WINTER– 2017 EXAMINATION

Subject **17301**

Model Answer

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any TEN of the following:**	**20**
	a)	Find the radius of curvature of the curve $y=x^3$ at $(2,8)$	**02**
	Ans	$y=x^3$	
		$\therefore \frac{dy}{dx}=3x^2$	
		$\therefore \frac{d^2y}{dx^2}=6x$	½
		at $(2,8)$	
		$\frac{dy}{dx}=12$	
		$\frac{d^2y}{dx^2}=12$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(12)^2\right]^{\frac{3}{2}}}{12}$	½
		$\therefore \rho=145.50$	½
		--	
	b)	Find the point on the curve $y=7x-3x^2$, where the inclination of the tangent is 45^0	**02**

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	$y=7x-3x^2$	
	Ans	$\therefore \frac{dy}{dx}=7-6x$	½
		$\therefore m=7-6x$	
		Given inclination of the tangent is 45^0	
		$\therefore m=\tan 45^0$	
		$\therefore m=1$	½
		$7-6x=1$	
		$-6x=-6$	
		$x=1$	½
		$\therefore y=4$	½
		$\therefore$ point is $(1,\ 4)$	
		--	
	c)	Evaluate: $\int x.\sin x\ dx$	**02**
	Ans	$\int x.\sin x\ dx$	
		$=x\int \sin xdx-\int\left(\int \sin xdx.\frac{d}{dx}x\right)dx$	½
		$=x(-\cos x)-\int(-\cos x).1\ dx$	½+½
		$=-x\cos x+\sin x+c$	½
	d)	Evaluate: $\int e^{2\log x}\ dx$	**02**
	Ans	$\int e^{2\log x}\ dx$	
		$=\int e^{l\text{og}x^2}\ dx$	½
		$=\int x^2\ dx$	½
		$=\frac{x^3}{3}+c$	1
		--	
	e)	Evaluate $\int \sin^2 x\ dx$	**02**
	Ans	$\int \sin^2 xdx$	
		$=\int \frac{1-\cos 2x}{2}\ dx$	1
		$=\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	Evaluate: $\int \frac{dx}{\sqrt{4-9x^2}}$	**02**
	Ans	$\int \frac{dx}{\sqrt{4-9x^2}}$	
		$= \int \frac{dx}{\sqrt{(2)^2-(3x)^2}}$	1
		$= \sin^{-1}\left(\frac{3x}{2}\right).\frac{1}{3}+c$	1
		OR	
		$\int \frac{dx}{\sqrt{4-9x^2}}$	
		$= \frac{1}{3}\int \frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2-x^2}}$	½
		$= \frac{1}{3}\sin^{-1}\left(\frac{x}{2/3}\right)+c$	1
		$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+c$	½
		--	
	g)	Evaluate: $\int_0^{\frac{\pi}{2}} \sin x \ \cos x \ dx$	**02**
	Ans	$\int_0^{\frac{\pi}{2}} \sin x \ \cos x \ dx$	
		$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin x \ \cos x \ dx$	
		$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2x \ dx$	½
		$= \frac{1}{2}\left(-\cos 2x \cdot \frac{1}{2}\right)_0^{\frac{\pi}{2}}$	½
		$= -\frac{1}{4}(\cos\pi - \cos 0)$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	g)	$=-\frac{1}{4}(-1-1)$	
		$=\frac{1}{2}$	½
		OR	
		$\int_0^{\frac{\pi}{2}} \sin x \ \cos x \ dx$	
		Put $\sin x = t$ when $x \to 0$ to $\frac{\pi}{2}$, $t \to 0$ to 1	½
		$\therefore \cos x dx = dt$	½
		$\therefore \int_0^1 t \ dt$	
		$=\left[\frac{t^2}{2}\right]_0^1$	½
		$=\frac{1}{2}\left[t^2\right]_0^1$	
		$=\frac{1}{2}\left(1^2-0\right)$	
		$=\frac{1}{2}$	½
	h)	Find the area under the curve $y = x^2$ from $x=0$ to $x=3$ with x-axis	**02**
