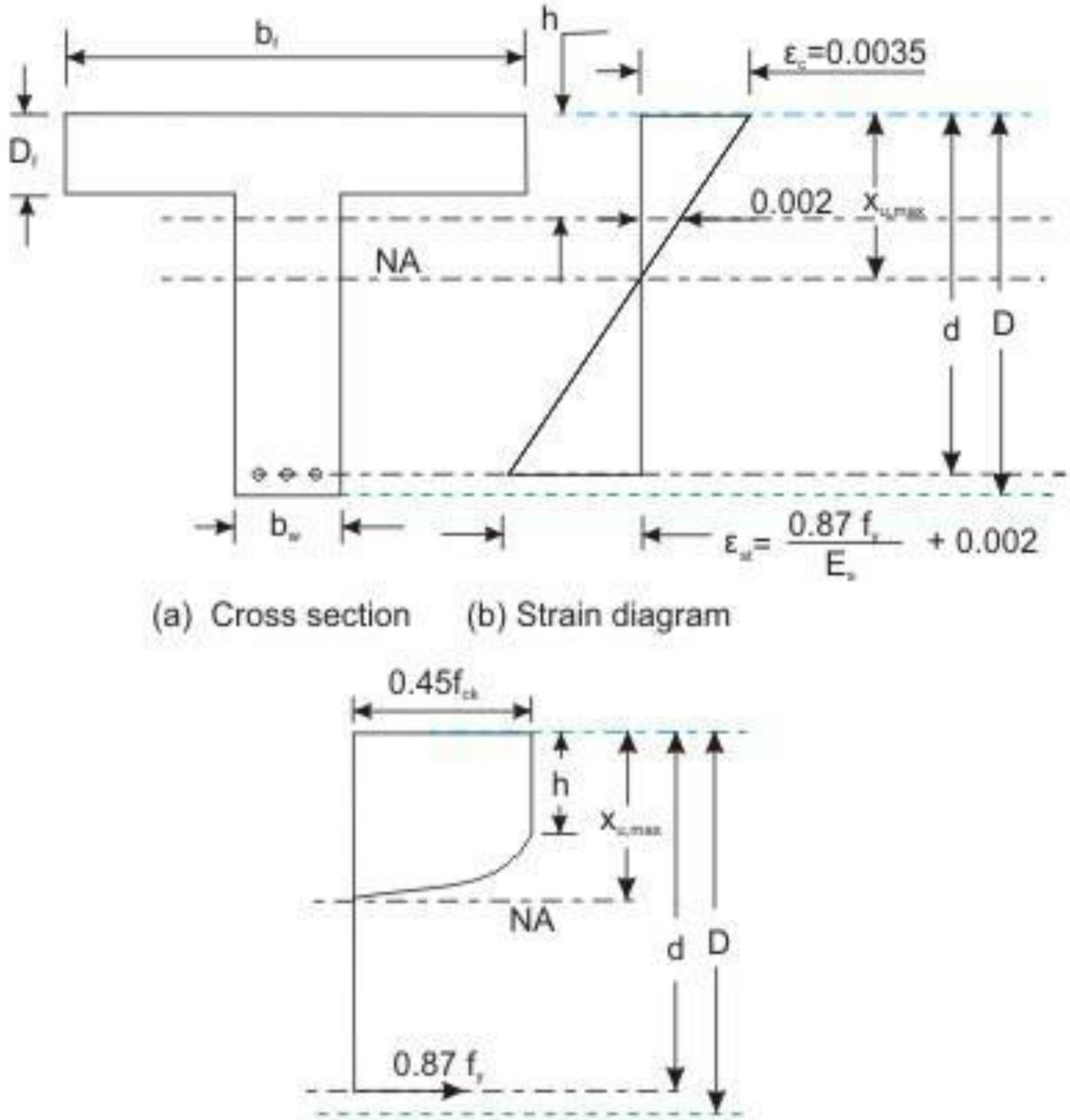


Fig. 5.10.6: T-beam, case (i), when $x_u < D_f$

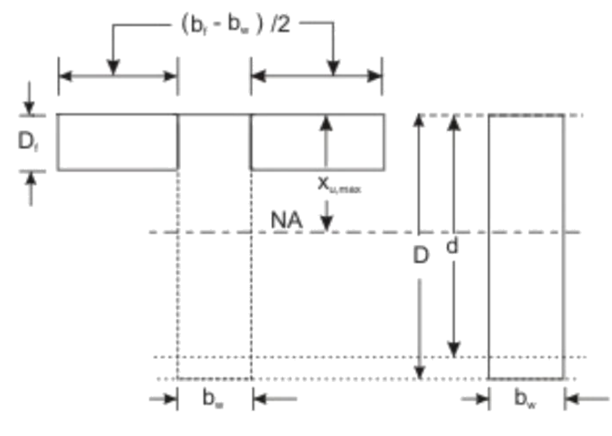
(i) Neutral axis is in the flange ($x_u < D_f$), (Fig. 5.10.6 a to c)



(a) Cross section (b) Strain diagram

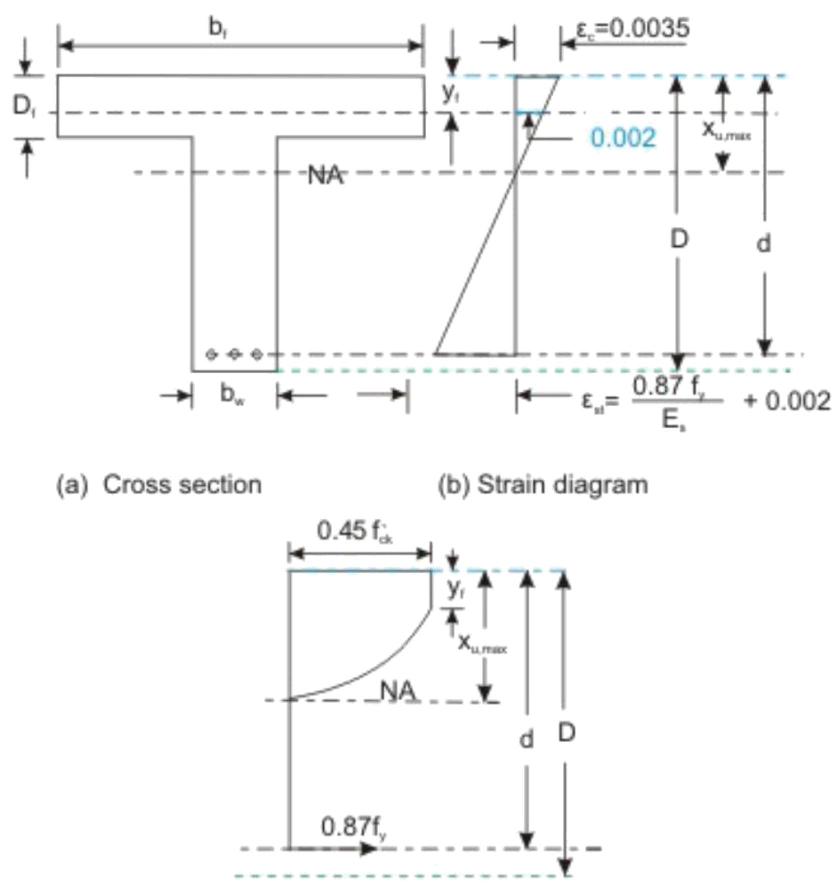
(c) Stress diagram

T-beam, case (ii a), when D_f/d <= 0.2 and balanced $x_{u,max} > D_f$



(d) Flange (e) Web

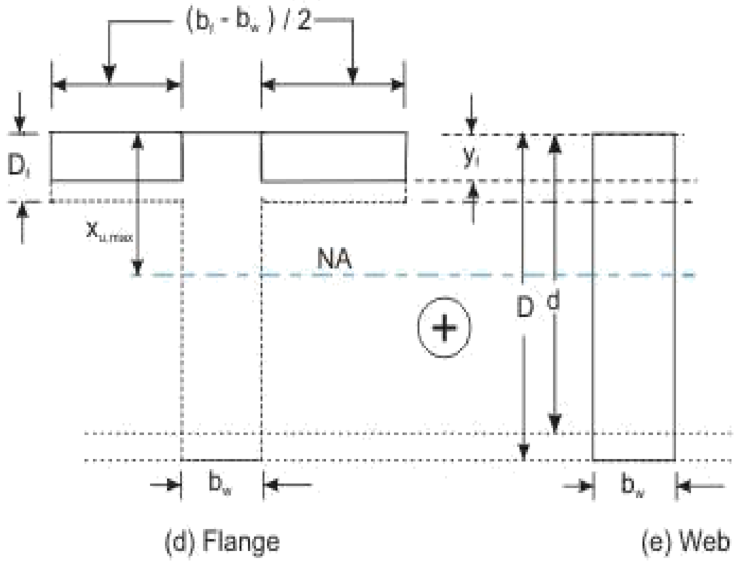
T-beam, case (ii a), when D_f/d <= 0.2 and balanced $x_{u,max} > D_f$



(a) Cross section (b) Strain diagram

(c) Stress diagram

T-beam, case (ii b), when $D_f/d > 0.2$ and balanced $x_{u,max} > D_f$



T-beam, case (ii b), when $D_f/d > 0.2$ and balanced $x_{u,max} > D_f$

1. Neutral axis is in the web and the section is balanced ($x_u = x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a to e)

It has two situations: (a) when D_f/d does not exceed 0.2, the constant stress block is for the entire depth of the flange (Fig. 5.10.7), and
(b) when $D_f/d > 0.2$, the constant stress block is for a part of the depth of flange

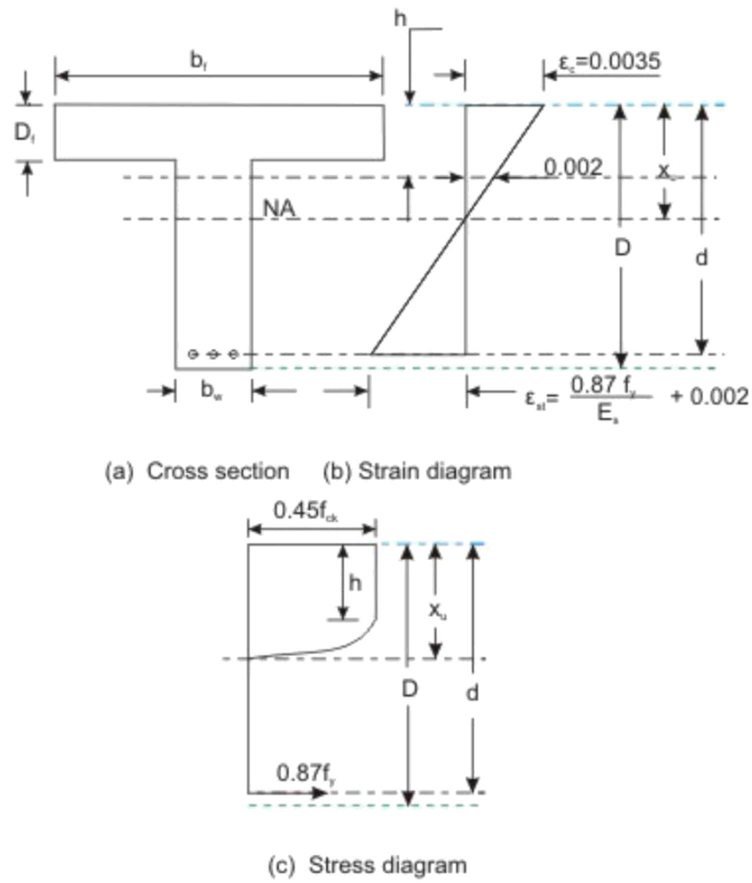


Fig. 5.10.9: T-beam, case (iii a), when $D_f / x_u <= 0.43$ and under-reinforced $x_u > D_f$

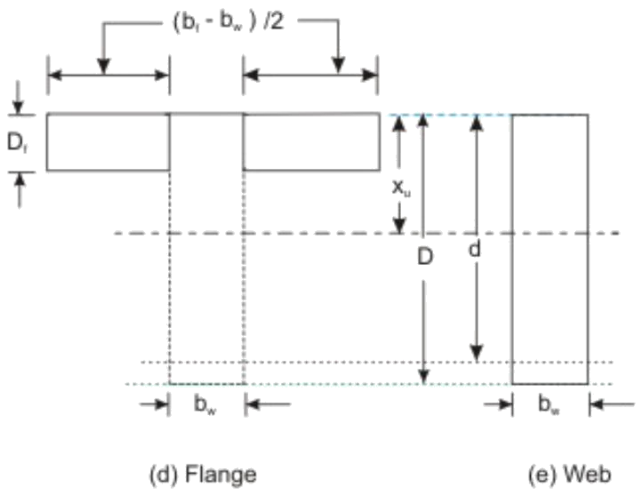


Fig. 5.10.9: T-beam, case (iii a), when $D_f / x_u <= 0.43$ and under-reinforced $x_u > D_f$

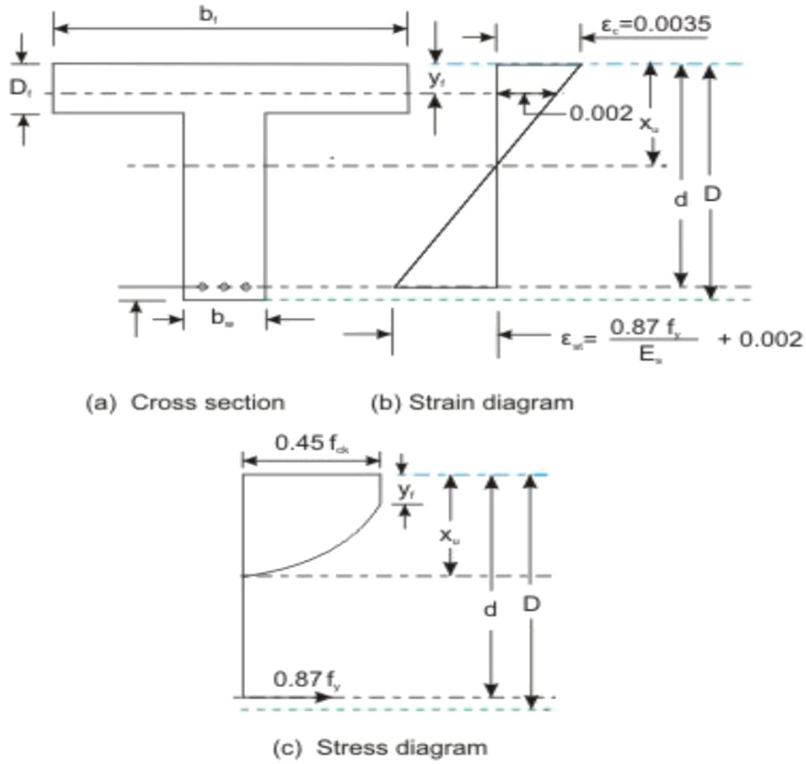


Fig. 5.10.10: T-beam, case (iii b), when $D_f/x_u > 0.43$ and under-reinforced $x_u > D_f$

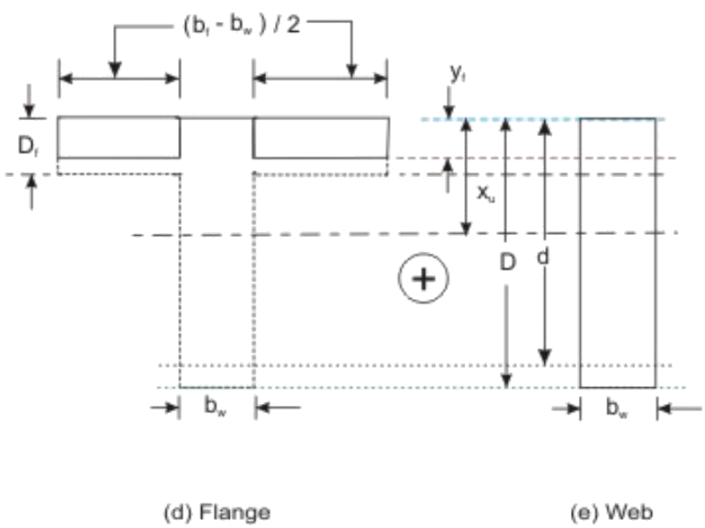


Fig. 5.10.10: T-beam, case (iii b), when $D_f/x_u > 0.43$ and under-reinforced $x_u > D_f$

14. Neutral axis is in the web and the section is under-reinforced ($x_{u,max} > x_u > D_f$), (Figs. 5.10.9 and 10 a to e)

This has two situations: (a) when D_f / x_u does not exceed 0.43, the full depth of flange is having the constant stress (Fig. 5.10.9), and (b) when $D_f / x_u > 0.43$, the constant stress is for a part of the depth of flange

(Fig. 5.10.10).

(i) Neutral axis is in the web and the section is over-reinforced ($x_u > x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a to e)

As mentioned earlier, the value of x_u is then taken as $x_{u,max}$ when $x_u > x_{u,max}$. Therefore, this case also will have two situations depending on D_f /d not exceeding 0.2 or > 0.2 as in (ii) above. The governing equations of the four different cases are now taken up.

Governing Equations

The following equations are only for the singly reinforced *T*-beams.

Additional terms involving $M_{u,lim}$, M_{u2}, A_{sc} , A_{st1} and A_{st2} are to be included from Eqs. 4.1 to 4.8 of sec. 4.8.3 of Lesson 8 depending on the particular case.

Applications of these terms are explained through the solutions of numerical problems of doubly reinforced *T*-beams in Lessons 11 and 12.

Case (i): When the neutral axis is in the flange ($x_u < D_f$), (Figs. 5.10.6 a to c)

Concrete below the neutral axis is in tension and is ignored. The steel reinforcement takes the tensile force (Fig. 5.10.6). Therefore, *T* and *L*-beams are considered as rectangular beams of width b_f and effective depth d. All the equations of singly and doubly reinforced rectangular beams derived in Lessons 4 to 5 and 8 respectively, are also applicable here.

Case (ii): When the neutral axis is in the web and the section is balanced ($x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a to e)

(a) When D_f /d does not exceed 0.2, (Figs. 5.10.7 a to e)

As explained in sec. 5.10.3, the depth of the rectangular portion of the stress block (of constant stress = 0.446 f_{ck}) in this case is greater than D_f (Figs. 5.10.7 a, b and c). The section is split into two parts: (i) rectangular web of width b_w and effective depth d, and (ii) flange of width (b_f - b_w) and depth D_f (Figs. 5.10.7 d and e).

Total compressive force = Compressive force of rectangular beam of width b_w and depth d + Compressive force of rectangular flange of width (b_f - b_w) and depth D_f .

Thus, total compressive force

$$C = 0.36\, f_{ck} b_w x_{u,\,max} + 0.45 f_{ck}\,(b_f - b_w)\, D_f \qquad (5.5)$$

(Assuming the constant stress of concrete in the flange as0.45 f_{ck} in place of 0.446 f_{ck} ,as per G-2.2 of IS 456), and the tensile force

$$T = 0.87 f_y A_{st} \qquad (5.6)$$

The lever arm of the rectangular beam (web part) is (d - 0.42 $x_{u,\,max}$) and the same for the flanged part is (d - 0.5 D_f).

So, the total moment = Moment due to rectangular web part + Moment due to rectangular flange part

or $\quad M_u = 0.36 f_{ck}\, b_w\, x_{u,\,max}\,(d - 0.42\, x_{u,\,max}) + 0.45 f_{ck}\,(b_f - b_w)\, D_f\,(d - D_f/2)$

or $\quad M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45 f_{ck}(b_f - b_w)\, D_f(d - D_f/2) \qquad (5.7)$

Equation 5.7 is given in G-2.2 of IS 456.

(b) When $D_f/d > 0.2$, (Figs. 5.10.8 a to e)

In this case, the depth of rectangular portion of stress block is within the flange (Figs. 5.10.8 a, b and c). It is assumed that this depth of constant stress (0.45 f_{ck}) is y_f, where

$$y_f = 0.15\, x_{u,\,max} + 0.65\, D_f, \text{ but not greater than } D_f \qquad (5.8)$$

The above expression of y_f is derived in sec. 5.10.4.5.