	Ans	Area $A=\int_a^b ydx$	
		$=\int_0^3 x^2 dx$	½
		$=\left[\frac{x^3}{3}\right]_0^3$	½
		$=\left[\frac{3^3}{3}-0\right]$	½
		$=\frac{27}{3}$	
		$=9$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	i)	Find the order and degree of the equation $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{5}{3}} = 2\frac{d^2y}{dx^2}$	**02**
	Ans	$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{5}{3}} = 2\frac{d^2y}{dx^2}$	
		$\therefore Order = 2$	1
		$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{5} = 8\left(\frac{d^2y}{dx^2}\right)^3$	
		$\therefore Degree = 3$	1
		---	
	j)	Verify that y = log x is a solution of $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	**02**
	Ans	$y=\log x$	
		$\therefore \frac{dy}{dx}=\frac{1}{x}$	½
		$\therefore \frac{d^2y}{dx^2}=-\frac{1}{x^2}$	½
		$L.H.S.=x\frac{d^2y}{dx^2}+\frac{dy}{dx}$	
		$=x\left(-\frac{1}{x^2}\right)+\frac{1}{x}$	
		$=-\frac{1}{x}+\frac{1}{x}$	
		$=0=R.H.S.$	1
		OR	
		$y=\log x$	
		$\therefore \frac{dy}{dx}=\frac{1}{x}$	½
		$\therefore x\frac{dy}{dx}=1$	½
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}(1)=0$	1
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	

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1.	k)	Find the probability of getting sum of numbers is 9 with two dice.	**02**
	Ans	$S=\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$ $(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$ $(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$ $(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$ $(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$ $(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$	
		$n(S)=36$	½
		sum of numbers is 9 $\therefore A=\{(4,5)\ (5,4)\ (3,6)\ (6,3)\}$ $\therefore n(A)=4$	½
		$p(A)=\dfrac{n(A)}{n(S)}$ $p(A)=\dfrac{4}{36}$ or 0.111	1
	l)	Three fair coins are tossed. Find the probability that atleast two heads appear.	**02**
	Ans	$S=\{HHH,HTT,THT,TTH,HTH,HHT,THH,TTT\}$ $\therefore n(S)=8$	½
		atleast two heads $A=\{HHH,HTH,HHT,THH\}$ $n(A)=4$	½
		$\therefore p(A)=\dfrac{n(A)}{n(S)}$ $\therefore p(A)=\dfrac{4}{8}=\dfrac{1}{2}$ or 0.5	1
2.		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	Evaluate: $\int \dfrac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx$	**04**
	Ans	$\int \dfrac{\sec^2 x}{(1+\tan x)(2+\tan x)}\,dx$ Put $\tan x=t$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	$\sec^2 x dx = dt$	1
		$\therefore \int \frac{dt}{(1+t)(2+t)}$	
		Let $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$	
		$1 = A(2+t) + B(1+t)$	
		Put $t = -1$	
		$1 = A(1)$	
		$\therefore A = 1$	1
		Put $t = -2$	
		$1 = B(-1)$	
		$\therefore B = -1$	1
		$\therefore \frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$	