As in the previous case (ii a), when D_f/d does not exceed 0.2, equations of C, T and M_u are obtained from Eqs. 5.5, 6 and 7 by changing D_f to y_f. Thus, we have (Figs. 5.10.8 d and e)

$$C = 0.36\, f_{ck}\, b_w\, x_{u,\,max} + 0.45\, f_{ck}\,(b_f - b_w)\, y_f$$

$T = 0.87 \quad f_y \quad A_{st}$ (5.10)

The lever arm of the rectangular beam (web part) is (d - 0.42 $x_{u,\ max}$ same for the flange part is (d - 0.5 y_f). Accordingly, the expression of follows:

$$M_u = 0.36(x_{u,\ max}/d)\{1 - 0.42(x_{u,\ max}/d)\}\ f_{ck}\ b_w\ d^2 + 0.45 \quad f_{ck}(b_f - b_w)\ y_f(d - y_f/2)$$

(5.11)

Case (iii): When the neutral axis is in the web and the section is under-reinforced ($x_u > D_f$), (Figs. 5.10.9 and 10 a to e)

(a) When D_f / x_u does not exceed 0.43, (Figs. 5.10.9 a to e)

Since D_f does not exceed 0.43 x_u and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the entire flange will be under a constant stress of 0.45 f_{ck} (Figs. 5.10.9 a, b and c). The equations of C, T and M_u can be written in the same manner as in sec. 5.10.4.2, case (ii a). The final forms of the equations are obtained from Eqs. 5.5, 6 and 7 by replacing $x_{u,\ max}$ by x_u. Thus, we have (Figs. 5.10.9 d and e)

$$C = 0.36\ f_{ck} \quad b_w \quad x_u + 0.45 \quad f_{ck}\ (b_f - b_w)\ D_f$$

(5.12)

$$T = 0.87 \quad f_y \quad A_{st}$$

(5.13)

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\}\ f_{ck}\ b_w\ d^2 + 0.45 \quad f_{ck}(b_f - b_w)\ D_f\ (d - D_f/2)$$

(5.14)

(b) When D_f / x_u > 0.43, (Figs. 5.10.10 a to e)

Since D_f > 0.43 x_u and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the part of the flange having the constant stress of 0.45 f_{ck} is assumed as y_f (Fig. 5.10.10 a, b and c). The expressions of y_f , C, T and M_u can be written from Eqs. 5.8, 9, 10 and 11 of sec. 5.10.4.2, case (ii b), by replacing $x_{u,max}$ by x_u. Thus, we have (Fig. 5.10.10 d and e)

$$y_f = 0.15\, x_u + 0.65\, D_f, \text{ but not greater than } D_f$$

(5.15)

$$C = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$$

(5.16)

$$T = 0.87\, f_y\, A_{st}$$

(5.17)

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f\, (d - y_f/2)$$

(5.18)

Case (iv): When the neutral axis is in the web and the section is over-reinforced ($x_u > D_f$), (Figs. 5.10.7 and 8 a to e)

For the over-reinforced beam, the depth of neutral axis x_u is more than $x_{u,\,max}$ as in rectangular beams. However, x_u is restricted up to $x_{u,max}$. Therefore, the corresponding expressions of C, T and Mu for the two situations (a) when D_f / d does not exceed 0.2 and (b) when $D_f / d >$ 0.2 are written from Eqs. 5.5 to 5.7 and 5.9 to 5.11, respectively of sec. 5.10.4.2 (Figs. 5.10.7 and 8). The expression of y_f for (b) is the same as that of Eq. 5.8.

(a) When D_f/d does not exceed 0.2 (Figs. 5.10.7 a to e)

The equations are:

$$C = 0.36\, f_{ck} b_w x_{u,\,max} + 0.45 f_{ck}\,(b_f - b_w)\,D_f \qquad (5.5)$$

$$T = 0.87 f_y A_{st} \qquad (5.6)$$

$$M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, D_f(d - D_f/2) \qquad (5.7)$$

(b) When $D_f/d > 0.2$ (Figs. 5.10.8 a to e)

$$y_f = 0.15\, x_{u,\,max} + 0.65\, D_f, \text{ but not greater than } D_f \qquad (5.8)$$

$$C = 0.36\, f_{ck} b_w x_{u,\,max} + 0.45 f_{ck}\,(b_f - b_w)\,y_f \qquad (5.9)$$

$$T = 0.87 f_y A_{st} \qquad (5.10)$$

$$M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f(d - y_f/2) \qquad (5.11)$$

It is clear from the above that the over-reinforced beam will not have additional moment of resistance beyond that of the balanced one. Moreover, it will prevent steel failure. It is, therefore, recommended either to re-design or to go for doubly reinforced flanged beam than designing over-reinforced flanged beam.

Derivation of the equation to determine y_f, Eq. 5.8, Fig. 5.10.11

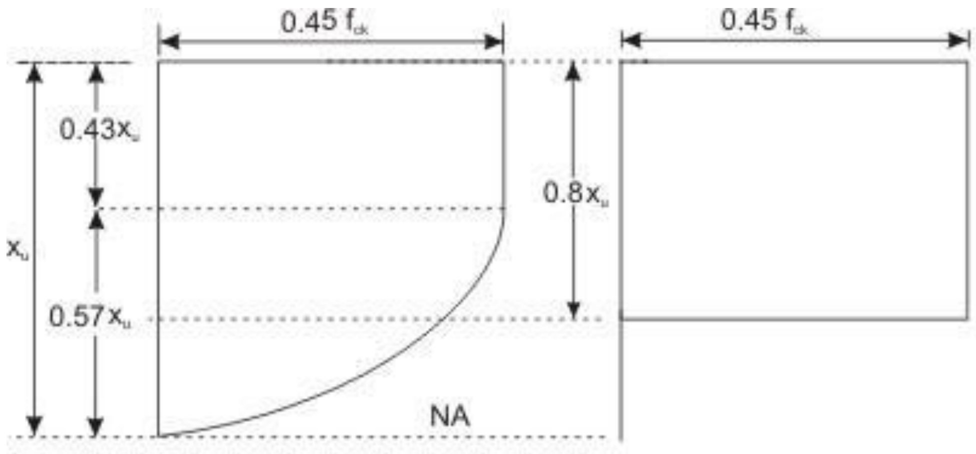


Fig. 5.10.11: (a) IS 456 stress block (b) Whitney's stress block

Whitney's stress block has been considered to derive Eq. 5.8. Figure 5.10.11 shows the two stress blocks of IS code and of Whitney.

y_f = Depth of constant portion of the stress block when $D_f/d > 0.2$. As y_f is a function of x_u and D_f and let us assume

$$y_f = A\ x_u + B\ D_f \tag{5.19}$$

where A and B are to be determined from the following two conditions:

(i) $y_f = 0.43x_u$, when $D_f = 0.43x_u$ (5.20)

(ii) $y_f = 0.8\ x_u$, when $D_f = x_u$ (5.21)

Using the conditions of Eqs. 5.20 and 21 in Eq. 5.19, we get A = 0.15 and B = 0.65. Thus, we have

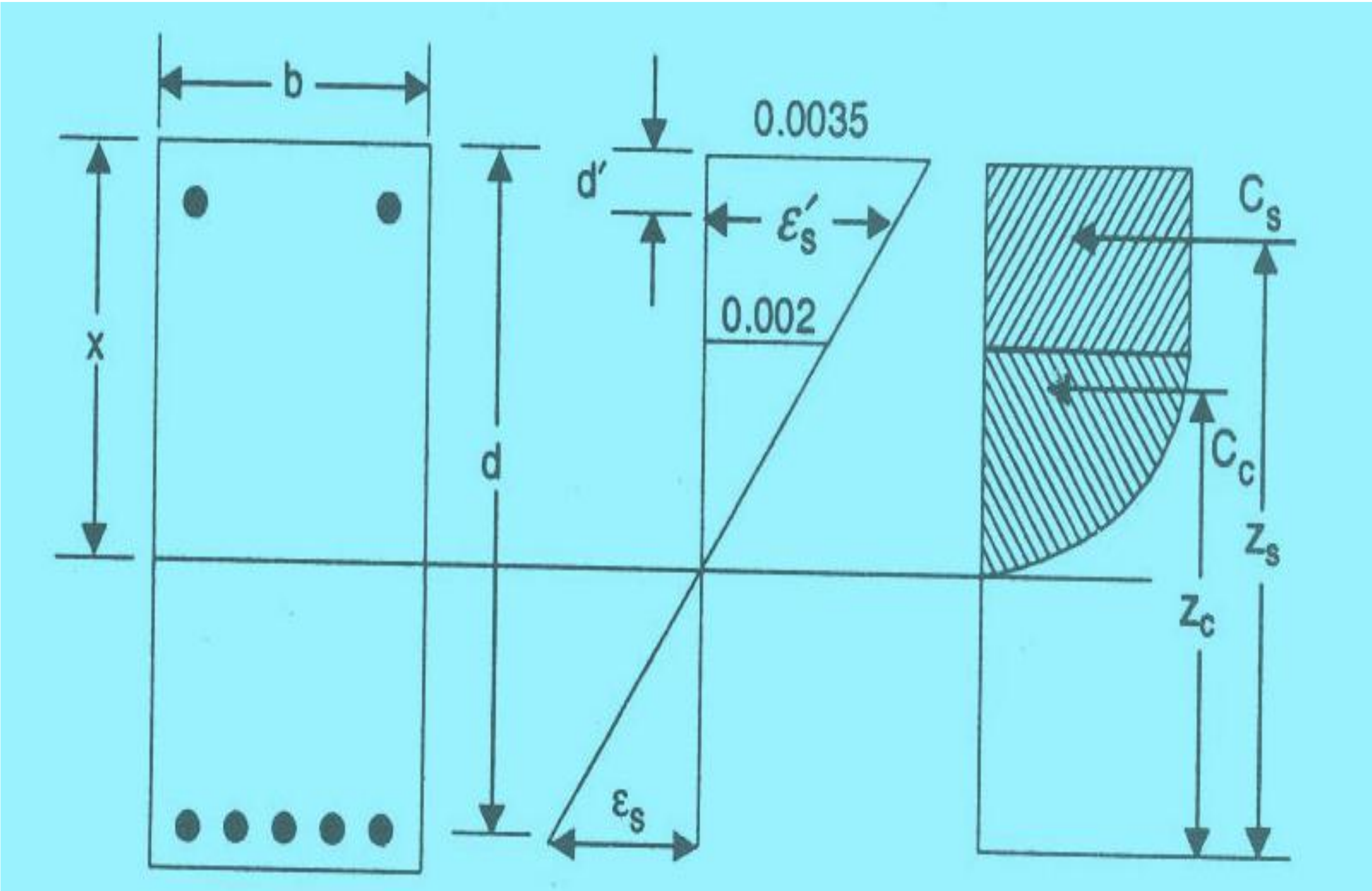
$$y_f = 0.15\ x_u + 0.65\ D_f \tag{5.8}$$

DESIGN OF DOUBLY REINFORCED BEAMS

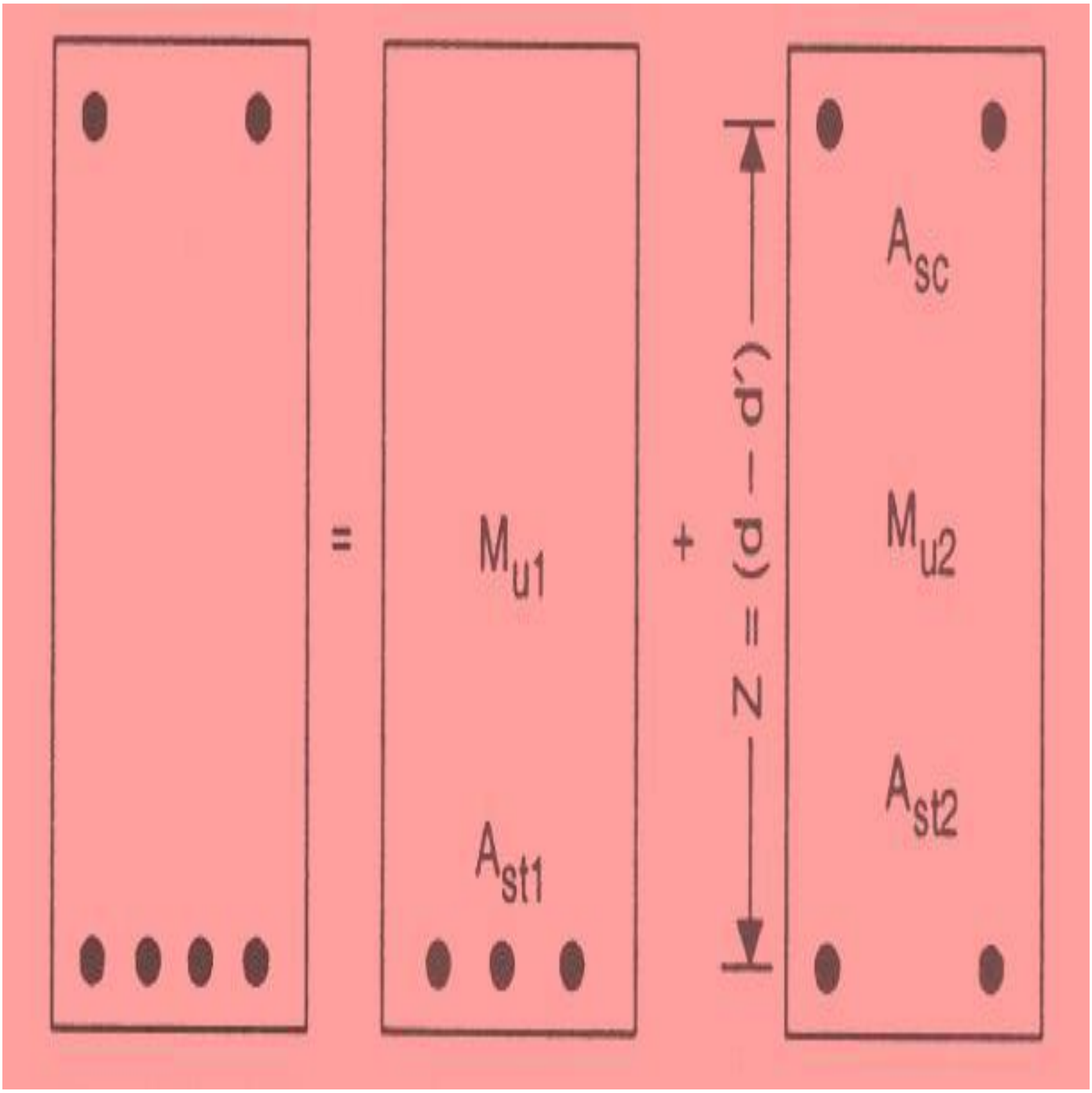
Doubly Reinforced Beams

- ***When beam depth is restricted and the moment the beam has to carry is greater than the moment capacity of the beam in concrete failure.***
- ***When B.M at the section can change sign.***
- ***When compression steel can substantially improve the ductility of beams and its use is therefore advisable in members when larger amount of tension steel becomes necessary for its strength.***
- ***Compression steel is always used in structures in earthquake regions to increase their ductility.***
- ***Compression reinforcement will also aid significantly in reducing the long-term deflections of beams.***

Doubly Reinforced Concrete Beam



Steel Beam Theory



Doubly Reinforced Beams

(iv) **A doubly reinforced concrete beam is reinforced in both compression and tension faces.**

4.8.5 When depth of beam is restricted, strength available from a singly reinforced beam is inadequate.

4.8.6 At a support of a continuous beam, the bending moment changes sign, such a situation may also arise in design of a ring beam.

2 Analysis of a doubly reinforced section involves determination of moment of resistance with given beam width, depth, area of tension and compression steels and their covers.

3 In doubly reinforced concrete beams the compressive force consists of two parts; both in concrete and steel in compression.

4 Stress in steel at the limit state of collapse may be equal to yield stress or less depending on position of the neutral axis.

Design Steps

Determine the limiting moment of resistance M_{um} for the given cross-section using the equation for a singly reinforced beam

$$M_{lim} = 0.87 f_y . A_{st,1} [d - 0.42 x_m] = 0.36\, f_{ck} . b . x_m\, [d - 0.42 x_m]$$

(ii) If the factored moment M_u exceeds M_{lim}, a doubly reinforced section is required $(M_u - M_{lim}) = M_{u2}$

Additional area of tension steel A_{st2} is obtained by considering the equilibrium of force of compression in comp. steel and force of tension T_2 in the additional tension steel

A_{sc} = compression steel.

σ_{cc} = Comp. stress in conc at the level of comp. steel = 0.446fck.

Reasons

(iii) When beam section is shallow in depth, and the flexural strength obtained using balanced steel is insufficient i.e. the factored moment is more than the limiting ultimate moment of resistance of the beam section. Additional steel enhances the moment capacity.

(iv) Steel bars in compression enhances ductility of beam at ultimate strength.

(v) Compression steel reinforcement reduces deflection as moment of inertia of the beam section also increases.

(vi) Long-term deflections of beam are reduced by compression steel.

(vii) Curvature due to shrinkage of concrete are also reduced.

(viii) Doubly reinforced beams are also used in reversal of external load

Examples

(ii) ***A single reinforced rectangular beam is 400mm wide. The effective depth of the beam section is 560mm and its effective cover is 40mm. The steel reinforcement consists of 4 MS 18mm diameter bars in the beam section. The grade of concrete is M20. Locate the neutral axis of the beam section.***

(iii) ***In example 1, the bending moment at a transverse section of beam is 105 kN-m. Determine the strains at the extreme fibre of concrete in compression and steel bars provided as reinforcement in tension. Also determine the stress in steel bars.***

(iv) ***In example 2, the strain in concrete at the extreme fibre in compression ε_{cu} is 0.00069 and the tensile stress in bending in steel is 199.55 N/mm^2. Determine the depth of neutral axis and the moment of resistance of the beam section.***

(v) ***Determine the moment of resistance of a section 300mm wide and 450mm deep up to the centre of reinforcement. If it is reinforced with (i) 4-12mm fe415 grade bars, (ii) 6-18mm fe415 grade bars.***

(i) **A rectangular beam section is 200mm wide and 400mm deep up to the centre of reinforcement. Determine the reinforcement required at the bottom if it has to resist a factored moment of 40kN-m. Use M20 grade concrete and fe415 grade steel.**

(ii) **A rectangular beam section is 250mm wide and 500mm deep up to the centre of tension steel which consists of 4-22mm dia. bars. Find the position of the neutral axis, lever arm, forces of compression and tension and safe moment of resistance if concrete is M20 grade and steel is Fe500 grade.**

(iii) **A rectangular beam is 200mm wide and 450 mm overall depth with an effective cover of 40mm. Find the reinforcement required if it has to resist a moment of 35 kN.m. Assume M20 concrete and Fe250 grade steel.**

Limit State of Serviceability

- explain the need to check for the limit state of serviceability after designing the structures by limit state of collapse,
- differentiate between short- and long-term deflections,
- state the influencing factors to both short- and long-term deflections,
- select the preliminary dimensions of structures to satisfy the requirements as per IS 456,
- calculate the short- and long-term deflections of designed beams.

Introduction

Structures designed by limit state of collapse are of comparatively smaller sections than those designed employing working stress method. They, therefore, must be checked for deflection and width of cracks. Excessive deflection of a structure or part thereof adversely affects the appearance and efficiency of the structure, finishes or partitions. Excessive cracking of concrete also seriously affects the appearance and durability of the structure. Accordingly, cl. 35.1.1 of IS 456 stipulates that the designer should consider all relevant limit states to ensure an adequate degree of safety and serviceability. Clause 35.3 of IS 456 refers to the limit state of serviceability comprising deflection in cl. 35.3.1 and cracking in cl. 35.3.2. Concrete is said to be durable when it performs satisfactorily in the working environment during its anticipated exposure conditions during service. Clause 8 of IS 456 refers to the durability aspects of concrete. Stability of the structure against overturning and sliding (cl. 20 of IS

456), and fire resistance (cl. 21 of IS 456) are some of the other importance issues to be kept in mind while designing reinforced concrete structures.

This lesson discusses about the different aspects of deflection of beams and the requirements as per IS 456. In addition, lateral stability of beams is also taken up while selecting the preliminary dimensions of beams. Other requirements, however, are beyond the scope of this lesson.

Short- and Long-term Deflections

As evident from the names, short-term deflection refers to the immediate deflection after casting and application of partial or full service loads, while the long-term deflection occurs over a long period of time largely due to shrinkage and creep of the materials. The following factors influence the short-term deflection of structures:

(v) magnitude and distribution of live loads,

(vi) span and type of end supports,
(vii) cross-sectional area of the members,
(viii) amount of steel reinforcement and the stress developed in the reinforcement,
(ix) characteristic strengths of concrete and steel, and
(x) amount and extent of cracking.

The long-term deflection is almost two to three times of the short-term deflection. The following are the major factors influencing the long-term deflection of the structures.

4.8.7 humidity and temperature ranges during curing,

4.8.8 age of concrete at the time of loading, and
(c) type and size of aggregates, water-cement ratio, amount of compression reinforcement, size of members etc., which influence the creep and shrinkage of concrete.

Control of Deflection

Clause 23.2 of IS 456 stipulates the limiting deflections under two heads as given below:

5 The maximum final deflection should not normally exceed span/250 due to all loads including the effects of temperatures, creep and shrinkage and measured from the as-cast level of the supports of floors, roof and all other horizontal members.

6 The maximum deflection should not normally exceed the lesser of span/350 or 20 mm including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes.

It is essential that both the requirements are to be fulfilled for every structure.

Selection of Preliminary Dimensions

The two requirements of the deflection are checked after designing the members. However, the structural design has to be revised if it fails to satisfy any one of the two or both the requirements. In order to avoid this, IS 456 recommends the guidelines to assume the initial dimensions of the members which will generally satisfy the deflection limits. Clause 23.2.1 stipulates different span to effective depth ratios and cl. 23.3 recommends limiting slenderness of

beams, a relation of *b* and *d* of the members, to ensure lateral stability. They are given below:

(A) For the deflection requirements

Different basic values of span to effective depth ratios for three different support conditions are prescribed for spans up to 10 m, which should be modified under any or all of the four different situations: (i) for spans above 10 m, (ii) depending on the amount and the stress of tension steel reinforcement, (iii) depending on the amount of compression reinforcement, and (iv) for flanged beams. These are furnished in Table 7.1.

(B) For lateral stability

The lateral stability of beams depends upon the slenderness ratio and the support conditions. Accordingly cl. 23.3 of IS code stipulates the following:

4.8.4 For simply supported and continuous beams, the clear distance between the lateral restraints shall not exceed the lesser of 60*b* or $250b^2/d$, where *d* is the effective depth and *b* is the breadth of the compression face midway between the lateral restraints.

4.8.5 For cantilever beams, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed the lesser of 25*b* or $100b^2/d$.

Table 7.1 Span/depth ratios and modification factors

Sl. No.	Items	Cantilever	Simply supported	Continuous
1	Basic values of span to effective depth ratio for spans up to 10 m	7	20	26
2	Modification factors for spans > 10 m	Not applicable as deflection calculations are to be done.	Multiply values of row 1 by 10/span in metres.	
3	Modification factors depending on area and stress of steel	Multiply values of row 1 or 2 with the modification factor from Fig.4 of IS 456.		
4	Modification factors depending as area of compression steel	Further multiply the earlier respective value with that obtained from Fig.5 of IS 456.		
5	Modification factors for flanged beams	(i)Modify values of row 1 or 2 as per Fig.6 of IS 456. (ii)Further modify as per row 3 and/or 4 where reinforcement percentage to be used on area of section equal to $b_f\, d$.		

Calculation of Short-Term Deflection

Clause C-2 of Annex C of IS 456 prescribes the steps of calculating the short-term deflection. The code recommends the usual methods for elastic deflections using the short-term modulus of elasticity of concrete E_c and effective moment of inertia I_{eff} given by the following equation:

$$I_{eff} = \frac{I_r}{1.2 - (M_r/M)(z/d)(1 - x/d)(b_w/b)};\text{but } I_r \le I_{eff} \le I_{gr}$$

(7.1)

where I_r =moment of inertia of the cracked section,

M_r = cracking moment equal to (f_{cr} I_{gr})/y_t , where f_{cr} is the modulus of rupture of concrete, I_{gr} is the moment of inertia of the gross section about the centroidal axis neglecting the reinforcement, and y_t is the distance from centroidal axis of gross section, neglecting the reinforcement, to extreme fibre in tension,

M = maximum moment under service loads,

z = lever arm,

x = depth of neutral axis,

d = effective depth,

b_w = breadth of web, and

b = breadth of compression face.

For continuous beams, however, the values of I_r, I_{gr} and M_r are to be modified by the following equation:

$$X_e = k_1 \frac{X_1 + X_2}{2} + (1 - k_1) X_o$$

(7.2)

where X_e = modified value of X,

X_1, X_2 = values of X at the supports,

X_o = value of X at mid span,

k_1 = coefficient given in Table 25 of IS 456 and in Table 7.2 here, and

X = value of I_r, I_{gr} or M_r as appropriate.

Table 7.2 Values of coefficient k_1

k_1	0.5 or less	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
k_2	0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97	1.0

Note: k_2 is given by $(M_1 + M_2)/(M_{F1} + M_{F2})$, where M_1 and M_2 = support moments, and M_{F1} and M_{F2} = fixed end moments.

Deflection due to Shrinkage

Clause C-3 of Annex C of IS 456 prescribes the method of calculating the deflection due to shrinkage α_{cs} from the following equation:

$$\alpha_{cs} = k_3 \psi_{cs} l^2 \qquad (7.3)$$

where k_3 is a constant which is 0.5 for cantilevers, 0.125 for simply supported members, 0.086 for members continuous at one end, and 0.063 for fully continuous members; ψ_{cs} is shrinkage curvature equal to $k4\ \varepsilon_{cs}/D$ where ε_{cs} is the ultimate shrinkage strain of concrete. For ε_{cs}, cl. 6.2.4.1 of IS 456 recommends an approximate value of 0.0003 in the absence of test data.

$$k_4 = 0.72(p_t - p_c)/\sqrt{p_t} \leq 1.0, \text{ for } 0.25 \leq p_t - p_c < 1.0$$
$$= 0.65(p_t - p_c)/\sqrt{p_t} \leq 1.0, \text{ for } p_t - p_c \geq 1.0 \qquad (7.4)$$

where $p_t = 100A_{st}/bd$ and $p_c = 100A_{sc}/bd$, D is the total depth of the section, and l is the length of span.

Deflection Due to Creep

Clause C-4 of Annex C of IS 456 stipulates the following method of calculating deflection due to creep. The creep deflection due to permanent loads

$\alpha_{cc(perm)}$ is obtained from the following equation:

$$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(perm)}$$

(7.5)

where $\alpha_{1cc(perm)}$ =initial plus creep deflection due to permanent loads obtained usinganelasticanalysiswithaneffectivemodulusof elasticity,

$E_{ce} = E_c / (1 + \theta)$, θ being the creep coefficient, and

$\alpha_{1(perm)}$ = short-term deflection due to permanent loads using E_c.

(iii) Numerical Problems

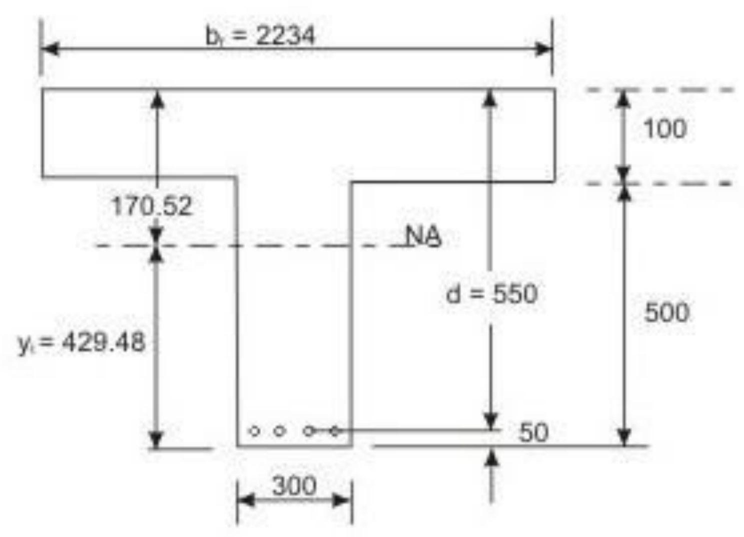


Fig. 7.17.1: Problem 1 (uncracked section)

Example Problem 1:

Figures 7.17.1 and 2 present the cross-section and the tensile steel of a simply supported *T*-beam of 8 m span using M 20 and Fe 415 subjected to dead

load of 9.3 kN/m and imposed loads of 10.7 kN/m at service. Calculate the short-and long-term deflections and check the requirements of IS 456.

Solution 1:

Step 1: Properties of plain concrete section

Taking moment of the area about the bottom of the beam

$$y_t = \frac{(300)(600)(300) + (2234 - 300)(100)(550)}{(300)(600) + (2234 - 300)(100)} = 429.48 \text{ mm}$$

$$I_{gr} = \frac{300(429.48)^3}{3} + \frac{2234(170.52)^3}{3} - \frac{1934(70.52)^3}{3} = (11.384)\,(10)^9 \ \text{mm}^4$$

This can also be computed from SP-16 as explained below:

Here, $b_f / b_w = 7.45$, $D_f / D = 0.17$. Using these values in chart 88 of SP-16, we get $k_1 = 2.10$.

$$I_{gr} = k_1 b_w D^3/12 = (2.10)(300)(600)^3/12 = (11.384)(10)^9 \ \text{mm}^4$$

Step 2: Properties of the cracked section (Fig.7.17.2)

$M_r = f_{cr}\, I_{gr} / y_t = 3.13(11.384)(10)^9/429.48 = 82.96$ kNm

$E_s = 200000$ N/mm^2

$E_c = 5000 \sqrt{f_{ck}}$ (cl. 6.2.3.1 of IS 456) $= 22360.68$ N/mm^2

$m = E_s / E_c = 8.94$

Taking moment of the compressive concrete and tensile steel about the neutral axis, we have (Fig.7.17.2)

$b_f\, x^2/2 \;=m\, A_{st}\,(d - x)$ gives $(2234)(x^2/2)=(8.94)(1383)(550 - x)$

or $x^2 + 11.07\,x - 6087.92 = 0$ .Solving the equation, we get x = 72.68 mm.

z =lever arm= $d - x/3$ =525.77 mm

$$I_r = \frac{2234(72.68)^3}{3} + 8.94(1383)(550 - 72.68)^2 = 3.106(10)^9 \text{ mm}^4$$

$$M = wl^2/8 = (9.3 + 10.7)(8)(8)/8 = 160\text{kNm}$$

$$I_{eff} = \frac{I_r}{1.2 - \frac{M_r}{M}\frac{z}{d}(1 - \frac{x}{d})(\frac{b_w}{b})} \quad \ldots. \quad \text{(Eq. 7.1)}$$

$$= \frac{I_r}{1.2 - (\frac{82.96}{160})(\frac{525.77}{550})(1 - \frac{72.68}{550})(\frac{300}{2234})} = 0.875\, I_r\text{. But } I_r \le I_{eff} \le I_{gr}$$

So, $Ieff = I_r = 3.106(10)^9$ mm^4.

Step 3: Short-term deflection (sec. 7.17.5)

$E_c = 5000\sqrt{f_{ck}}$ (cl. 6.2.3.1 of IS 456) = 22360.68 N/mm^2

Short-term deflection = $(5/384)\, wl^4/E_cI_{eff}$

= $(5)(20)(8)^4(10^{12})/(384)(22360.68)(3.106)(10^9)$ = 15.358 mm

(1)

Step 4: Deflection due to shrinkage (sec. 7.17.6)

$k_4 = 0.72(\,p_t - p_c\,)/p_t = 0.72(0.84)0.84 = 0.6599$

$\psi_{cs} = k_4\, \varepsilon_{cs} / D = (0.6599)(0.0003)/600 = 3.2995(10)^{-7}$

k_3 =0.125 (from sec. 7.17.6)

$\alpha_{cs} = k_3\, \psi_{cs}\, l^2$ (Eq. 7.3) = (0.125)(3.2995)$(10)^{-7}$(64)(10^6) = 2.64 mm (2)

Step 5:Deflection due to creep (sec. 7.17.7)

Equation 7.5 reveals that the deflection due to creep $\alpha_{cc(perm)}$ can be obtained after calculating $\alpha_{1cc(perm)}$ and $\alpha_{1(perm)}$. We calculate $\alpha_{1cc(perm)}$ in the next step.

Step 5a:Calculation of $\alpha_{1cc(perm)}$

Assuming the age of concrete at loading as 28 days, cl. 6.2.5.1 of IS 456 gives θ = 1.6. So, $E_{cc} = E_c/(1 + \theta)$ = 22360.68/(1 + 1.6) = 8600.2615 N/mm^2 and $m = E_s/E_{cc}$ = 200000/8600.2615 = 23.255

Step 5b: Properties of cracked section

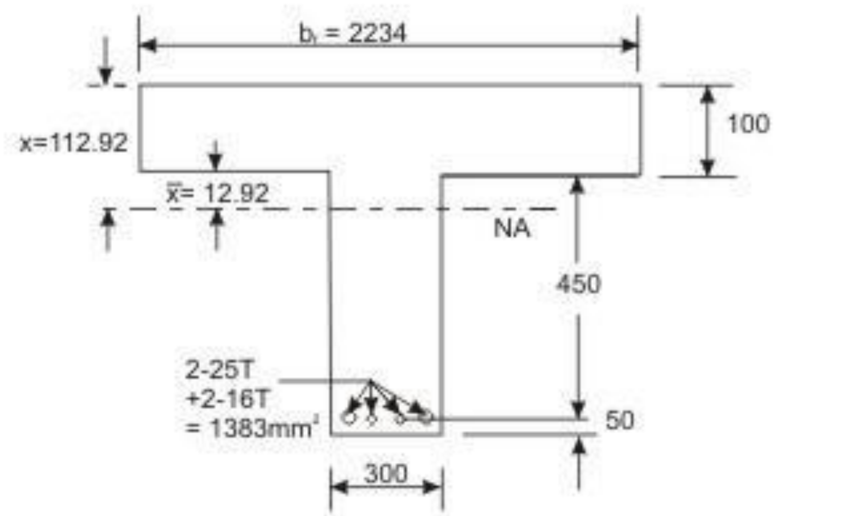


Fig. 7.17.3: Problem 1 (cracked section, $E_c = E_{cc}$)

Taking moment of compressive concrete and tensile steel about the neutral axis (assuming at a distance of $\bar{x}$ from the bottom of the flange as shown in Fig.7.17.3):

$$2234(100)(50 + \bar{x}) = (23.255)(1383)(450 - \bar{x}) \text{ or } \bar{x} = 12.92 \text{ mm}$$

which gives x=112.92 mm. Accordingly, z=leverarm $= d - x/3 =$ 512.36mm.

I_r=2234(100)^3/12+ 2234(100)(62.92)^2 + 23.255(1383)(550 – 112.92)^2

+ 300(12.92)^3/3 =7.214(10^9)mm^4

M_r = 82.96 kNm (see Step 2)

M = w_{perm} l^2/8 = 9.3(8)(8)/8 = 74.4 kNm.

$$I_{eff} = \frac{I_r}{(1.2) - \left(\frac{82.96}{74.4}\right)\left(\frac{512.36}{550}\right)\left(1-\frac{112.92}{550}\right)\left(\frac{300}{2234}\right)} = 0.918\, I_r$$

However, to satisfy $I_r \le I_{eff} \le I_{gr}$, I_{eff} should be equal to I_{gr}. So, I_{eff} = I_{gr} = 11.384(10^9). For the value of I_{gr} please see Step 1.

Step 5c:Calculation of $\alpha_{1cc(perm)}$

$\alpha_{1cc(perm)}$ = 5wl^4/384(E_{cc})(I_{eff}) = 5(9.3)(8)^4(10)^{12}/384(8600.2615)(11.384)(10^9)

=5.066mm

(3)

Step 5d:Calculation of $\alpha_{1(perm)}$

$\alpha_{1(perm)}$ = 5wl^4/384(E_c)(I_{eff}) = 5(9.3)(8)^4(10)^{12}/384(22360.68)(11.384)(10^9)

= 1.948 mm

(4)

Step 5e: Calculation of deflection due to creep

$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(perm)}$

$= 5.066 – 1.948 = 3.118$ mm

(5)

It is important to note that the deflection due to creep $\alpha_{cc(perm)}$ can be obtained even without computing $\alpha_{1cc(perm)}$. The relationship of $\alpha_{cc(perm)}$ and

is given below.

$$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(perm)}$$

$$= \{5wl^4/384(Ec)(Ieff)\}\{(Ec/Ecc) - 1\} = \alpha_{1(perm)}(\theta)$$

Hence, the deflection due to creep, for this problem is:

$$\alpha_{cc(perm)} = \alpha_{1(perm)}(\theta) = 1.948(1.6) = 3.116 \text{ mm}$$

Step 6: Checking of the requirements of IS 456

The two requirements regarding the control of deflection are given in sec. 7.17.3. They are checked in the following:

Step 6a: Checking of the first requirement

The maximum allowable deflection $= 8000/250 = 32$ mm

The actual final deflection due to all loads

(ix) 15.358 (see Eq.1 of Step 3) + 2.64 (see Eq.2 of Step 4)

+ 3.118 (see Eq.5 ofStep 5e) =21.116 mm<32mm. Hence,

o.k.

Step 6b: Checking of the second requirement

The maximum allowable deflection is the lesser of span/350 or 20 mm. Here, span/350 = 22.86 mm. So, the maximum allowable deflection = 20 mm. The actual final deflection = 1.948 (see Eq.4 of Step 5d) + 2.64 (see Eq.2 of Step

4) + 3.118 (see Eq.5 of step 5e) $= 7.706$ mm $<$ 20 mm. Hence, o.k.

Thus, both the requirements of cl.23.2 of IS 456 and as given in sec. 7.17.3 are satisfied.

Calculation of deflection

Step 1: Properties of concrete section

y_t = $D/2$ = 300 mm, I_{gr} = $bD^3/12$ = $300(600)^3/12$ = $5.4(10^9)$ mm^4

Step 2: Properties of cracked section

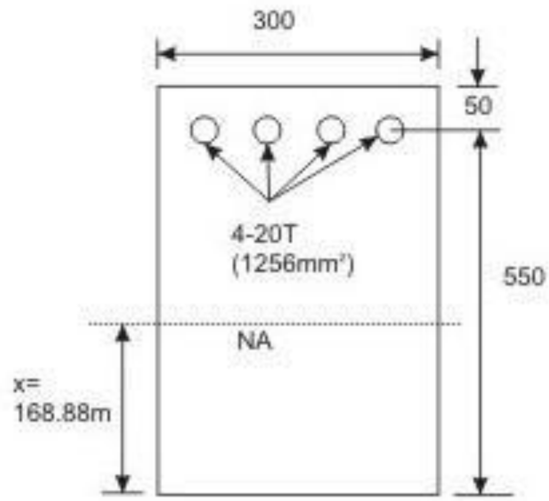


Fig. 7.17.5: TQ. 3 (cracked section, E_c=E_c)

fcr = 0.720$\sqrt{}$ (cl. 6.2.2 of IS 456)=3.13 N/mm^2

y_t =300 mm

M_r = fcr Igr $/yt$ =3.13(5.4)(10^9)/300=5.634(10^7)Nmm

E_s= 200000N/mm^2

E_c = 5000 $\sqrt{f_{ck}}$(cl. 6.2.3.1 of IS 456)= 22360.68N/mm^2

m= E_s /E_c= 8.94

Taking moment of the compressive concrete and tensile steel about the neutral axis (Fig.7.17.5):

$300\,x^2/2 = (8.94)(1256)(550 - x)$ or $x^2 + 74.86\,x - 41171.68 = 0$

This gives $x = 168.88$ mm and $z = d - x/3 = 550 - 168.88/3 = 493.71$ mm.

$I_r = 300(168.88)^3/3 + 8.94(1256)(550 - 168.88)^2 = 2.1126(10^9)\text{mm}^4$

$M = wl^2/2 = 20(4)(4)/2 = 160\text{kNm}$

$$I_{eff} = \frac{I_r}{(1.2) - \left(\frac{5.634}{16}\right)\left(\frac{493.71}{550}\right)\left(1 - \frac{168.88}{550}\right)(1)} = 1.02\, I_r = 2.1548\,(10^9)\ \text{mm}^4$$

This satisfies $I_r \le I_{eff} \le I_{gr}$. So, $I_{eff} = 2.1548(10^9)\text{mm}^4$.

Step 3: Short-term deflection (sec. 7.17.5)

$E_c = 22360.68$ N/mm^2 (cl. 6.2.3.1 of IS 456) Short-term deflection = $wl^4/8E_cI_{eff}$

$= 20(4^4)(10^{12})/8(22360.68)(2.1548)(10^9) = 13.283$ mm

So, short-term deflection = 13.283 mm (1)

Step 4: Deflection due to shrinkage (sec. 7.17.6)

$k_4 = 0.72(0.761)/\sqrt{0.761} = 0.664$

$\psi_{cs} = k_4\ \varepsilon_{cs} / D = (0.664)(0.0003)/600 = 3.32(10)^{-7}$

$k_3 = 0.5$ (from sec. 7.17.6)

$\alpha_{cs} = k_3\, \psi_{cs}\, l^2 = (0.5)(3.32)(10)^{-7}(16)(10^6) = 2.656$ mm

(2)

Step 5: Deflection due to creep (sec. 7.17.7)

Step 5a: Calculation of $\alpha_{1cc(perm)}$

Assuming the age of concrete at loading as 28 days, cl. 6.2.5.1 of IS 456

gives

$\theta = 1.6$

So, $E_{cc} = E_c/(1+\theta) = 8600.2615$ N/mm^2

$m = E_s/E_{cc} = 200000/8600.2615 = 23.255$

Step 5b: Properties of cracked section

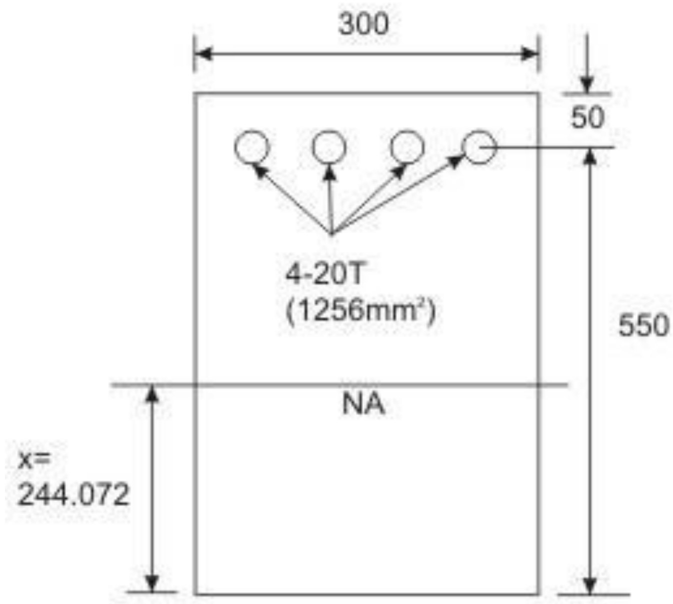


Fig. 7.17.6: TQ. 3 (cracked section, E_c=E_{cc})

From Fig.7.17.6, taking moment of compressive concrete and tensile steel about the neutral axis, we have:

$300\,x^2/2=(23.255)(1256)(550 - x)$

or $x^2 + 194.72\,x - 107097.03=0$

solving we get x=244.072 mm

$z=d - x/3=468.643$ mm

$I_r=300(244.072)^3/3+(23.255)(1256)(550 - 468.643)^2$

$=1.6473(10)^9$ mm^4

M_r = 5.634(10^7) Nmm (see Step 2)

$M = w_{perm}\, l^2/2 = 4.5(4^2)/2 = 36$ kNm

$$I_{eff} = \frac{I_r}{1.2 - \left(\frac{5.634}{3.6}\right)\left(\frac{468.643}{550}\right)\left(1 - \frac{244.072}{550}\right)(1)} = 2.1786\, I_r = 3.5888(10^9)\ \text{mm}^4$$

Since this satisfies $I_r \le I_{eff} \le I_{gr}$, we have, I_{eff} = 3.5888(10^9) mm^4. For the value of I_{gr} please see Step 1.