		$\therefore \int \frac{dt}{(1+t)(2+t)} = \int \left(\frac{1}{1+t} + \frac{-1}{2+t}\right) dt$	
		$\therefore = \log(1+t) - \log(2+t) + c$	½
		$\therefore = \log(1+\tan x) - \log(2+\tan x) + c$ or $= \log\left(\frac{1+\tan x}{2+\tan x}\right) + c$	½
		OR	
		$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$	
		Put $\tan x = t$	
		$\sec^2 x dx = dt$	1
		$\therefore \int \frac{dt}{(1+t)(2+t)}$	
		$= \int \frac{dt}{t^2 + 3t + 2}$	
		Third term $= \frac{(3)^2}{4} = \frac{9}{4}$	
		$= \int \frac{dt}{t^2 + 3t + \frac{9}{4} - \frac{9}{4} + 2}$	½
		$= \int \frac{dt}{\left(t + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	$=\frac{1}{2\left(\frac{1}{2}\right)}\log\left(\frac{t+\frac{3}{2}-\frac{1}{2}}{t+\frac{3}{2}+\frac{1}{2}}\right)+c$	1
		$=\log\left(\frac{t+1}{t+2}\right)+c$	½
		$=\log\left(\frac{1+\tan x}{2+\tan x}\right)+c$	½
		--	
	b)	Evaluate: $\int\cos(\log x)\ dx$	**04**
	Ans	$\int\cos(\log x)\ dx$	
		Put $\log x=t\Rightarrow x=e^t$	
		$\therefore\frac{1}{x}dx=dt$	
		$\therefore dx=xdt$	
		$\therefore dx=e^t dt$	1
		$\therefore\int e^t\cos t dt$	
		$=\frac{e^t}{1+1}(1\cos t+1\sin t)+c$	2
		$=\frac{x}{2}(\cos(\log x)+\sin(\log x))+c$	1
		OR	
		$\int\cos(\log x)\ dx$	
		Put $\log x=t\Rightarrow x=e^t$	
		$\therefore\frac{1}{x}dx=dt$	
		$\therefore dx=xdt$	
		$\therefore dx=e^t dt$	1
		$\therefore I=\int e^t\cos t dt$	
		$=\cos t\int e^t dt-\int\left(\int e^t dt\frac{d}{dt}\cos t\right)dt$	
		$=\cos t\ e^t-\int e^t(-\sin t)dt$	
		$=\cos t\ e^t+\int e^t\sin t dt+c$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	b)	$= \cos t\ e^t + e^t \sin t - \int e^t \cos t dt + c$	
		$\therefore I = \cos t\ e^t + e^t \sin t - I + c$	1
		$\therefore 2I = \cos t\ e^t + e^t \sin t + c$	
		$\therefore I = \frac{e^t}{2}(\cos t + \sin t) + c$	
		$\therefore I = \frac{x}{2}(\cos(\log x) + \sin(\log x)) + c$	1
		OR	
		$I = \int \cos(\log x)\ dx$	
		$\therefore I = \int \cos(\log x).1\ dx$	½
		$\therefore I = \cos(\log x)\int 1dx - \int\left(\int 1dx \frac{d}{dx}\cos(\log x)\right) dx$	½
		$\therefore I = \cos(\log x)x - \int x\left(\frac{-\sin(\log x)}{x}\right) dx$	½
		$\therefore I = x\cos(\log x) + \int \sin(\log x)\ dx$	½
		$\therefore I = x\cos(\log x) + \int \sin(\log x).1dx$	½
		$\therefore I = x\cos(\log x) + \sin(\log x)x - \int x\left(\frac{\cos(\log x)}{x}\right) dx$	½
		$\therefore I = x\cos(\log x) + x\sin(\log x) - \int \cos(\log x)\ dx$	
		$\therefore I = x\cos(\log x) + x\sin(\log x) - I + c$	½
		$\therefore 2I = x(\cos(\log x) + \sin(\log x)) + c$	
		$\therefore I = \frac{x}{2}(\cos(\log x) + \sin(\log x)) + c$	½
	c)	Evaluate $\int x \tan^{-1} x dx$	**04**
	Ans	$\int x \tan^{-1} x dx$	
		$= \tan^{-1} x \int x dx - \int\left(\int x dx\ \frac{d}{dx}\tan^{-1} x\right)dx$	1