Step 5c: Calculation of $\alpha_{1cc(perm)}$

$\alpha_{1cc(perm)} = (w_{perm})(l^4)/(8E_{cc}\, I_{eff}) = 4.5(4)^4(10)^{12}/8(8600.2615)(3.5888)(10^9)$

$= 4.665$ mm

(3)

Step 5d: Calculation of $\alpha_{1(perm)}$

$\alpha_{1(perm)} = (w_{perm})(l^4)/(8E_c\, I_{eff}) = 4.5(4)^4(10)^{12}/8(22360.68)(3.5888)(10^9)$

$= 1.794$ mm

(4)

Step 5e: Calculation of deflection due to creep

$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(perm)}$

$= 4.665 - 1.794 = 2.871$ mm

(5)

Moreover: $\alpha_{cc(perm)} = \alpha_{1cc(perm)}\,(\theta)$ gives $\alpha_{cc(perm)}$ = 1.794(1.6) = 2.874 mm.

Step 6: Checking of the two requirements of IS 456

Step 6a: First requirement

Maximum allowable deflection = 4000/250 = 16 mm

The actual deflection = 13.283 (Eq.1 of Step 3) + 2.656 (Eq.2 of Step 4)

+ 2.871 (Eq.5 of Step 5e) = 18.81 > Allowable 16 mm.

Step 6b: Second requirement

The allowable deflection is lesser of span/350 or 20 mm. Here, span/350 = 11.428 mm is the allowable deflection. The actual deflection = 1.794 (Eq.4 of Step 5d) + 2.656 (Eq.2 of Step 4) + 2.871 (Eq.5 of step 5e) = 7.321 mm < 11.428 mm.

Ex.3: Determine the moment of resistance of the beam of Fig. 5.11.4 when A_{st} = 2,591 mm^2 (4-25 T and 2- 20 T). Other parameters are the same as those of Ex.1: b_f = 1,000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u to be in the flange and the beam is under-reinforced, we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, b_f f_{ck}} = \frac{0.87\,(415)\,(2591)}{0.36\,(1000)\,(20)} = 129.93 \text{ mm} > 100 \text{ mm}$$

Since $x_u > D_f$, the neutral axis is in web.Here, D_f/d=100/450=0.222>0.2.
So, we have to substitute the term y_f from Eq. 5.15 of Lesson 10, assuming $D_f / x_u >$ 0.43 in the equation of $C = T$ from Eqs. 5.16 and 17 ofsec. 5.10.4.3 b of Lesson 10. Accordingly, we get:

$$0.36\, f_{ck} b_w x_u + 0.45 f_{ck}\, (b_f - b_w)\, y_f = 0.87\, f_y\, A_{st}$$

or $$0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\, x_u + 0.65\,(100)\}$$

$$= 0.87\,(415)\,(2591)$$

or x_u =169.398mm<216 mm ($x_{u,max}$ = 0.48 x_u = 216 mm)

So, the section is under-reinforced.

Step 2: To determine M_u

$$D_f / x_u = 100/169.398 = 0.590 > 0.43$$

This is the problem of case (iii b) of sec. 5.10.4.3 b. The corresponding equations are Eq. 5.15 of Lesson 10 for y_f and Eqs. 5.16 to 18 of Lesson 10 for C, T and M_u, respectively. From Eq. 5.15 of Lesson 10, we have:

$$y_f = 0.15\, x_u + 0.65 \quad D_f = 0.15\,(169.398) + 0.65\,(100) = 90.409 \text{ mm}$$

From Eq. 5.18 of Lesson 10, we have

$$M_u = 0.36(x_u /d)\{1 - 0.42(x_u /d)\}\, f_{ck}\, b_w\, d^2 + 0.45 \quad f_{ck}(b_f - b_w)\, y_f\, (d - y_f /2)$$

or $$M_u = 0.36\,(169.398/450)\,\{1 - 0.42\,(169.398/450)\}\,(20)\,(300)\,(450)\,(450)$$

$$\text{(iii)} \quad 0.45\,(20)\,(1000 - 300)\,(90.409)\,(450 - 90.409/2)$$

7 138.62 + 230.56=369.18kNm.

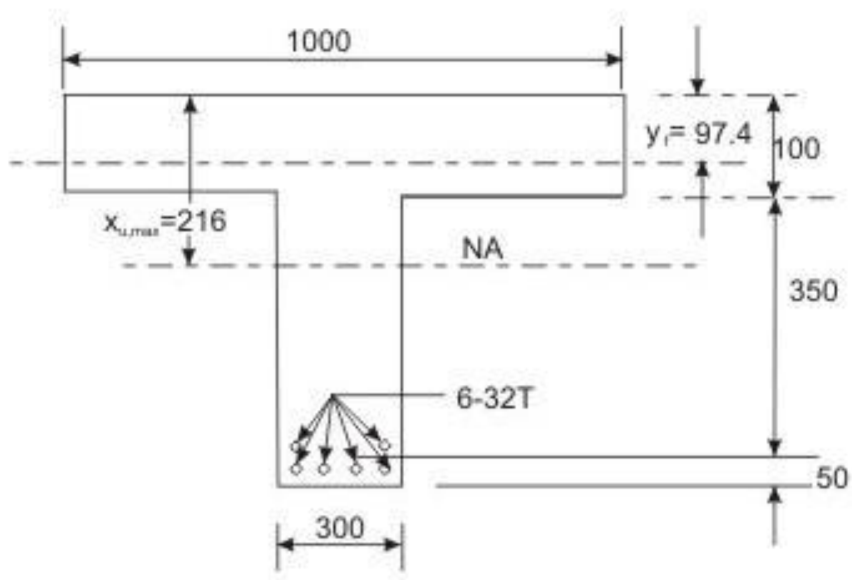


Fig. 5.11.5: Example 4, case (iv b)

Ex.4: Determine the moment of resistance of the flanged beam of Fig. 5.11.5 with A_{st} = 4,825 mm^2 (6- 32 T). Other parameters and data are the same as those of Ex.1: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u in the flange of under-reinforced rectangular beam we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, b_f\, f_{ck}} = \frac{0.87\,(415)\,(4825)}{0.36\,(1000)\,(20)} = 241.95 \text{ mm} > D_f$$

Here, D_f/d=100/450= 0.222> 0.2. So, we have to determine y_f from Eq. 5.15 and equating C and T from Eqs. 5.16 and 17 of Lesson 10.

$$y_f = 0.15\, x_u + 0.65 D_f \qquad (5.15)$$

$$0.36\, f_{ck} b_w x_u + 0.45 f_{ck}\,(b_f - b_w)\, y_f = 0.87\, f_y\, A_{st} \qquad (5.16 \text{ and } 5.17)$$

or $\quad 0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\, x_u + 0.65\,(100)\}$

$\quad = 0.87\,(415)\,(4825)$

or $\quad 2160\, x_u + 945\, x_u = -409500 + 1742066$

or $\quad x_u = 1332566/3105 = 429.17$ mm

$\quad x_{u,max} = 0.48\,(450) = 216$ mm

Since $x_u > x_{u,max}$, the beam is over-reinforced. Accordingly.

$\quad x_u = x_{u,max} = 216$ mm.

Step 2: To determine M_u

This problem belongs to case (iv b), explained in sec.5.10.4.4 b of Lesson 10. So, we can determine Mu from Eq. 5.11 of Lesson 10.

$$M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45 f_{ck}(b_f - b_w)\, y_f\, (d - y_f/2) \qquad (5.11)$$

where $y_f = 0.15\, x_{u,\,max} + 0.65 D_f = 97.4$ mm $\qquad$ (5.8)

From Eq. 5.11, employing the value of y_f = 97.4 mm, we get:

$$M_u = 0.36\,(0.48)\,\{1 - 0.42\,(0.48)\}\,(20)\,(300)\,(450)\,(450)$$

$$+ 0.45 (20) (1000 - 300) (97.4) (450 - 97.4/2)$$

$$= 167.63 + 246.24 \quad = \quad 413.87 \quad \text{kNm}$$

It is seen that this over-reinforced beam has the same M_u as that of the balanced beam of Example 2.

Summary of Results of Examples 1-4

The results of four problems (Exs. 1-4) are given in Table 5.1 below. All the examples are having the common data except A_{st}.

Table 5.1 Results of Examples 1-4 (Figs. 5.11.2 – 5.11.5)

Ex. No.	*Ast* (mm^2)	Case	Section No.	M_u (kNm)	Remarks
1	1,963	(i)	5.10.4.1	290.06	x_u = 98.44 mm < $x_{u,\ max}$ (= 216 mm), x_u<D_f (= 100 mm), Under-reinforced, (NA in the flange).
2	3,066	(ii b)	5.10.4.2 (b)	413.87	x_u =$x_{u,\ max}$= 216 mm, D_f /d = 0.222 > 0.2, Balanced, (NA in web).
3	2,591	(iii b)	5.10.4.3 (b)	369.18	x_u = 169.398 mm < $x_{u,\ max}$(= 216 mm), D_f /x_u= 0.59 > 0.43, Under-reinforced, (NA in the web).
4	4,825	(iv b)	5.10.4.4 (b)	413.87	x_u =241.95 mm > $x_{u,\ max}$ (= 216 mm), D_f /d=0.222 > 0.2, Over-reinforced, (NA in web).

It is clear from the above table (Table 5.1), that Ex.4 is an over-reinforced flanged beam. The moment of resistance of this beam is the same as that of balanced beam of Ex.2. Additional reinforcement of 1,759 mm^2 (= 4,825 mm^2 – 3,066 mm^2) does not improve the M_u of the over-reinforced beam. It rather prevents the beam from tension failure. That is why over-reinforced beams are to be avoided. However, if the M_u has to be increased beyond 413.87 kNm, the flanged beam may be doubly reinforced.

Use of SP-16 for the Analysis Type of Problems

Using the two governing parameters (b_f /b_w) and (D_f /d), the $M_{u,lim}$ of balanced flanged beams can be determined from Tables 57-59 of SP-16 for the three grades of steel (250, 415 and 500). The value of the moment coefficient $M_{u,lim}/b_w d^2 f_{ck}$ of Ex.2, as obtained from SP-16, is presented in Table 5.2 making linear interpolation for both the parameters, wherever needed. $M_{u,lim}$ is then calculated from the moment coefficient.

Table 5.2 $M_{u,lim}$ of Example 2 using Table 58 of SP-16

Parameters:(i) b_f/b_w = 1000/300 =3.33

(ii) D_f/d = 100/450 = 0.222

$(M_{u,lim}/b_w d^2 f_{ck})$ in N/mm^2			
D_f/d	b_f/b_w		
	3	4	3.33
0.22	0.309	0.395	
0.23	0.314	0.402	
0.222	0.31*	0.3964*	0.339*

*by linear interpolation

So, from Table 5.2, $\dfrac{M_{u,lim}}{b_w d^2 f_{ck}} = 0.339$

$Mu,lim = 0.339\, b_w d^2 f_{ck} = 0.339\,(300)\,(450)\,(450)\,(20)\,10^{-6} = 411.88$ kNm

$M_{u,lim}$ as obtained from SP-16 is close to the earlier computed value of $M_{u,lim}$ = 413.87 kNm (see Table 5.1).

5.11.6 Practice Questions and Problems with Answers

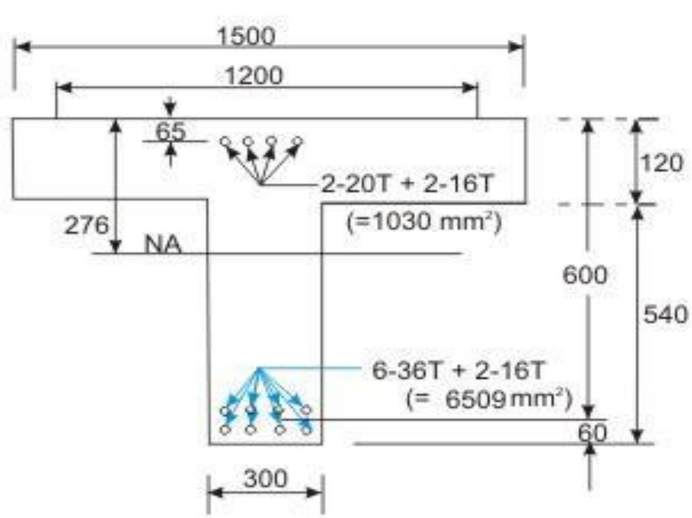


Fig. 5.11.6: Q. 1

Q.1: Determine the moment of resistance of the simply supported doubly reinforced flanged beam (isolated) of span 9 m as shown in Fig. 5.11.6. Assume M 30 concrete and Fe 500 steel.

A.1:Solution of Q.1:

$$\text{Effective width } b_f = \frac{l_o}{(l_o/b)+4} + b_w = \frac{9000}{(9000/1500)+4} + 300 = 1200 \text{ mm}$$

Step 1: To determine the depth of the neutral axis

Assuming neutral axis to be in the flange and writing the equation $C = T$, we have:

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_f\, x_u + (f_{sc}\, A_{sc} - f_{cc}\, A_{sc})$$

Here, $d' / d = 65/600 = 0.108 = 0.1$ (say). We, therefore, have $fsc = 353 \text{ N/mm}^2$.

From the above equation, we have:

$$x_u = \frac{0.87\,(500)\,(6509) - \{(353)\,(1030) - 0.446\,(30)\,(1030)\}}{0.36\,(30)\,(1200)} =$$

So, the neutral axis is in web.

$D_f /d = 120/600 = 0.2$

Assuming $D_f /x_u < 0.43$, and Equating $C = T$

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\, (b_f - b_w)\, D_f$$

$$x_u = \frac{0.87\,(500)\,(6509) - 1030\{353 - 0.446\,(30)\} - 0.446\,(30)\,(1200 - 300)\,(120)}{0.36\,(30)\,(300)}$$

$= 319.92 > 276$ mm ($x_{u,max} = 276$ mm)

So, $x_u = x_{u,max} = 276$ mm (over-reinforced beam).

$$D_f /x_u = 120/276 = 0.4347 > 0.43$$

Let us assume $D_f /x_u > 0.43$. Now, equating $C = T$ with y_f as the depth of flange having constant stress of $0.446\, f_{ck}$. So, we have:

$$y_f = 0.15\, x_u + 0.65\, D_f = 0.15\, x_u + 78$$

$$0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\, (b_f - b_w)\, y_f + A_{sc}\, (f_{sc} - f_{cc}) = 0.87\, f_y\, A_{st}$$

$0.36\ (30)\ (300)\ x_u + 0.446\ (30)\ (900)\ (0.15\ x_u + 78)$

$= 0.87\ (500)\ (6509) – 1030\ \{353 – 0.446\ (30)\}$

or $x_u = 305.63$ mm > $x_{u,max}$. ($x_{u,max} = 276$ mm)

The beam is over-reinforced. Hence, $x_u = x_{u,max} = 276$ mm. This is a problem of case (iv), and we, therefore, consider the case (ii) to find out the moment of resistance in two parts: first for the balanced singly reinforced beam and then for the additional moment due to compression steel.

Step 2: Determination of $x_{u,lim}$ for singly reinforced flanged beam

Here, $D_f /d = 120/600 = 0.2$, so y_f is not needed. This is a problem of case (ii a) of sec. 5.10.4.2 of Lesson 10. Employing Eq. 5.7 of Lesson 10, we have:

$M_{u,lim} = 0.36\ (x_{u,max} /d)\ \{1 – 0.42\ (x_{u,max} /d)\}\ f_{ck}\ b_w\ d^2$

i $0.45\ f_{ck}\ (b_f – b_w)\ D_f\ (d – D_f /2)$

4.8.6 $0.36(0.46)\ \{1 – 0.42(0.46)\}\ (30)\ (300)\ (600)\ (600) +$
$0.45(30)\ (900)\ (120)\ (540)$

4.8.7 1,220.20kNm

$$A_{st,lim} = \frac{M_{u,lim}}{0.87\ f_y\ d\ \{1 - 0.42\ (x_{u,max}\ /\ d)\}}$$

$$= \frac{(1220.20)\ (10^6)}{(0.87)\ (500)\ (600)\ (0.8068)} = 5{,}794.6152 \text{mm}^2$$

Step 3:Determination ofM_{u2}

TotalA_{st}=6,509mm^2,$A_{st,lim}$ =5,794.62mm^2

$A_{st2} = 714.38$ mm^2 and $A_{sc} = 1{,}030$ mm^2

It is important to find out how much of the total A_{sc} and A_{st2} are required effectively. From the equilibrium of C and T forces due to additional steel (compressive and tensile), we have:

$(A_{st2})\ (0.87)\ (f_y) = (A_{sc})\ (f_{sc})$

If we assume $A_{sc} = 1{,}030$ mm^2

$A_{st\,2} = \frac{1030 \; (353)}{0.87\;(500)}$ = 835.84 mm^2 > 714.38 mm^2 , (714.38 mm^2 is the total

A_{st2} provided). So, this is not possible.

Now, using A_{st2} = 714.38 mm^2 , we get A_{sc} from the above equation.

$$A_{sc} = \frac{(714.38)\;(0.87)\;(500)}{353} = 880.326 < 1{,}030 \text{ mm}^2,$$

(1,030 mm^2 is the total A_{sc} provided).

$$M_{u\,2} = A_{sc}\, f_{sc}\, (d - d') = (880.326)\;(353)\;(600 - 60) = 167.807 \text{ kNm}$$

Total moment of resistance= $Mu,lim + Mu2$= 1,220.20 + 167.81= 1,388.01 kNm

Total A_{st} required=$A_{st,lim}$ + A_{st2} =5,794.62 + 714.38= 6,509.00mm^2 ,

(provided A_{st} = 6,509 mm^2)

A_{sc} required=880.326mm^2 (provided 1,030mm^2).

Flanged Beams – Theory and Numerical Problems

Introduction

Lesson 10 illustrates the governing equations of flanged beams. It is now necessary to apply them for the solution of numerical problems. Two types of numerical problems are possible: (i) Analysis and (ii) Design types. This lesson explains the application of the theory of flanged beams for the analysis type of problems. Moreover, use of tables of SP-16 has been illustrated to determine the limiting moment of resistance of sections quickly for the three grades of steel.

Besides mentioning the different steps of the solution, numerical examples are also taken up to explain their step-by-step solutions.

Analysis Type of Problems

The dimensions of the beam b_f, b_w, D_f, d, D, grades of concrete and steel and the amount of steel A_{st} are given. It is required to determine the moment of resistance of the beam.

To determine the depth of the neutral axis x_u

The depth of the neutral axis is determined from the equation of equilibrium $C = T$. However, the expression of C depends on the location of neutral axis, D_f /d and D_f / x_u parameters. Therefore, it is required to assume first that the x_u is in the flange. If this is not the case, the next step is to assume x_u in the web and the computed value of x_u will indicate if the beam is under-reinforced, balanced or over-reinforced.

Other steps:

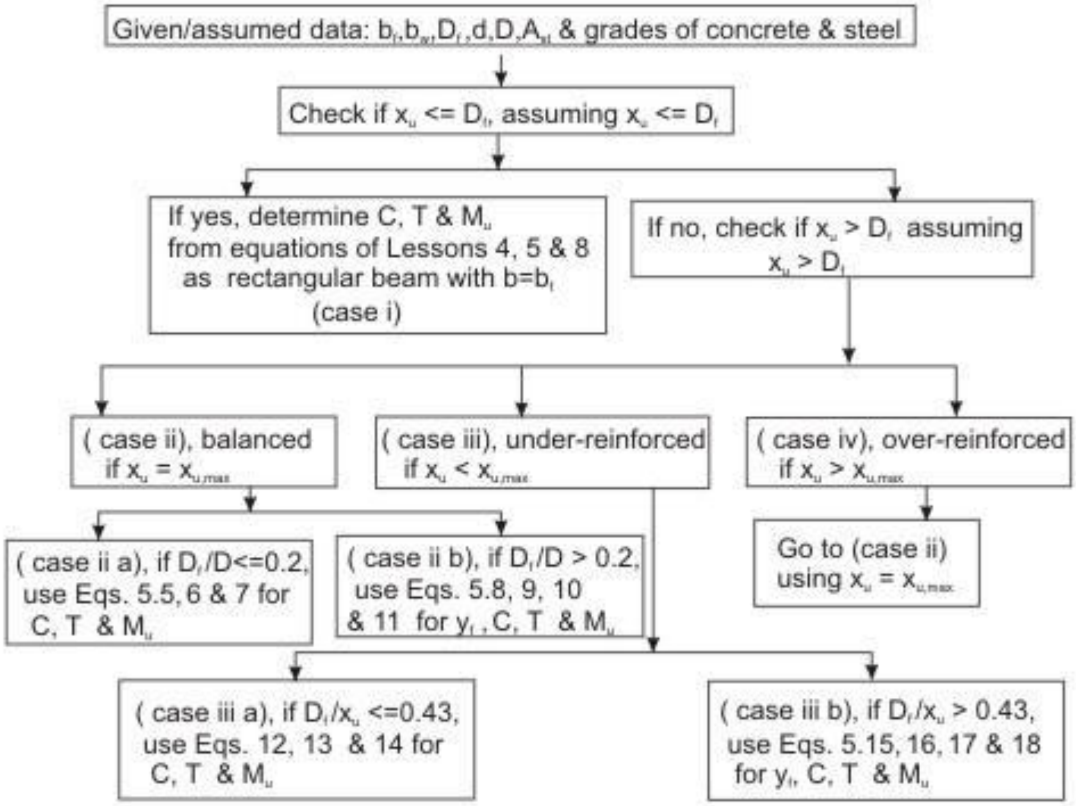


Fig. 5.11.1: Steps of solution of analysis type of problems

After knowing if the section is under-reinforced, balanced or over-reinforced, the respective parameter D_f/d or D_f/x_u is computed for the under-reinforced, balanced or over-reinforced beam. The respective expressions of C,
T and M_u, as established in Lesson 10, are then employed to determine their values. Figure 5.11.1 illustrates the steps to be followed.

4.8.9 Numerical Problems (Analysis Type)

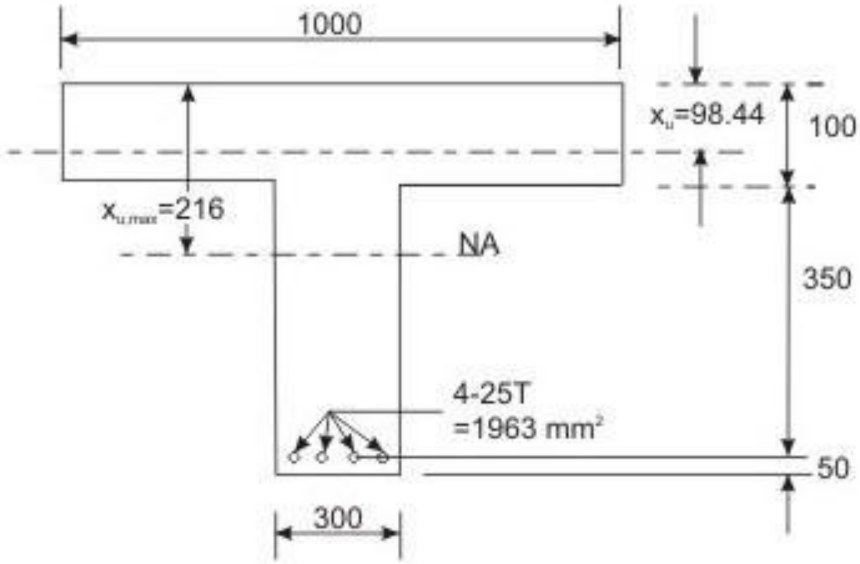


Fig. 5.11.2: Example 1, case (i)

Ex.1: Determine the moment of resistance of the *T*-beam of Fig. 5.11.2. Given data: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm, d = 450 mm and A_{st} = 1963 mm^2 (4- 25 T). Use M 20 and Fe 415.

Step 1: To determine the depth of the neutral axis x_u

Assuming x_u in the flange and equating total compressive and tensile forces from the expressions of *C* and *T* (Eq. 3.16 of Lesson 5) as the *T*-beam can be treated as rectangular beam of width b_f and effective depth *d*, we get:

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, b_f\, f_{ck}} = \frac{0.87\,(415)\,(1963)}{0.36\,(1000)\,(20)} = 98.44 \text{ mm} < 100 \text{ mm}$$

So, the assumption of x_u in the flange is correct.

$x_{u,\,max}$ for the balanced rectangular beam = 0.48 d = 0.48 (450) = 216 mm.

It is under-reinforced since $x_u < x_{u,max}$.

Step 2: To determine C, T and M_u

From Eqs. 3.9 (using $b = b_f$) and 3.14 of Lesson 4 for *C* and *T* and Eq. 3.23 of Lesson 5 for M_u, we have:

$C = 0.36 b_f x_u f_{ck}$

$= 0.36\ (1000)\ (98.44)\ (20) \quad = \quad 708.77 \quad \text{kN}$

$T = 0.87 \quad f_y A_{st}$

(3.14)

$= \quad 0.87\ (415)\ (1963) \quad = \quad 708.74\ \text{kN}$

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} b_f d}\right)$$

$$= 0.87\ (415)\ (1963)\ (450)\ \{1 - \frac{(1963)\ (415)}{(20)\ (1000)\ (450)}\} = 290.06\ \text{kNm}$$

This problem belongs to the case (i) and is explained in sec. 5.10.4.1 of Lesson 10.

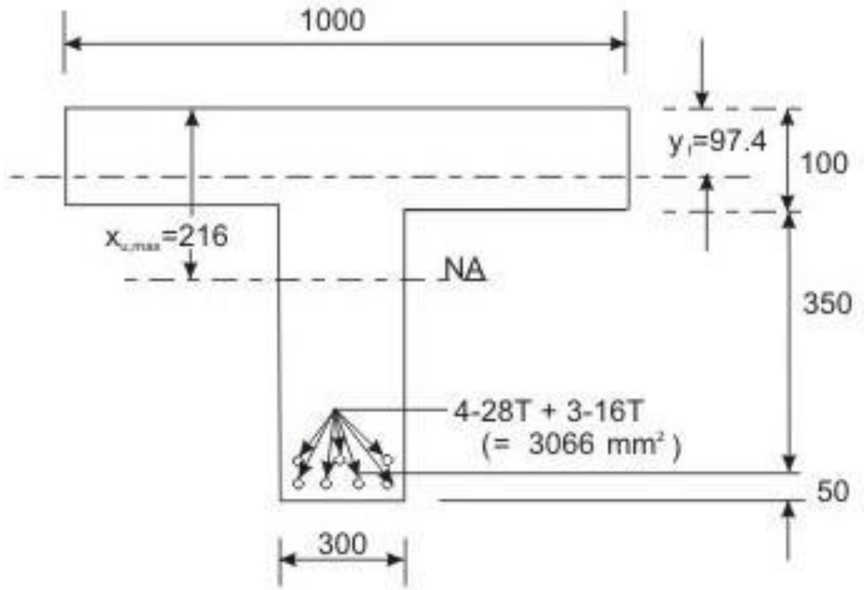


Fig. 5.11.3: Example 2, case (ii b)

Ex.2: Determine $A_{st,lim}$ and $M_{u,lim}$ of the flanged beam of Fig. 5.11.3. Given data are: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d

= 450 mm. Use M 20 and Fe 415.

Step 1: To determine D_f/d ratio

For the limiting case $x_u = x_{u,max}$ = 0.48 (450) = 216 mm > D_f. The ratio D_f/d is computed.

$D_f/d = 100/450 = 0.222 > 0.2$

Hence, it is a problem of case (ii b) and discussed in sec. 5.10.4.2 b of Lesson 10.

Step 2: Computations of y_f, C and T

First, we have to compute y_f from Eq.5.8 of Lesson 10 and then employ Eqs. 5.9, 10 and 11 of Lesson 10 to determine C, T and Mu, respectively.

$y_f = 0.15\, x_{u,max} + 0.65\, D_f = 0.15\,(216) + 0.65\,(100) = 97.4$ mm. (from Eq. 5.8)

$$C = 0.36\, f_{ck} b_w\, x_{u,max} + 0.45 f_{ck}\,(b_f - b_w)\, y_f \qquad (5.9)$$

$$= 0.36\,(20)\,(300)\,(216) + 0.45\,(20)\,(1000 - 300)\,(97.4) = 1{,}080.18 \text{ kN.}$$

$$T = 0.87\, f_y A_{st} = 0.87\,(415)\, A_{st} \qquad (5.10)$$

Equating C and T, we have

$$A_{st} = \frac{(1080.18)\,(1000)\text{ N}}{0.87\,(415)\text{ N/mm}^2} = 2{,}991.77 \text{ mm}^2$$

Provide 4-28 T (2463 mm^2) +3-16 T (603mm^2)=3,066mm^2

Step 3: Computation of M_u

$$M_{u,lim} = 0.36\left(\frac{x_{u,max}}{d}\right)\left\{1 - 0.42\left(\frac{x_{u,max}}{d}\right)\right\} f_{ck} b_w d^2 + 0.45 f_{ck}\,(b_f - b_w)\, y_f\,(d - y_f/2) \qquad (5.11)$$

$$= 0.36\,(0.48)\,\{1 - 0.42\,(0.48)\}\,(20)\,(300)\,(450)^2$$

$$+ 0.45\,(20)\,(1000 - 300)\,(97.4)\,(450 - 97.4/2) = 413.87 \text{ kNm}$$

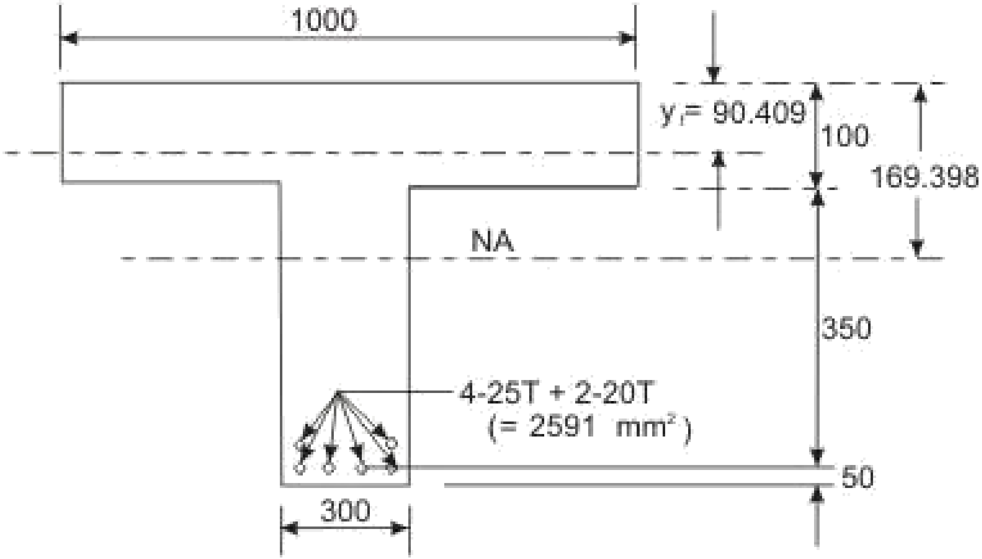


Fig. 5.11.4: Example 3, case (iii b)

Ex.3: Determine the moment of resistance of the beam of Fig. 5.11.4 when A_{st} = 2,591 mm^2 (4-25 T and 2- 20 T). Other parameters are the same as those of Ex.1: b_f = 1,000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u to be in the flange and the beam is under-reinforced, we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, b_f f_{ck}} = \frac{0.87\,(415)\,(2591)}{0.36\,(1000)\,(20)} = 129.93 \text{ mm} > 100 \text{ mm}$$

Since $x_u > D_f$, the neutral axis is in web.Here, D_f/d=100/450=0.222>0.2.
So, we have to substitute the term y_f from Eq. 5.15 of Lesson 10, assuming $D_f / x_u >$ 0.43 in the equation of $C = T$ from Eqs. 5.16 and 17 ofsec. 5.10.4.3 b of Lesson 10. Accordingly, we get:

$$0.36\, f_{ck} b_w x_u + 0.45 f_{ck}\,(b_f - b_w)\, y_f = 0.87\, f_y\, A_{st}$$

or $$0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\, x_u + 0.65\,(100)\}$$

$$= 0.87\,(415)\,(2591)$$

or x_u =169.398mm<216 mm ($x_{u,max}$ = 0.48 x_u = 216 mm)

So, the section is under-reinforced.

Step 2: To determine M_u

$$D_f / x_u = 100/169.398 = 0.590 > 0.43$$

This is the problem of case (iii b) of sec. 5.10.4.3 b. The corresponding equations are Eq. 5.15 of Lesson 10 for y_f and Eqs. 5.16 to 18 of Lesson 10 for C, T and M_u, respectively. From Eq. 5.15 of Lesson 10, we have:

$$y_f = 0.15\, x_u + 0.65\, D_f = 0.15\,(169.398) + 0.65\,(100) = 90.409 \text{ mm}$$

From Eq. 5.18 of Lesson 10, we have

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f\,(d - y_f/2)$$

or $M_u = 0.36\,(169.398/450)\,\{1 - 0.42\,(169.398/450)\}\,(20)\,(300)\,(450)\,(450)$

(iii) $0.45\,(20)\,(1000 - 300)\,(90.409)\,(450 - 90.409/2)$

8 138.62 + 230.56=369.18kNm.

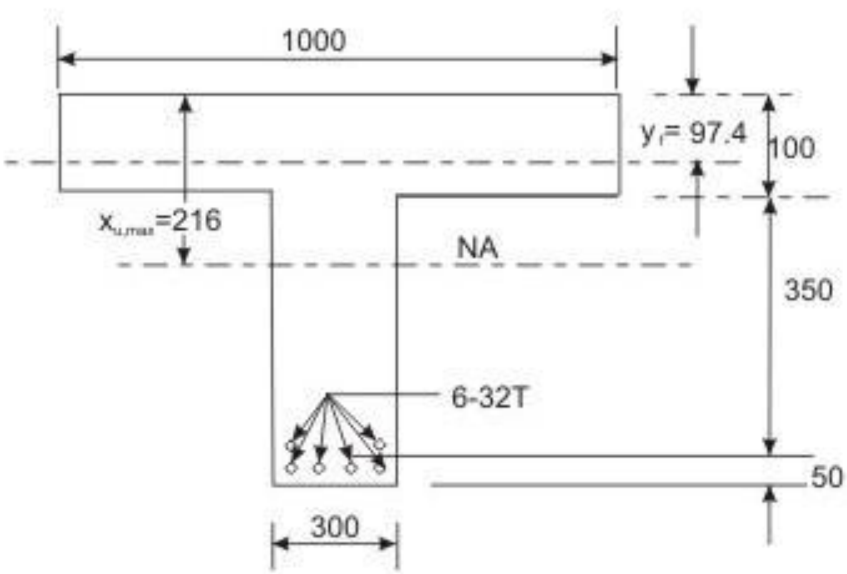


Fig. 5.11.5: Example 4, case (iv b)

Ex.4: Determine the moment of resistance of the flanged beam of Fig. 5.11.5 with A_{st} = 4,825 mm^2 (6- 32 T). Other parameters and data are the same as those of Ex.1: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u in the flange of under-reinforced rectangular beam we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, b_f\, f_{ck}} = \frac{0.87\,(415)\,(4825)}{0.36\,(1000)\,(20)} = 241.95 \text{ mm} > D_f$$

Here, D_f/d=100/450= 0.222> 0.2. So, we have to determine y_f from Eq. 5.15 and equating C and T from Eqs. 5.16 and 17 of Lesson 10.

$$y_f = 0.15\, x_u + 0.65 D_f \qquad (5.15)$$

$$0.36\, f_{ck} b_w x_u + 0.45 f_{ck}\,(b_f - b_w)\, y_f = 0.87\, f_y\, A_{st} \qquad (5.16 \text{ and } 5.17)$$

or $0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\, x_u + 0.65\,(100)\}$

$= 0.87\,(415)\,(4825)$

or $2160\, x_u + 945\, x_u = -409500 + 1742066$

or $x_u = 1332566/3105 = 429.17$ mm

$x_{u,max} = 0.48\,(450) = 216$ mm

Since $x_u > x_{u,max}$, the beam is over-reinforced. Accordingly.

$x_u = x_{u,max} = 216$ mm.

Step 2: To determine M_u

This problem belongs to case (iv b), explained in sec.5.10.4.4 b of Lesson 10. So, we can determine Mu from Eq. 5.11 of Lesson 10.

$$M_u = 0.36(x_{u,max}/d)\{1 - 0.42(x_{u,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45 f_{ck}(b_f - b_w)\, y_f\,(d - y_f/2) \qquad (5.11)$$

where $y_f = 0.15\, x_{u,max} + 0.65 D_f = 97.4$mm (5.8)

From Eq. 5.11, employing the value of y_f = 97.4 mm, we get:

M_u = 0.36 (0.48) {1 - 0.42 (0.48)} (20) (300) (450) (450)

$+ 0.45 (20) (1000 - 300) (97.4) (450 - 97.4/2)$

$= 167.63 + 246.24 \quad = 413.87$ kNm

It is seen that this over-reinforced beam has the same M_u as that of the balanced beam of Example 2.

5.11.4 Summary of Results of Examples 1-4

The results of four problems (Exs. 1-4) are given in Table 5.1 below. All the examples are having the common data except A_{st}.

Table 5.1 Results of Examples 1-4 (Figs. 5.11.2 – 5.11.5)

Ex. No.	*Ast* (mm^2)	Case	Section No.	M_u (kNm)	Remarks
1	1,963	(i)	5.10.4.1	290.06	x_u = 98.44 mm < $x_{u,\,max}$ (= 216 mm), x_u<D_f (= 100 mm), Under-reinforced, (NA in the flange).
2	3,066	(ii b)	5.10.4.2 (b)	413.87	x_u =$x_{u,\,max}$= 216 mm, D_f/d = 0.222 > 0.2, Balanced, (NA in web).
3	2,591	(iii b)	5.10.4.3 (b)	369.18	x_u = 169.398 mm < $x_{u,\,max}$(= 216 mm), D_f/x_u= 0.59 > 0.43, Under-reinforced, (NA in the web).
4	4,825	(iv b)	5.10.4.4 (b)	413.87	x_u =241.95 mm > $x_{u,\,max}$ (= 216 mm), D_f/d=0.222 > 0.2, Over-reinforced, (NA in web).

It is clear from the above table (Table 5.1), that Ex.4 is an over-reinforced flanged beam. The moment of resistance of this beam is the same as that of balanced beam of Ex.2. Additional reinforcement of 1,759 mm^2 (= 4,825 mm^2 – 3,066 mm^2) does not improve the M_u of the over-reinforced beam. It rather prevents the beam from tension failure. That is why over-reinforced beams are to be avoided. However, if the M_u has to be increased beyond 413.87 kNm, the flanged beam may be doubly reinforced.

Use of SP-16 for the Analysis Type of Problems

Using the two governing parameters (b_f/b_w) and (D_f/d), the $M_{u,lim}$ of balanced flanged beams can be determined from Tables 57-59 of SP-16 for the three grades of steel (250, 415 and 500). The value of the moment coefficient $M_{u,lim}\ /b_w d^2 f_{ck}$ of Ex.2, as obtained from SP-16, is presented in Table 5.2 making linear interpolation for both the parameters, wherever needed. $M_{u,lim}$ is then calculated from the moment coefficient.

Table 5.2 $M_{u,lim}$ of Example 2 using Table 58 of SP-16

Parameters:(i) b_f /b_w = 1000/300 =3.33

(ii) D_f /d = 100/450 = 0.222

$(M_{u,lim} /b_w\, d^2\, f_{ck})$inN/mm^2			
D_f /d	b_f /b_w		
	3	4	3.33
0.22	0.309	0.395	
0.23	0.314	0.402	
0.222	0.31*	0.3964*	0.339*

*by linear interpolation

So, from Table 5.2, $\dfrac{M_{u,\lim}}{b_w\ d^2\ f_{ck}} = 0.339$

$Mu,lim = 0.339\, b_w\, d^2\, f_{ck} = 0.339\ (300)\ (450)\ (450)\ (20)\ 10^{-6} = 411.88$ kNm

$M_{u,lim}$ as obtained from SP-16 is close to the earlier computed value of $M_{u,lim}$ = 413.87 kNm (see Table 5.1).

5.11.6 Practice Questions and Problems with Answers

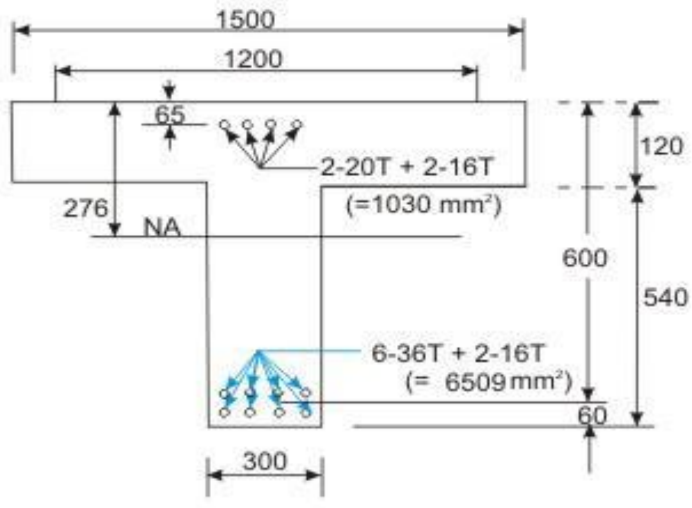


Fig. 5.11.6: Q. 1

Q.1: Determine the moment of resistance of the simply supported doubly reinforced flanged beam (isolated) of span 9 m as shown in Fig. 5.11.6. Assume M 30 concrete and Fe 500 steel.