		$= \tan^{-1} x\ \frac{x^2}{2} - \int \frac{x^2}{2}\frac{1}{1+x^2} dx$	1
		$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2} dx$	½
		$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)+c$	1
	d)	Find the maximum and minimum value of $y=2x^3-3x^2-36x+10$	**04**
	Ans	Let $y=2x^3-3x^2-36x+10$	
		$\therefore\frac{dy}{dx}=6x^2-6x-36$	½
		$\therefore\frac{d^2y}{dx^2}=12x-6$	½
		Consider $\frac{dy}{dx}=0$	
		$6x^2-6x-36=0$	½
		$x^2-x-6=0$	
		$\therefore x=-2$ or $x=3$	½
		at $x=-2$	
		$\frac{d^2y}{dx^2}=12(-2)-6=-30<0$	½
		$\therefore y$ is maximum at $x=-2$	
		$y_{\max}=2(-2)^3-3(-2)^2-36(-2)+10$	
		$=54$	½
		$at\ \ x=3$	
		$\frac{d^2y}{dx^2}=12(3)-6=30>0$	½
		$\therefore y$ is minimum at $x=3$	
		$y_{\min}=2(3)^3-3(3)^2-36(3)+10$	
		$=-71$	½
		--	
	e)	Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=1$ at $\left(\frac{1}{4},\frac{1}{4}\right)$	**04**
	Ans	$\sqrt{x}+\sqrt{y}=1$	
		$\therefore\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$	½
		$\therefore\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2	e)	$\frac{d^2y}{dx^2}=\frac{-\left[\sqrt{x}\frac{1}{2\sqrt{y}}\frac{dy}{dx}-\sqrt{y}\frac{1}{2\sqrt{x}}\right]}{\left(\sqrt{x}\right)^2}$	½
		$\frac{d^2y}{dx^2}=\frac{-\left[\sqrt{x}\frac{1}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right)-\sqrt{y}\frac{1}{2\sqrt{x}}\right]}{x}$	
		$\frac{d^2y}{dx^2}=\frac{-\left[-\frac{1}{2}-\frac{\sqrt{y}}{2\sqrt{x}}\right]}{x}$	½
		$\therefore$ at $\left(\frac{1}{4},\frac{1}{4}\right)$	
		$\frac{dy}{dx}=-\frac{\sqrt{\frac{1}{4}}}{\sqrt{\frac{1}{4}}}=-1$	½
		$\frac{d^2y}{dx^2}=\frac{-\left[-\frac{1}{2}-\frac{1}{2}\right]}{\frac{1}{4}}=4$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+\left(-1\right)^2\right]^{\frac{3}{2}}}{4}$	½
		$\therefore \rho=0.707$	½
		--	
	f)	Find the equation of tangent and normal to the curve $x^2+3xy+y^2=5$ at $(1,1)$	**04**
	Ans	$x^2+3xy+y^2=5$	
		$2x+3\left(x\frac{dy}{dx}+y\right)+2y\frac{dy}{dx}=0$	½
		$2x+3x\frac{dy}{dx}+3y+2y\frac{dy}{dx}=0$	
		$(3x+2y)\frac{dy}{dx}=-2x-3y$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	f)	$\frac{dy}{dx}=\frac{-2x-3y}{3x+2y}$	½
		at $(1,1)$	
		$\therefore \frac{dy}{dx}=\frac{-2(1)-3(1)}{3(1)+2(1)}=-1$	½
		$\therefore$ slope of tangent $m=-1$	
		Equation of tangent is	
		$y-y_1=m(x-x_1)$	
		$y-1=-1(x-1)$	
		$x+y-2=0$	1
		slope of normal is $=\frac{-1}{m}=\frac{-1}{-1}=1$	½
		Equation of normal at $(1,1)$ is	
		$y-1=1(x-1)$	
		$x-y=0$	1
		--	
3.		**Attempt any FOUR of the following:**	**16**
	a)	Solve : $\frac{dy}{dx}=(4x+y+1)^2$	**04**
	Ans	$\frac{dy}{dx}=(4x+y+1)^2$	
		Put $4x+y+1=v$	½
		$4+\frac{dy}{dx}=\frac{dv}{dx}$	½
		$\frac{dy}{dx}=\frac{dv}{dx}-4$	
		$\therefore \frac{dv}{dx}-4=v^2$	
		$\frac{dv}{dx}=v^2+4$	½
		$\therefore \frac{dv}{v^2+4}=dx$	½
		$\therefore \int \frac{dv}{v^2+4}=\int dx$	½