A.1:Solution of Q.1:

Effective width $b_f = \dfrac{l_o}{(l_o/b)+4} + b_w = \dfrac{9000}{(9000/1500)+4} + 300 = 1200$ mm

Step 1: To determine the depth of the neutral axis

Assuming neutral axis to be in the flange and writing the equation $C = T$, we have:

$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_f\, x_u + (f_{sc}\, A_{sc} - f_{cc}\, A_{sc})$

Here, $d'/d = 65/600 = 0.108 = 0.1$ (say). We, therefore, have $fsc = 353$ N/mm^2 .

From the above equation, we have:

$$x_u = \frac{0.87\,(500)\,(6509) - \{(353)\,(1030) - 0.446\,(30)\,(1030)\}}{0.36\,(30)\,(1200)} = 191.48 \text{ mm} > 120 \text{ mm}$$

So, the neutral axis is in web.

$D_f/d = 120/600 = 0.2$

Assuming $D_f/x_u < 0.43$, and Equating $C = T$

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\,(b_f - b_w)\, D_f + (f_{sc} - f_{cc})\, A_{sc}$$

$$x_u = \frac{0.87\,(500)\,(6509) - 1030\{353 - 0.446\,(30)\} - 0.446\,(30)\,(1200 - 300)\,(120)}{0.36\,(30)\,(300)}$$

$= 319.92 > 276$ mm ($x_{u,max} = 276$ mm)

So, $x_u = x_{u,max} = 276$ mm (over-reinforced beam).

$D_f/x_u = 120/276 = 0.4347 > 0.43$

Let us assume $D_f/x_u > 0.43$. Now, equating $C = T$ with y_f as the depth of flange having constant stress of $0.446\, f_{ck}$. So, we have:

$y_f = 0.15\, x_u + 0.65\, D_f = 0.15\, x_u + 78$

$0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\,(b_f - b_w)\, y_f + A_{sc}\,(f_{sc} - f_{cc}) = 0.87\, f_y\, A_{st}$

$$0.36\ (30)\ (300)\ x_u + 0.446\ (30)\ (900)\ (0.15\ x_u + 78)$$

$$= \ 0.87\ (500)\ (6509) - 1030\ \{353 - 0.446\ (30)\}$$

or $\quad x_u \ = \ 305.63 \text{ mm} > \quad x_{u,max}. \quad (x_{u,max} \ = \ 276 \text{ mm})$

The beam is over-reinforced. Hence, $x_u = x_{u,max} = 276$ mm. This is a problem of case (iv), and we, therefore, consider the case (ii) to find out the moment of resistance in two parts: first for the balanced singly reinforced beam and then for the additional moment due to compression steel.

Step 2: Determination of $x_{u,lim}$ for singly reinforced flanged beam

Here, $D_f/d = 120/600 = 0.2$, so y_f is not needed. This is a problem of case (ii a) of sec. 5.10.4.2 of Lesson 10. Employing Eq. 5.7 of Lesson 10, we have:

$$M_{u,lim} \ = \ 0.36\ (x_{u,max}/d)\ \{1 - 0.42\ (x_{u,max}/d)\}\ f_{ck}\ b_w\ d^2$$

$$i \ \ 0.45\ f_{ck}\ (b_f - b_w)\ D_f\ (d - D_f/2)$$

4.8.8 $\quad 0.36(0.46)\ \{1 - 0.42(0.46)\}\ (30)\ (300)\ (600)\ (600) +$
$0.45(30)\ (900)\ (120)\ (540)$

4.8.9 $\quad$ 1,220.20kNm

$$A_{st,lim} \ = \ \frac{M_{u,lim}}{0.87\ f_y\ d\ \{1 - 0.42\ (x_{u,max}/d)\}}$$

$$= \ \frac{(1220.20)\ (10^6)}{(0.87)\ (500)\ (600)\ (0.8068)} \ = \ 5{,}794.6152 \text{ mm}^2$$

Step 3:Determination of M_{u2}

Total A_{st} = 6,509 mm^2, $A_{st,lim}$ = 5,794.62 mm^2

$A_{st2} \ = \ 714.38 \ \text{mm}^2$ and $A_{sc} \ = \ 1{,}030 \ \text{mm}^2$

It is important to find out how much of the total A_{sc} and A_{st2} are required effectively. From the equilibrium of C and T forces due to additional steel (compressive and tensile), we have:

$$(A_{st2})\ (0.87)\ (f_y) \ = \ (A_{sc})\ (f_{sc})$$

If we assume $\quad A_{sc} \ = \ 1{,}030 \text{ mm}^2$

$A_{st\,2} = \frac{1030}{0.87\,(500)}(353) = 835.84\ mm^2 > 714.38\ mm^2$, ($714.38\ mm^2$ is the total A_{st2} provided). So, this is not possible.

Now, using $A_{st2} = 714.38\ mm^2$, we get A_{sc} from the above equation.

$$A_{sc} = \frac{(714.38)\,(0.87)\,(500)}{353} = 880.326 < 1{,}030\ mm^2,$$

($1{,}030\ mm^2$ is the total A_{sc} provided).

$$M_{u2} = A_{sc}\, f_{sc}\, (d - d') = (880.326)\,(353)\,(600 - 60) = 167.807\ kNm$$

Total moment of resistance= $M_{u,lim} + M_{u2}$= 1,220.20 + 167.81= 1,388.01 kNm

Total A_{st} required= $A_{st,lim} + A_{st2}$ =5,794.62 + 714.38= 6,509.00mm^2, (provided A_{st} = 6,509 mm^2)

A_{sc} required=880.326mm^2 (provided 1,030mm^2).

Test 11 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Determine $M_{u,lim}$ of the flanged beam of Ex. 2 (Fig. 5.11.3) with the help of SP-16 using (a) M 20 and Fe 250, (b) M 20 and Fe 500 and (c) compare the results with the $M_{u,lim}$ of Ex. 2 from Table 5.2 when grades of concrete and steel are M 20 and Fe 415, respectively. Other data are: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm.

(10 X 3 = 30 marks)

A.TQ.1: From the results of Ex. 2 of sec. 5.11.5 (Table 5.2), we have:

Parameters:(i) b_f / b_w = 1000/300 =3.33

(ii) D_f / d = 100/450= 0.222

For part (a): When Fe 250 is used, the corresponding table is Table 57 of SP-16. The computations are presented in Table 5.3 below:

Table 5.3 ($M_{u,lim} / b_w d^2 f_{ck}$) in N/mm^2 Of TQ.1 (PART a for M 20 and Fe 250)

($M_{u,lim} / b_w d^2 f_{ck}$) in N/mm^2			
D_f / d	b_f / b_w		
	3	4	3.33
0.22	0.324	0.411	
0.23	0.330	0.421	
0.222	0.3252*	0.413*	0.354174*

(vi) by linear interpolation

$M_{u,lim} / b_w d^2 f_{ck}$ = 0.354174 = 0.354 (say)

So, $M_{u,lim}$ = (0.354) (300) (450) (450) (20) N mm = 430.11 kNm

For part (b): When Fe 500 is used, the corresponding table is Table 59 of SP-

16. The computations are presented in Table 5.4 below:

Table 5.4 ($M_{u,lim}/b_w d^2 f_{ck}$) in N/mm^2 Of TQ.1 (PART b for M 20 and Fe 500)

($M_{u,lim}/b_w d^2 f_{ck}$) in N/mm^2

D_f/d	b_f/b_w		
	3	4	3.33
0.22	0.302	0.386	
0.23	0.306	0.393	
0.222	0.3028*	0.3874*	0.330718*

* by linear interpolation

$M_{u,lim}/b_w d^2 f_{ck}$= 0.330718 = 0.3307 (say)

So, Mu,lim =(0.3307) (300) (450) (450) (20) mm= 401.8kNm

For part (c): Comparison of results of this problem with that of Table 5.2 (M 20 and Fe

415) is given below in Table 5.5.

Table 5.5 Comparison of results of $M_{u,lim}$

Sl. No.	Grade of Steel	$M_{u,lim}$(kNm)
1	Fe 250	430.11
2	Fe 415	411.88
3	Fe 500	401.80

It is seen that $M_{u,lim}$ of the beam decreases with higher grade of steel for a particular grade of concrete.

TQ.2: With the aid of SP-16, determine separately the limiting moments of resistance and the limiting areas of steel of the simply supported isolated, singly reinforced and balanced flanged beam of Q.1 as shown in Fig. 5.11.6 if the span = 9 m. Use M 30 concrete and three grades of steel, Fe

250, Fe 415 and Fe 500, respectively. Compare the results obtained above with that of Q.1 of sec. 5.11.6, when balanced.

(15 + 5 = 20 marks)

A.TQ.2: From the results of Q.1 sec. 5.11.6, we have:

Parameters:(i)b_f/b_w = 1200/300 = 4.0

(ii) D_f/d = 120/600 = 0.2

For Fe 250, Fe 415 and Fe 500, corresponding tables are Table 57, 58 and 59, respectively of SP-16. The computations are done accordingly. After computing the limiting moments of resistance, the limiting areas of steel are determined as explained below. Finally, the results are presented in Table 5.6 below:

$$A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y \, d \, \{1 - 0.42 \, (x_{u,max} / d)\}}$$

Table 5.6 Values of $M_{u,lim}$ in N/mm^2 Of TQ.2

GradeofFe/Q.1of sec. 5.11.6	$(M_{u,lim}/b_w d^2 f_{ck})$ (N/mm^2)	$M_{u,lim}$ (kNm)	$A_{st,lim}$ (mm^2)
Fe 250	0.39	1, 263.60	12,455.32
Fe 415	0.379	1, 227.96	7,099.78
Fe 500	0.372	1, 205.28	5,723.76
Q.1ofsec.5.11.6(Fe 415)		1, 220.20	5,794.62

The maximum area of steel allowed is .04 $b\,D$ = (.04) (300) (660) = 7,920 mm^2 . Hence, Fe 250 is not possible in this case.

(iv) Summary of this Lesson

This lesson mentions about the two types of numerical problems (i) analysis and (ii) design types. In addition to explaining the steps involved in solving the analysis type of numerical problems, several examples of analysis type of problems are illustrated explaining all steps of the solutions both by direct computation method and employing SP- 16. Solutions of practice and test problems will give readers the confidence in applying the theory explained in Lesson 10 in solving the numerical problems.

UNIT-III

DESIGN OF SLAB

One-way and Two-way Slabs

Figures 2.1a and b explain the share of loads on beams supporting solid slabs along four edges when vertical loads are uniformly distributed. It is evident from the figures that the share of loads on beams in two perpendicular directions depends upon the aspect ratio ly /lx of the slab, lx being the shorter span. For large values of ly, the triangular area is much less than the trapezoidal area (Fig. 2.1a). Hence, the share of loads on beams along shorter span will gradually reduce with increasing ratio of ly /lx. In such cases, it may be said that the loads are primarily taken by beams along longer span. The deflection profiles of the slab along both directions are also shown in the figure. The deflection profile is found to be constant along the longer span except near the edges for the slab panel of Fig. 2.1a. These slabs are designated as one-way slabs as they span in one direction (shorter one) only for a large part of the slab when $ly /lx > 2$.

On the other hand, for square slabs of $l_y /l_x = 1$ and rectangular slabs of l_y /l_x up to 2, the deflection profiles in the two directions are parabolic (Fig. 2.1b). Thus, they are spanning in two directions and these slabs with *ly /lx* up to 2 are designated as two-way slabs, when supported on all edges.

It would be noted that an entirely one-way slab would need lack of support on short edges. Also, even for *ly /lx* < 2, absence of supports in two parallel edges will render the slab one-way. In Fig. 2.1b, the separating line at 45 degree is tentative serving purpose of design. Actually, this angle is a function of *ly /lx*

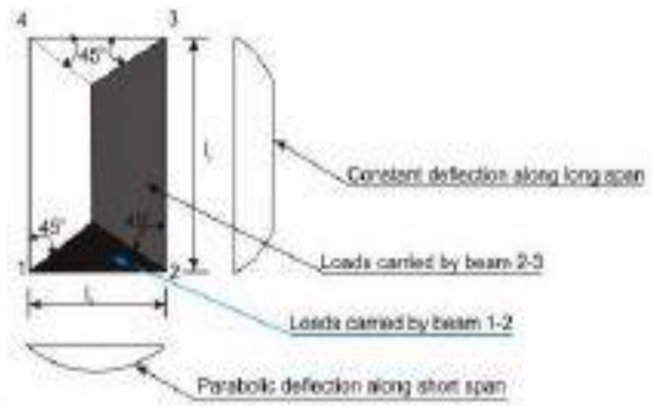


Figure 2.1(a) One-way slab (l_y/l_x>2)

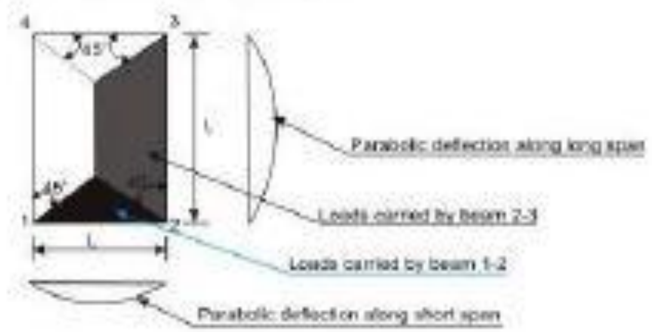


Figure 2.1(b) Two-way slab (l_y/l_x<=2)

Design of One-way Slabs

The procedure of the design of one-way slab is the same as that of beams. However, the amounts of reinforcing bars are for one metre width of the slab as to be determined from either the governing design moments (positive or negative) or from the requirement of minimum reinforcement. The different steps of the design are explained below.

Step 1: Selection of preliminary depth of slab

The depth of the slab shall be assumed from the span to effective depth ratios.

Step 2: Design loads, bending moments and shear forces

The total factored (design) loads are to be determined adding the estimated dead load of the slab, load of the floor finish, given or assumed live loads etc. after multiplying each of them with the respective partial safety factors. Thereafter, the design positive and negative bending moments and shear forces are to be determined using the respective coefficients given in Tables 12 and 13 of IS 456.

Step 3: Determination/checking of the effective and total depths of slabs

The effective depth of the slab shall be determined employing.

$M_{u,lim} = R_{,lim}\, bd2$

The total depth of the slab shall then be determined adding appropriate nominal cover (Table 16 and 16A of cl.26.4 of IS 456) and half of the diameter of the larger bar if the bars are of different sizes. Normally, the computed depth of the slab comes out to be much less than the assumed depth in Step 1. However, final selection of the depth shall be done after checking the depth for shear force.

Step 4: Depth of the slab for shear force

Theoretically, the depth of the slab can be checked for shear force if the design shear strength of concrete is known. Since this depends upon the percentage of tensile reinforcement, the design shear strength shall be assumed considering the lowest percentage of steel. The value of shall be modified after knowing the multiplying factor *k* from the depth tentatively selected for the slab in Step 3. If necessary, the depth of the slab shall be modified. c

Step 5: Determination of areas of steel

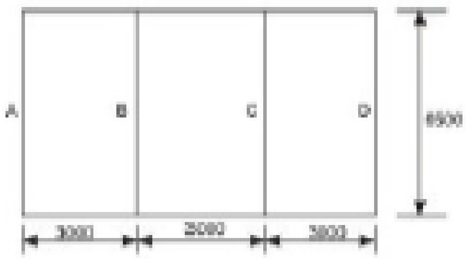
Area of steel reinforcement along the direction of one-way slab should be determined employing the following Eq.

$M_u = 0.87\, f_y\, A_{st}\, d\, \{1 - (A_{st})(f_y)/(f_{ck})(bd)\}$

The above equation is applicable as the slab in most of the cases is under-reinforced due to the selection of depth larger than the computed value in Step 3. The area of steel so determined should be checked whether it is at least the minimum area of steel as mentioned in cl.26.5.2.1 of IS 456.

Step 6: Selection of diameters and spacings of reinforcing bars (cls.26.5.2.2 and 26.3.3 of IS 456)

The diameter and spacing of bars are to be determined as per cls.26.5.2.2 and 26.3.3 of IS 456. As mentioned in Step 5, this step may be avoided when using the tables and charts of SP-16. Design the one-way continuous slab of Fig.8.18.6 subjected to uniformly distributed imposed loads of 5 kN/m2 using M 20 and Fe 415. The load of floor finish is 1 kN/m2. The span dimensions shown in the figure are effective spans. The width of beams at the support = 300 mm.



Step 1: Selection of preliminary depth of slab

The basic value of span to effective depth ratio for the slab having simple support at the end and continuous at the intermediate is (20+26)/2 = 23 (cl.23.2.1 of IS 456).
Modification factor with assumed p = 0.5 and fs = 240 N/mm2 is obtained as 1.18 from Fig.4 of IS 456.
Therefore, the minimum effective depth = 3000/23(1.18) = 110.54 mm. Let us take the effective depth d = 115 mm and with 25 mm cover, the total depth D = 140 mm.

Step 2: Design loads, bending moment and shear force

Dead loads of slab of 1 m width = 0.14(25) = 3.5 kN/m
Dead load of floor finish =1.0 kN/m
Factored dead load = 1.5(4.5) = 6.75 kN/m
Factored live load = 1.5(5.0) = 7.50 kN/m
Total factored load = 14.25 kN/m
Maximum moments and shear are determined from the coefficients given in Tables 12 and 13 of IS 456.
Maximum positive moment = 14.25(3)(3)/12 = 10.6875 kNm/m
Maximum negative moment = 14.25(3)(3)/10 = 12.825 kNm/m
Maximum shear Vu = 14.25(3)(0.4) = 17.1 Kn

Step 3: Determination of effective and total depths of slab

From Eq. Mu,lim = R,lim bd2 where R,lim is 2.76 N/mm2. So, d = {12.825(106)/(2.76)(1000)}0.5 = 68.17 mm
Since, the computed depth is much less than that determined in Step 1, let us keep D = 140 mm and d = 115 mm.

Step 4: Depth of slab for shear force

Table 19 of IS 456 gives = 0.28 N/mm2 for the lowest percentage of steel in the slab. Further for the total depth of 140 mm, let us use the coefficient k of cl. 40.2.1.1 of IS 456 as 1.3 to get = 1.3(0.28) = 0.364 N/mm2. c □ c c k□ □ □
Table 20 of IS 456 gives = 2.8 N/mm2. For this problem $bdVuv$/ =η = 17.1/115 = 0.148 N/mm2. Since, , the effective depth d = 115 mm is acceptable. ,max c□ u v Vbd □ □ ,max v c c□ □ □ □ □

Step 5: Determination of areas of steel

It is known that

$Mu = 0.87\ fy\ Ast\ d\ \{1 - (Ast)(fy)/(fck)(bd)\}$

(i) For the maximum negative bending moment

$12825000 = 0.87(415)(Ast)(115)\{1 - (Ast)(415)/(1000)(115)(20)\}$

or - 5542.16 $A2stAst$ + 1711871.646 = 0

Solving the quadratic equation, we have the negative Ast = 328.34 mm2

(ii) For the maximum positive bending moment

$10687500 = 0.87(415)\ Ast(115)\ \{1 - (Ast)(415)/(1000)(115)(20)\}$

or - 5542.16 $A2stAst$ + 1426559.705 = 0

Solving the quadratic equation, we have the positive Ast = 270.615 mm2

Distribution steel bars along longer span *ly*

Distribution steel area = Minimum steel area = 0.12(1000)(140)/100 = 168 mm2. Since, both positive and negative areas of steel are higher than the minimum area, we provide:

(a) For negative steel: 10 mm diameter bars @ 230 mm c/c for which Ast = 341 mm2 giving ps = 0.2965.

(b) For positive steel: 8 mm diameter bars @ 180 mm c/c for which Ast = 279 mm2 giving ps = 0.2426

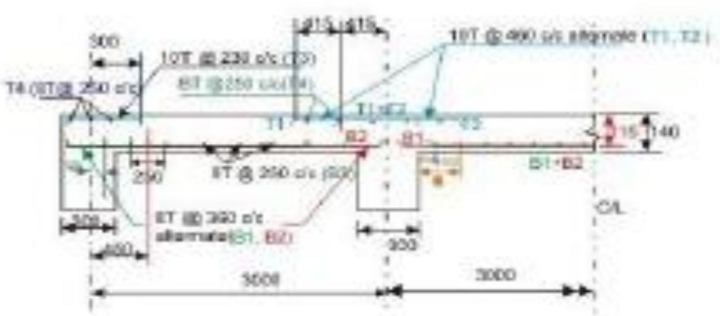
(c) For distribution steel: Provide 8 mm diameter bars @ 250 mm c/c for which Ast (minimum) = 201 mm2.

Step 6: Selection of diameter and spacing of reinforcing bars

The diameter and spacing already selected in step 5 for main and distribution bars are checked below:

For main bars (cl. 26.3.3.b.1 of IS 456), the maximum spacing is the lesser of 3d and 300 mm i.e., 300 mm. For distribution bars (cl. 26.3.3.b.2 of IS 456), the maximum spacing is the lesser of 5d or 450 mm i.e., 450 mm. Provided spacings, therefore, satisfy the requirements.

Maximum diameter of the bars (cl. 26.5.2.2 of IS 456) shall not exceed 140/8 = 17 mm is also satisfied with the bar diameters selected here.



UNIT-IV

DESIGN OF COLUMNS

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. Examples of compression member pedestal, column, wall and strut

Definitions

(a) Effective length: The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length *le* of that compression member in that plane. The effective length is different from the unsupported length *l* of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is

$le = k\, l$ (1)

Where *k* is the ratio of effective to the unsupported lengths. Clause 25.2 of IS 456 stipulates the effective lengths of compression members (vide Annex E of IS 456). This parameter is needed in classifying and designing the compression members.

(b) Pedestal: Pedestal is a vertical compression member whose effective length *le* does not exceed three times of its least horizontal dimension *b* (cl. 26.5.3.1h, Note). The other horizontal dimension *D* shall not exceed four times of *b*.

(c) Column: Column is a vertical compression member whose unsupported length *l* shall not exceed sixty times of *b* (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed 100*b*2/*D*, where *D* is the larger lateral dimension which is also restricted up to four times of *b* (vide cl. 25.3 of IS 456).

(d) Wall: Wall is a vertical compression member whose effective height *Hwe* to thickness *t* (least lateral dimension) shall not exceed 30 (cl. 32.2.3 of IS 456). The larger horizontal dimension i.e., the length of the wall *L* is more than 4t.

Classification of Columns Based on Types of Reinforcement

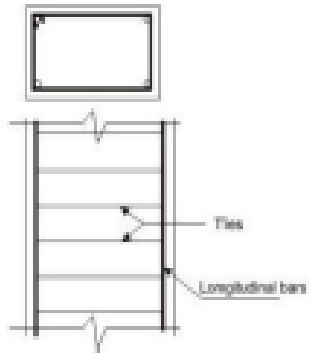


Figure 3.1(a) Tied Column

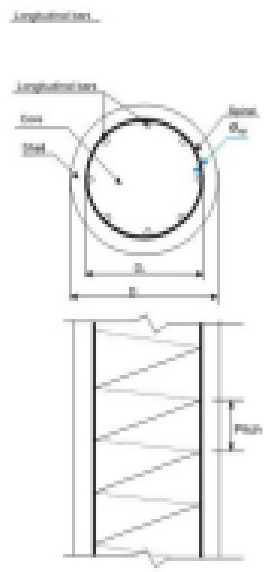


Figure 3.1(b) Column with helical reinforcement

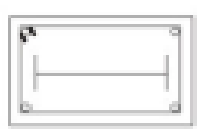


Figure 3.1(c) Composite column (steel section)

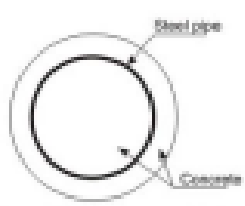


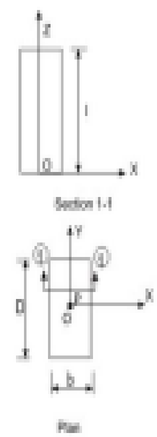
Figure 3.1(d) Composite column (steel pipe)

Figure 3.1 Tied, helically bound and composite columns

Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:

Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:

(i) Tied columns: The main longitudinal reinforcement bars are enclosed within closely spaced lateral ties (Fig.3.1a).
(ii) Columns with helical reinforcement: The main longitudinal reinforcement bars are enclosed within closely spaced and continuously wound spiral reinforcement. Circular and octagonal columns are mostly of this type (Fig. 3.1b).
(iii) Composite columns: The main longitudinal reinforcement of the composite columns consists of structural steel sections or pipes with or without longitudinal bars (Fig. 3.1c and d).
Out of the three types of columns, the tied columns are mostly common with different shapes of the cross-sections viz. square, rectangular etc. Helically bound columns are also used for circular or octagonal shapes of cross-sections.



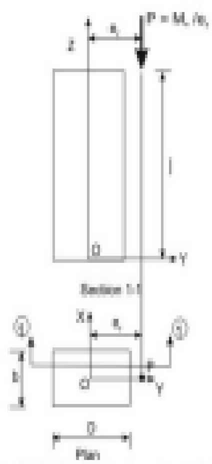


Figure 3.2(a) Axial loading (concentric)   Figure 3.2(b) Axial loading with uniaxial bending

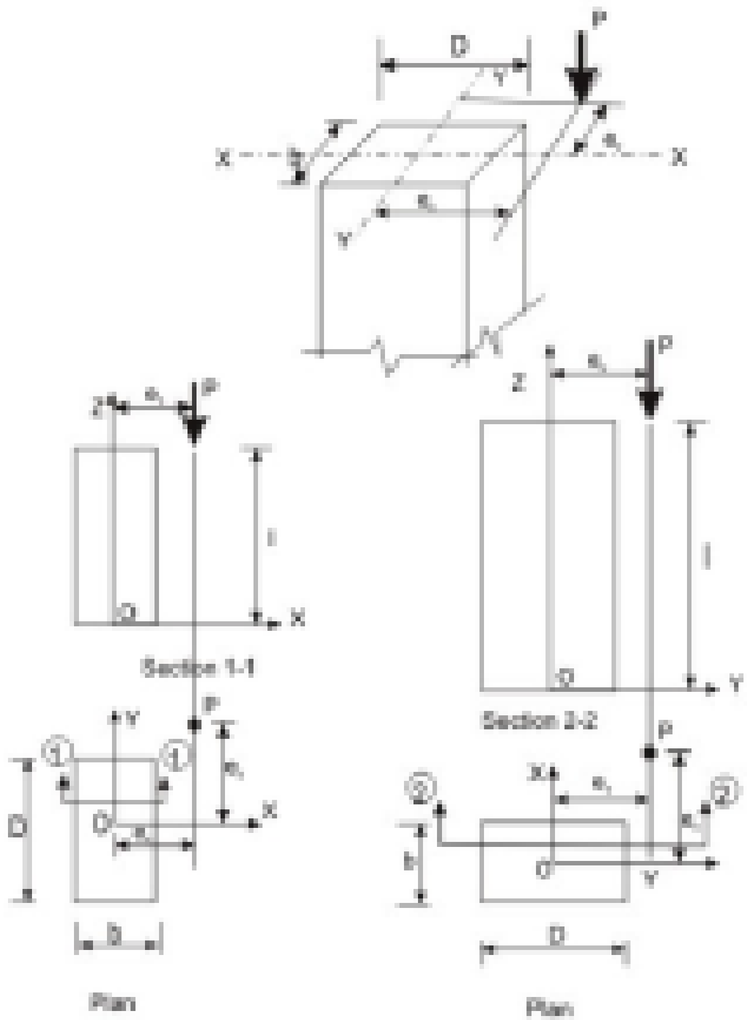


Figure 3.2(c) Axial loading with biaxial bending

Columns are classified into the three following types based on the loadings:
(i) Columns subjected to axial loads only (concentric), as shown in Fig. 3.2a.
(ii) Columns subjected to combined axial load and uniaxial bending, as shown in Fig. 3.2b.
(iii) Columns subjected to combined axial load and bi-axial bending, as shown in Fig. 3.2c.

Classification of Columns Based on Slenderness Ratios

Columns are classified into the following two types based on the slenderness ratios:
(i) Short columns

(ii) Slender or long columns

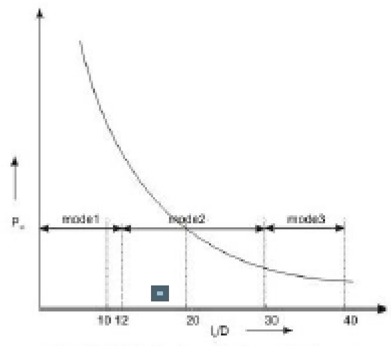


Figure 3.3 Modes of failure of columns

Figure 3.3 presents the three modes of failure of columns with different slenderness ratios when loaded axially. In the mode 1, column does not undergo any lateral deformation and collapses due to material failure. This is known as compression failure. Due to the combined effects of axial load and moment a short column may have material failure of mode 2. On the other hand, a slender column subjected to axial load only undergoes deflection due to beam-column effect and may have material failure under the combined action of direct load and bending moment. Such failure is called combined compression and bending failure of mode 2. Mode 3 failure is by elastic instability of very long column even under small load much before the material reaches the yield stresses. This type of failure is known as elastic buckling.

The slenderness ratio of steel column is the ratio of its effective length *le* to its least radius of gyration *r*. In case of reinforced concrete column, however, IS 456 stipulates the slenderness ratio as the ratio of its effective length *le* to its least lateral dimension. As mentioned earlier in sec. 3.1(a), the effective length *le* is different from the unsupported length, the rectangular reinforced concrete column of cross-sectional dimensions *b* and *D* shall have two effective lengths in the two directions of *b* and *D*. Accordingly, the column may have the possibility of buckling depending on the two values of slenderness ratios as given below:

Slenderness ratio about the major axis = *lex/D*

Slenderness ratio about the minor axis = *ley/b*

Based on the discussion above, cl. 25.1.2 of IS 456 stipulates the following:

A compression member may be considered as short when both the slenderness ratios *lex/D* and *ley/b* are less than 12 where *lex* = effective length in respect of the major axis, *D* = depth in respect of the major axis, *ley* = effective length in respect of the minor axis, and *b* = width of the member. It shall otherwise be considered as a slender compression member.

Further, it is essential to avoid the mode 3 type of failure of columns so that all columns should have material failure (modes 1 and 2) only. Accordingly, cl. 25.3.1 of IS 456 stipulates the maximum unsupported length between two restraints of a column to sixty times its least lateral dimension. For cantilever columns, when one end of the column is unrestrained, the unsupported length is restricted to 100*b*2*/D* where *b* and *D* are as defined earlier.

Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. Clause 26.5.3.1 stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

(a) The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.

(b) The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.

(c) Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.

(d) The diameter of the longitudinal bars should be at least 12 mm.

(e) Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.

(f) The bars shall be spaced not exceeding 300 mm along the periphery of the column.

(g) The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

Transverse Reinforcement

Transverse reinforcing bars are provided in forms of circular rings, polygonal links (lateral ties) with internal angles not exceeding 135o or helical reinforcement. The transverse reinforcing bars are provided to ensure that every longitudinal bar nearest to the compression face has effective lateral support against buckling. Clause 26.5.3.2 stipulates the guidelines of the arrangement of transverse reinforcement. The salient points are:

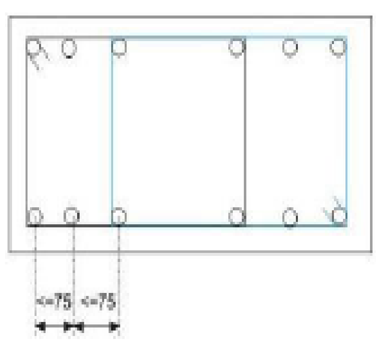


Figure 3.4 Lateral tie (Arrangement 1)

(a) Transverse reinforcement shall only go round corner and alternate bars if the longitudinal bars are not spaced more than 75 mm on either side (Fig.3.4).

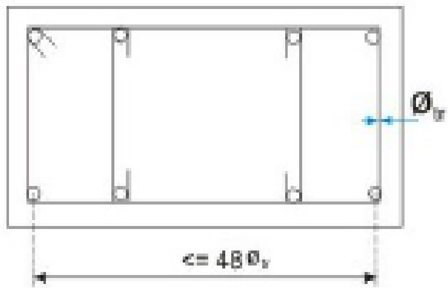


Figure 3.5 Lateral tie (Arrangement 2)

(b) Longitudinal bars spaced at a maximum distance of 48 times the diameter of the tie shall be tied by single tie and additional open ties for in between longitudinal bars (Fig.3.5).

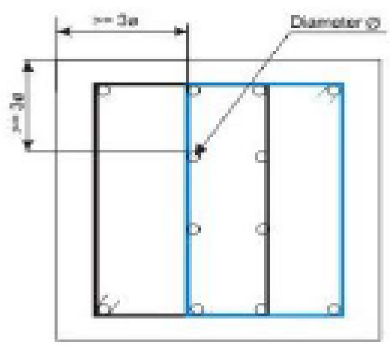


Figure 3.6 Lateral tie (Arrangement 3)

(c) For longitudinal bars placed in more than one row (Fig.10.21.9): (i) transverse reinforcement is provided for the outer-most row in accordance with (a) above, and (ii) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.

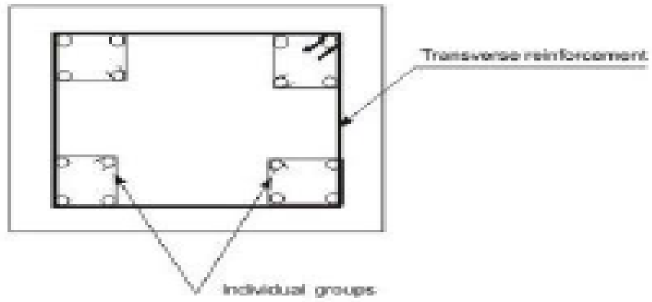


Figure 3.7 Lateral tie (Arrangement 4)

Pitch and Diameter of Lateral Ties

(a) Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:
(i) the least lateral dimension of the compression members;
(ii) sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and
(iii) 300 mm.
(b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

Assumptions in the Design of Compression Members by Limit State of Collapse

The following are the assumptions in addition to given in 38.1 (a) to (e) for flexure for the design of compression members (cl. 39.1 of IS 456).
(i) The maximum compressive strain in concrete in axial compression is taken as 0.002.
(ii) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

Minimum Eccentricity

In practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity as stipulated in cl. 25.4 of IS 456 and given below (Fig.3.2c)
$e_{x\,min} \geq$ greater of ($l/500 + D/30$) or 20 mm
$e_{y\,min} \geq$ greater of ($l/500 + b/30$) or 20 mm
where l, D and b are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area A_c and longitudinal steel of areas A_{sc} effectively held by lateral ties at intervals. Assuming the design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, we can write
$P_u = 0.4f_{ck} A_c + 0.67f_y A_{sc}$ (1)

Where Pu = factored axial load on the member,
fck = characteristic compressive strength of the concrete,
Ac = area of concrete,
fy = characteristic strength of the compression reinforcement, and
Asc = area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns Ac and Asc to be determined from one equation. The equation is recast in terms of Ag, the gross area of concrete and p, the percentage of compression reinforcement employing

$Asc = pAg/100$ (2)

$Ac = Ag(1 - p/100)$ (3)

Accordingly, we can write

$Pu/Ag = 0.4fck + (p/100)(0.67fy - 0.4fck)$ (4)

Equation 4 can be used for direct computation of Ag when Pu, fck and fy are known by assuming p ranging from 0.8 to 4 as the minimum and maximum percentages of longitudinal reinforcement. Equation 10.4 also can be employed to determine Ag and p in a similar manner by assuming p.

UNIT-V

DESIGN OF FOOTING AND STAIR CASE

The superstructure is placed on the top of the foundation structure, designated as substructure as they are placed below the ground level. The elements of the superstructure transfer the loads and moments to its adjacent element below it and finally all loads and moments come to the foundation structure, which in turn, transfers them to the underlying soil or rock. Thus, the foundation structure effectively supports the superstructure. However, all types of soil get compressed significantly and cause the structure to settle. Accordingly, the major requirements of the design of foundation structures are the two as given below (see cl.34.1 of IS 456)
:

1. Foundation structures should be able to sustain the applied loads, moments, forces and induced reactions without exceeding the safe bearing capacity of the soil.

2. The settlement of the structure should be as uniform as possible and it should be within the tolerable limits. It is well known from the structural analysis that differential settlement of supports causes additional moments in statically indeterminate structures. Therefore, avoiding the differential settlement is considered as more important than maintaining uniform overall settlement of the structure.

Types of Foundation Structures

1. Shallow Foundation

Shallow foundations are used when the soil has sufficient strength within a short depth below the ground level. They need sufficient plan area to transfer the heavy loads to the base soil. These heavy loads are sustained by the reinforced concrete columns or walls (either of bricks or reinforced concrete) of much less areas of cross-section due to high strength of bricks or reinforced concrete when compared to that of soil. The strength of the soil, expressed as the safe bearing capacity of the soil is normally supplied by the geotechnical experts to the structural engineer. Shallow foundations are also designated as footings. The different types of shallow foundations or footings are discussed below.

(i) Plain concrete pedestal footings
(ii) Isolated footings
(iii) Combined footings
(iv) Strap footings
(v) Strip foundation or wall footings
(vi) Raft or mat foundation

2. Deep foundations

As mentioned earlier, the shallow foundations need more plan areas due to the low strength of soil compared to that of masonry or reinforced concrete. However, shallow foundations are selected when the soil has moderately good strength, except the raft foundation which is good in poor condition of soil also. Raft foundations are under the category of shallow foundation as they have comparatively shallow depth than that of deep foundation. It is worth mentioning that the depth of raft foundation is much larger than those of other types of shallow foundations.

However, for poor condition of soil near to the surface, the bearing capacity is very less and foundation needed in such situation is the pile foundation. Piles are, in fact, small diameter columns which are driven or cast into the ground by suitable means. Precast piles are driven and cast-in-situ are cast. These piles support the structure by the skin friction between the pile surface and the surrounding soil and end bearing force, if such resistance is available to provide the bearing force. Accordingly, they are designated as frictional and end bearing piles. They are normally provided in a group with a pile cap at the top through which the loads of the superstructure are transferred to the piles.

Piles are very useful in marshy land where other types of foundation are impossible to construct. The length of the pile which is driven into the ground depends on the availability of hard soil/rock or the actual load test. Another advantage of the pile foundations is that they can resist uplift also in the same manner as they take the compression forces just by the skin friction in the opposite direction.

However, driving of pile is not an easy job and needs equipment and specially trained persons or agencies. Moreover, one has to select pile foundation in such a situation where the adjacent buildings are not likely to be damaged due to the driving of piles. The choice of driven or bored piles, in this regard, is critical.
Exhaustive designs of all types of foundations mentioned above are beyond the scope of this course. Accordingly, this module is restricted to the design of some of the shallow footings, frequently used for normal low rise buildings only.

Isolated Footing

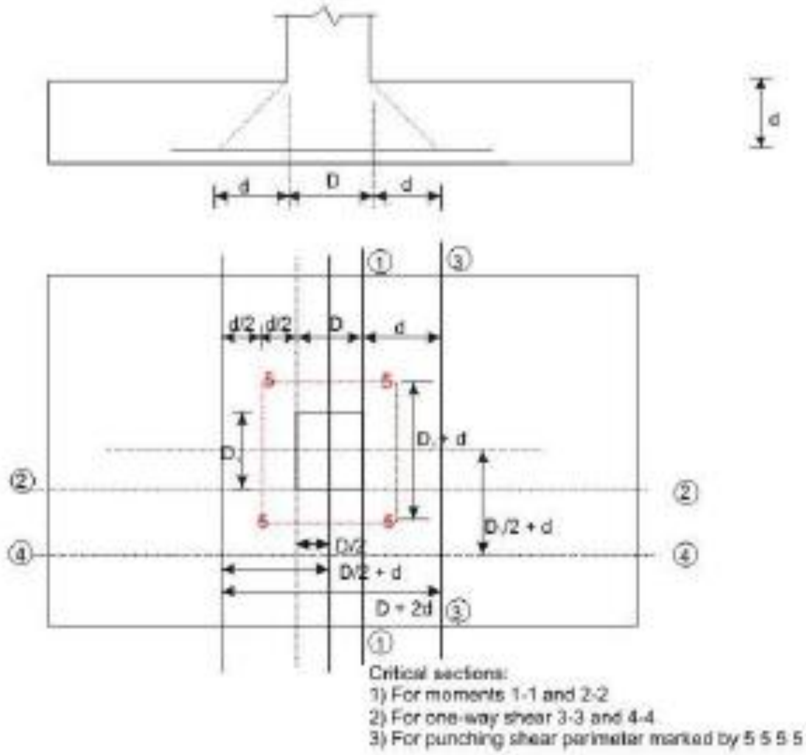


Figure 3.8: Uniform and rectangular footing

Design Considerations

(a) Minimum nominal cover (cl. 26.4.2.2 of IS 456)

The minimum nominal cover for the footings should be more than that of other structural elements of the superstructure as the footings are in direct contact with the soil. Clause 26.4.2.2 of IS 456 prescribes a minimum cover of 50 mm for footings. However, the actual cover may be even more depending on the presence of harmful chemicals or minerals, water table etc.

(b) Thickness at the edge of footings (cls. 34.1.2 and 34.1.3 of IS 456)

The minimum thickness at the edge of reinforced and plain concrete footings shall be at least 150 mm for footings on soils and at least 300 mm above the top of piles for footings on piles, as per the stipulation in cl.34.1.2 of IS 456.

For plain concrete pedestals, the angle α (see Fig.11.28.1) between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be determined from the following expression (cl.34.1.3 of IS 456)

0.5 tan0.9{(100/) 1} a ck qf□ □ □

where qa = calculated maximum bearing pressure at the base of pedestal in N/mm2, and
fck = characteristic strength of concrete at 28 days in N/mm2.

(c) Bending moments (cl. 34.2 of IS 456)

1. It may be necessary to compute the bending moment at several sections of the footing depending on the type of footing, nature of loads and the distribution of pressure at the base of the footing. However, bending moment at any section shall be determined taking all forces acting over the entire area on one side of the section of the footing, which is obtained by passing a vertical plane at that section extending across the footing (cl.34.2.3.1 of IS 456).

2. The critical section of maximum bending moment for the purpose of designing an isolated concrete footing which supports a column, pedestal or wall shall be:
(i) at the face of the column, pedestal or wall for footing supporting a concrete column, pedestal or reinforced concrete wall, and
(ii) halfway between the centre-line and the edge of the wall, for footing under masonry wall. This is stipulated in cl.34.2.3.2 of IS 456.
The maximum moment at the critical section shall be determined as mentioned in 1 above.
For round or octagonal concrete column or pedestal, the face of the column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal (see cl.34.2.2 of IS 456 and Figs.11.28.13a and b).

(d) Shear force (cl. 31.6 and 34.2.4 of IS 456)

Footing slabs shall be checked in one-way or two-way shears depending on the nature of bending. If the slab bends primarily in one-way, the footing slab shall be checked in one-way vertical shear. On the other hand, when the bending is primarily two-way, the footing slab shall be checked in two-way shear or punching shear. The respective critical sections and design shear strengths are given below:

1. One-way shear (cl. 34.2.4 of IS 456)
One-way shear has to be checked across the full width of the base slab on a vertical section located from the face of the column, pedestal or wall at a distance equal to
(i) effective depth of the footing slab in case of footing slab on soil, and
(ii) half the effective depth of the footing slab if the footing slab is on piles.
The design shear strength of concrete without shear reinforcement is given in Table 19 of cl.40.2 of IS 456.

2. Two-way or punching shear (cls.31.6 and 34.2.4)
Two-way or punching shear shall be checked around the column on a perimeter half the effective depth of the footing slab away from the face of the column or pedestal.
The permissible shear stress, when shear reinforcement is not provided, shall not exceed , where ks = (0.5 + $c\beta$), but not greater than one, $c\beta$ being the ratio of short side to long side of the column, and = 0.25(fck)1/2 in limit state method of design, as stipulated in cl.31.6.3 of IS 456. sc
k c
Normally, the thickness of the base slab is governed by shear. Hence, the necessary thickness of the slab has to be provided to avoid shear reinforcement.

(e) Bond (cl.34.2.4.3 of IS 456)

The critical section for checking the development length in a footing slab shall be the same planes as those of bending moments in part (c) of this section. Moreover, development length shall be checked at all other sections where they change abruptly. The critical sections for checking the development length are given in cl.34.2.4.3 of IS 456, which further recommends to check the anchorage requirements if the reinforcement is curtailed, which shall be done in accordance with cl.26.2.3 of IS 456

.

(f) Tensile reinforcement (cl.34.3 of IS 456)

The distribution of the total tensile reinforcement, calculated in accordance with the moment at critical sections, as specified in part (c) of this section, shall be done as given below for one-way and two-way footing slabs separately.

(i) In one-way reinforced footing slabs like wall footings, the reinforcement shall be distributed uniformly across the full width of the footing i.e., perpendicular to the direction of wall. Nominal distribution reinforcement shall be provided as per cl. 34.5 of IS 456 along the length of the wall to take care of the secondary moment, differential settlement, shrinkage and temperature effects.

(ii) In two-way reinforced square footing slabs, the reinforcement extending in each direction shall be distributed uniformly across the full width/length of the footing.

iii) In two-way reinforced rectangular footing slabs, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing slab. In the short direction, a central band equal to the width of the footing shall be marked along the length of the footing, where the portion of the reinforcement shall be determined as given in the equation below. This portion of the reinforcement shall be distributed across the central band

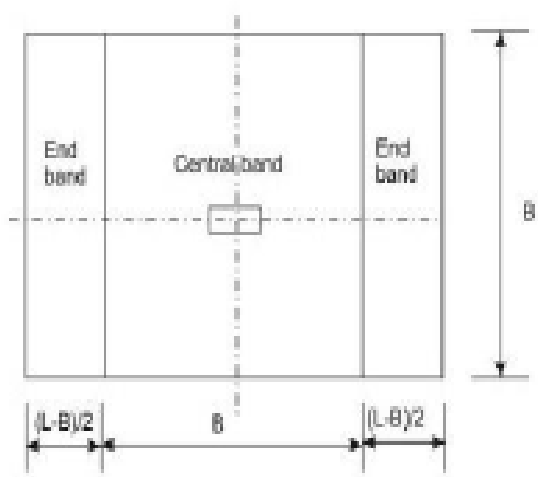


Figure 3.10 Bands for reinforcement in rectangular footing

Reinforcement in the central band = {2/(β+1)} (Total reinforcement in the short direction)
Where β is the ratio of longer dimension to shorter dimension of the footing slab (Fig.3.10).
Each of the two end bands shall be provided with half of the remaining reinforcement, distributed
uniformly across the respective end band.

(g) Transfer of load at the base of column (cl.34.4 of IS 456)

All forces and moments acting at the base of the column must be transferred to the pedestal,
if any, and then from the base of the pedestal to the footing, (or directly from the base of the column to the footing if there is no pedestal) by compression in concrete and steel and tension in steel. Compression forces are transferred through direct bearing while tension forces are transferred through developed reinforcement. The permissible bearing stresses on full area of concrete shall be taken as given below from cl.34.4 of IS 456:

□ br = 0.25fck , in working stress method, and

□ br = 0.45fck , in limit state method

The stress of concrete is taken as 0.45fck
while designing the column. Since the area of
footing is much larger, this bearing stress of concrete in column may be increased considering the dispersion of the concentrated load of column to footing. Accordingly, the permissible bearing stress of concrete in footing is given by (cl.34.4 of IS 456):

□ br = 0.45fck($A$1/$A$2)1/2

with a condition that

($A$1/$A$2)1/22.0 (11.8) ≤ 2

where $A$1= maximum supporting area of footing for bearing which is geometrically imilar to and concentric with the loaded area $A$2
.$A$2= loaded area at the base of the column.

The above clause further stipulates that in sloped or stepped footings, $A$1 may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal.
If the permissible bearing stress on concrete in column or in footing is exceeded, reinforcement shall be provided for developing the excess force (cl.34.4.1 of IS 456), either by extending the longitudinal bars of columns into the footing (cl.34.4.2 of IS 456) or by providing dowels as stipulated in cl.34.4.3 of IS 456 and given below:

(i) Sufficient development length of the reinforcement shall be provided to transfer the compression or tension to the supporting member in accordance with cl.26.2 of IS 456, when transfer of force is accomplished by reinforcement of column (cl.34.4.2 of IS 456).
(ii) Minimum area of extended longitudinal bars or dowels shall be 0.5 per cent of the cross-sectional area of the supported column or pedestal (cl.34.4.3 of IS 456).
(iii) A minimum of four bars shall be provided (cl.34.4.3 of IS 456).
(iv) The diameter of dowels shall not exceed the diameter of column bars by more than 3 mm.

(v) Column bars of diameter larger than 36 mm, in compression only can be doweled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel, as stipulated in cl.34.4.4 of IS 456.

(h) Nominal reinforcement (cl. 34.5 of IS 456)

Clause 34.5.1 of IS 456 stipulates the minimum reinforcement and spacing of the bars in footing slabs as per the requirements of solid slab (cls.26.5.2.1 and 26.3.3b(2) of IS 456, respectively).

Design of Staircase

The staircase is an important component of a building, and often the only means of access between the various floors in the building. It consists of a *flight* of steps, usually with one or more intermediate *landings* (horizontal slab platforms) provided between the floor levels. The horizontal top portion of a step (where the foot rests) is termed *tread* and the vertical projection of the step (i.e., the vertical distance between two neighbouring steps) is called *riser* [Fig. 2.10]. Values of 300 mm and 150 mm are ideally assigned to the tread and riser respectively — particularly in public buildings. However, lower values of tread (up to 250 mm) combined with higher values of riser (up to 190 mm) are resorted to in residential and factory buildings. The *width* of the stair is generally around 1.1 – 1.6*m*, and in any case, should normally not be less than 850 mm; large stair widths are encountered in entrances to public buildings. The horizontal projection (plan) of an inclined flight of steps, between the first and last risers, is termed *going*. A typical flight of steps consists of two landings and one going, as depicted in Fig. 2.10(a). Generally, risers in a flight should not exceed about 12 in number. The steps in the flight can be designed in a number of ways: with *waist slab*, with *tread-riser* arrangement (without waist slab) or with *isolated tread slabs* — as shown in Fig. 2.10(b), (c), (d) respectively.

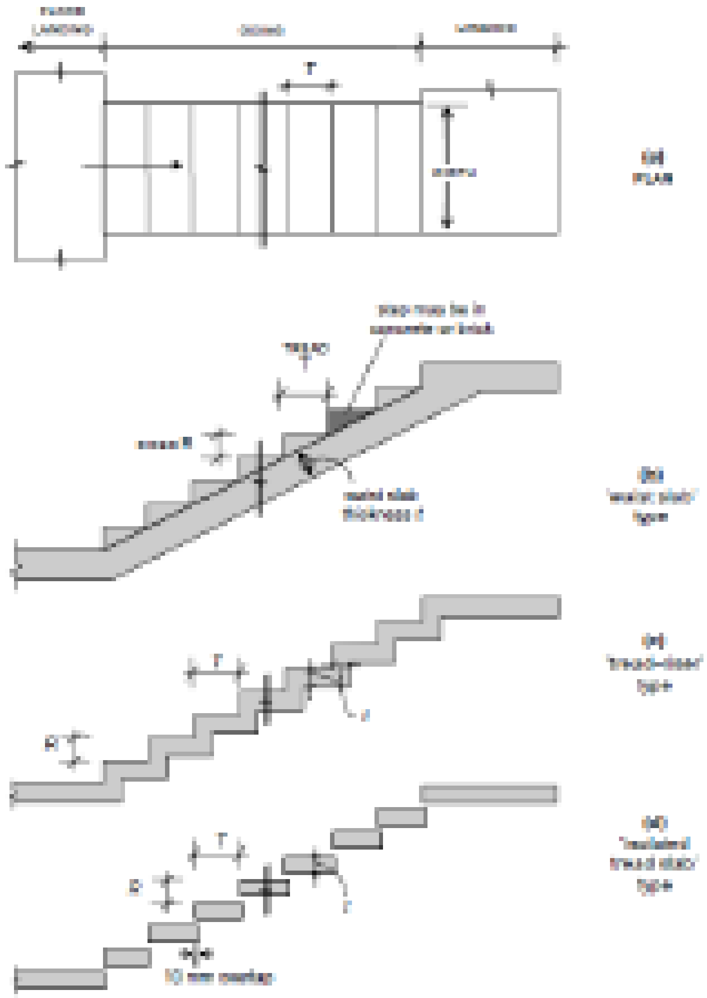


Figure 2.10 A typical flight in a staircase

TYPES OF STAIRCASES

Geometrical Configurations

A wide variety of staircases are met with in practice. Some of the more common geometrical configurations are depicted in Fig. 2.11. These include:

• straight stairs (with or without intermediate landing) [Fig. 2.11 (a)]
• quarter-turn stairs [Fig. 2.11 (b)]
• dog-legged stairs [Fig. 2.11 (c)]
open well stairs [Fig. 2.11 (d)]
• spiral stairs [Fig. 2.11 (e)]
• helicoidal stairs [Fig. 2.11 (f)]

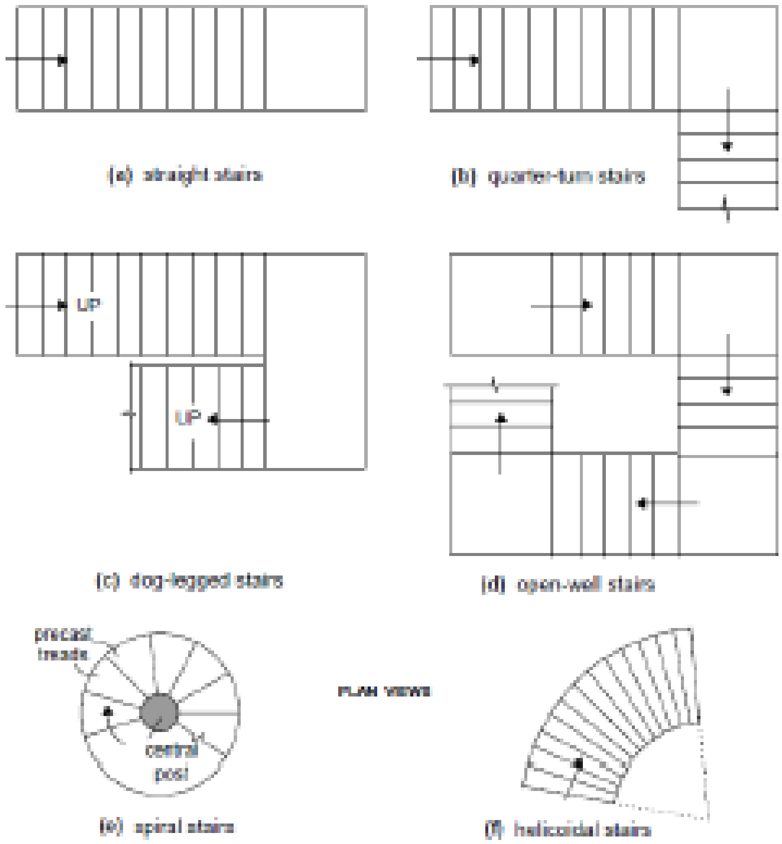


Fig. 2.11 Common geometrical configurations of stairs

Structural Classification

Structurally, staircases may be classified largely into two categories, depending on the predominant direction in which the slab component of the stair undergoes flexure:
1. Stair slab spanning transversely (stair widthwise);
2. Stair slab spanning longitudinally (along the incline).

Stair Slab Spanning Transversely

The slab component of the stair (whether comprising an isolated tread slab, a tread-riser unit or a waist slab) is supported on its side(s) or cantilevers laterally from a central support. The slab supports gravity loads by bending essentially in a *transverse vertical plane*, with the span along the *width* of the stair.

In the case of the cantilevered slabs, it is economical to provide isolated treads (without risers). However, the tread-riser type of arrangement and the waist slab type are also sometimes employed in practice, as cantilevers. The spandrel beam is subjected to torsion (_equilibrium torsion'), in addition to flexure and shear.

When the slab is supported at the two sides by means of ‗stringer beams' or masonry walls, it may be designed as simply supported, but reinforcement at the top should be provided near the supports to resist the ‗negative' moments that may arise on account of possible partial fixity.

Stair Slab Spanning Longitudinally

In this case, the supports to the stair slab are provided parallel to the riser at two or more locations, causing the slab to bend longitudinally between the supports. It may be noted that longitudinal bending can occur in configurations other than the straight stair configuration, such as quarter-turn stairs, dog-legged stairs, open well stairs and helicoidal stairs .

The slab arrangement may either be the conventional ‗waist slab' type or the ‗tread-riser' type. The slab thickness depends on the ‗effective span', which should be taken as the centre-to-centre distance between the beam/wall supports, according to the Code (Cl. 33.1a, c).In certain situations, beam or wall supports may not be available parallel to the riser at the landing. Instead, the flight is supported between the landings, which span transversely, parallel to the risers. In such cases, the Code(Cl. 33.1b) specifies that the effective span for the flight (spanning longitudinally) should be taken as *the going of the stairs plus at each end either half the width of the landing or one metre, whichever is smaller.*

Numerical Problem

Design a (‗waist slab' type) dog-legged staircase for an office building, given the following data:

• Height between floor = 3.2 m;

• Riser = 160 mm, tread = 270 mm;

• Width of flight = landing width = 1.25 m

• Live load = 5.0 kN/m2

• Finishes load = 0.6 kN/m2

Assume the stairs to be supported on 230 mm thick masonry walls at the outer edges of the landing, parallel to the risers [Fig. 12.13(a)]. Use M 20 concrete and Fe 415 steel. Assume *mild* exposure conditions.

Solution

Given: R = 160 mm, T = 270 mm ⇒+$RT$22

= 314 mm Effective span = c/c distance between supports = 5.16 m [Fig below].

• Assume a waist slab thickness ≈l20 = 5160/20 = 258 →260 mm.

Assuming 20 mm clear cover (*mild* exposure) and 12 θ main bars,

effective depth d = 260 – 20 – 12/2 = 234 mm.

The slab thickness in the landing regions may be taken as 200 mm, as the bending moments are relatively low here.

Loads on going [fig. below] on projected plan area:

(1) self-weight of waist slab @ 25 × 0.26 × 314/270 = 7.56 kN/m2

(2) self-weight of steps @ 25 × (0.5x0.16) = 2.00 kN/m2

(3) finishes (given) = 0.60 kN/m2

(4) live load (given) = 5.00 kN/m2

Total =15.16 kN/m2

⇒Factored load = 15.16 × 1.5 = 22.74 kN/m2

• *Loads on landing*
(1) self-weight of slab @ 25 × 0.20 = 5.00 kN/m2
(2) finishes @ 0.6 kN/m2
(3) live loads @ 5.0 kN/m2
Total =10.60 kN/*m*2
⇒Factored load = 10.60 × 1.5 = 15.90 kN/m2
• *Design Moment* [Fig. below]
Reaction *R*= (15.90x1.365)+(22.74x2.43)/2 = 49.33 kN/m
Maximum moment at midspan:
Mu = (49.33 × 2.58) – (15.90 × 1.365) × (2.58 – 1.365/2)
– (22.74) × (2.58 – 1.365)2/2
= 69.30 kNm/m

• *Maiinforcement*

= 1.265 MPa R bd
Assuming *fck* = 20 MPa, *fy* = 415 MPa,
2 0.381 10 100 100 t st pAx
⇒ 2 3 2 ()(0.38110)10 234 892/ st req Axxxmm m
Required spacing of 12 θ bars = 127 mm
Required spacing of 16 θ bars = 225 mm
Provide 16 θ @ 220c/c

• *Distributors*
2 ()0.0012312/ st req Abt mm m
spacing 10 θ bars = 251 mm
Provide 10 θ @ 250c/c as distributors.

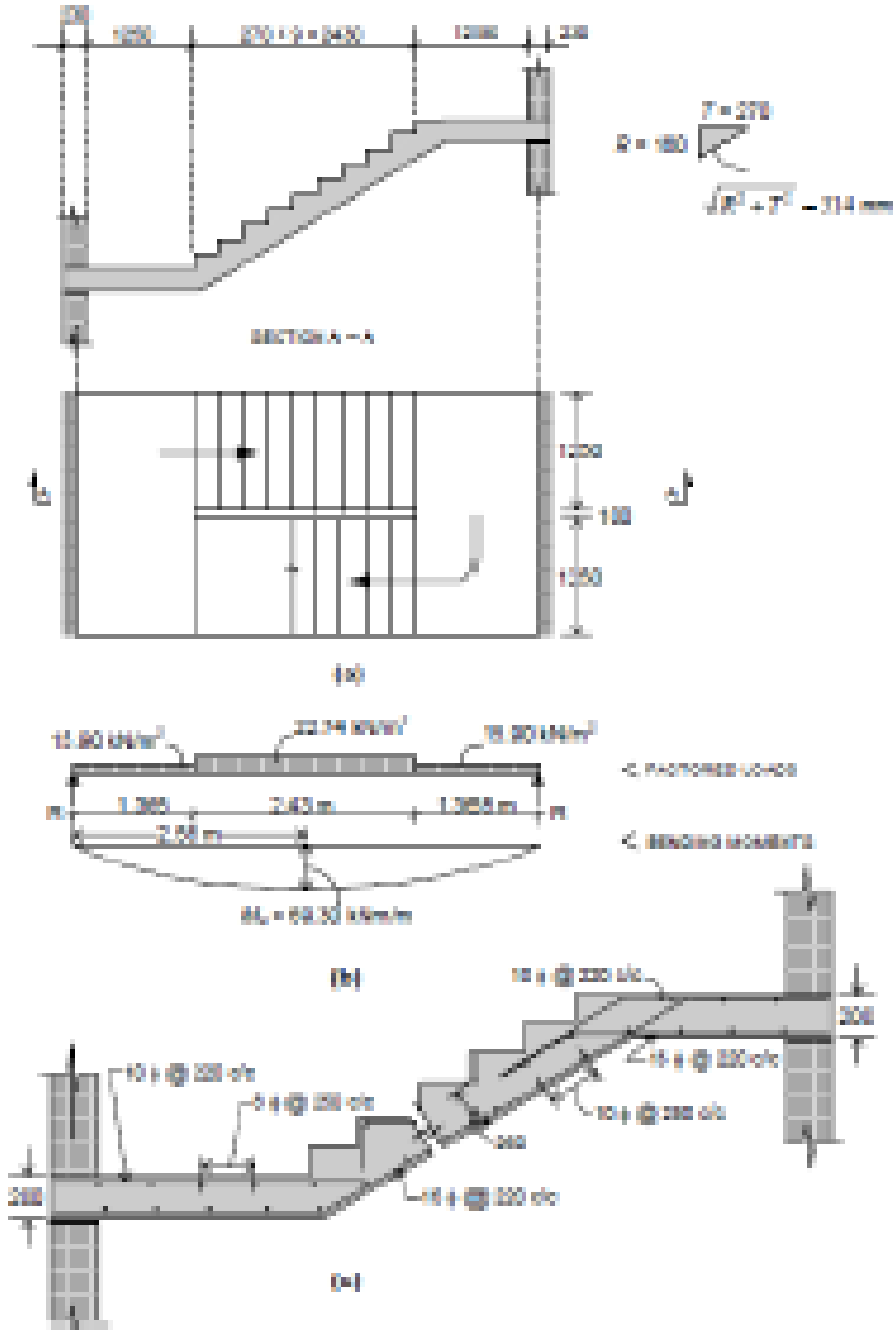


Figure for numerical problem