		$\therefore \int \frac{dv}{v^2+2^2}=\int dx$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	$\therefore \frac{1}{2}\tan^{-1}\left(\frac{v}{2}\right) = x + c$	1
		$\therefore \frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + c$	½
		--	
	b)	Solve : $\left(x^2+y^2\right)dx - 2xydy = 0$	**04**
	Ans	$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$	
		Put $y = vx$	½
		$\frac{dy}{dx} = v + x\frac{dv}{dx}$	½
		$\therefore v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2x(vx)}$	½
		$\therefore v + x\frac{dv}{dx} = \frac{x^2\left(1+v^2\right)}{2vx^2}$	
		$\therefore v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$	
		$\therefore x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$	
		$\therefore x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$	
		$\therefore x\frac{dv}{dx} = \frac{1-v^2}{2v}$	
		$\therefore \frac{2v}{1-v^2}dv = \frac{1}{x}dx$	½
		$\therefore \int \frac{2v}{1-v^2}dv = \int \frac{1}{x}dx$	½
		$\therefore -\log\left(1-v^2\right) = \log x + c$	1
		$\therefore -\log\left(1-\frac{y^2}{x^2}\right) = \log x + c$	½
		--	
	c)	Solve: $\left(2xy+y^2\right)dx + \left(x^2+2xy+\sin y\right)dy = 0$	**04**
	Ans	$\left(2xy+y^2\right)dx + \left(x^2+2xy+\sin y\right)dy = 0$	
		$M = 2xy + y^2$, $N = x^2 + 2xy + \sin y$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$\frac{\partial M}{\partial y}=2x+2y$, $\frac{\partial N}{\partial x}=2x+2y$	½ +½
		$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$	
		$\therefore$ D.E. exact	1
		Solution is	
		$\int_{y-constant} M\ dx+\int_{terms\ not\ containing\ 'x'} Ndy=c$	
		$\therefore \int_{y-constant}\left(2xy+y^2\right)\ dx+\int \sin y\ dy=c$	1
		$\therefore 2y\frac{x^2}{2}+y^2x+\left(-\cos y\right)=c$	1
		$\therefore x^2y+xy^2-\cos y=c$	
		--	
	d)	Find the area of the circle $x^2+y^2=16$ using integration .	**04**
	Ans	$x^2+y^2=16$	
		$\therefore y^2=16-x^2$	
		$\therefore y=\sqrt{4^2-x^2}$	
		$\therefore At\ y=0,\ 4^2-x^2=0$	
		$\therefore x=-4\ ,\ 4$	1
		$\therefore\ A=4\int_a^b ydx$	
		$A=4\left[\int_0^4\sqrt{4^2-x^2}dx\right]$	½
		$A=4\left[\frac{x}{2}\sqrt{(4)^2-x^2}+\frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$	1
		$A=4\left[\frac{16}{2}\sin^{-1}(1)-0\right]$	½
		$A=32\frac{\pi}{2}$	½
		$A=16\pi$	½
		--	
	e)	Evaluate: $\int_0^{\frac{\pi}{2}}\frac{dx}{5+4\cos x}$	**04**

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3	e)	Put $\tan\frac{x}{2}=t$	
		$\cos x=\frac{1-t^2}{1+t^2}$, $dx=\frac{2dt}{1+t^2}$	
		when $x\to 0$ to $\frac{\pi}{2}$ $t\to 0$ to 1	1
		$\therefore I=\int_0^1\frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}\frac{2dt}{1+t^2}$	½
		$\therefore I=2\int_0^1\frac{1}{5\left(1+t^2\right)+4\left(1-t^2\right)}dt$	
		$\therefore I=2\int_0^1\frac{1}{5+5t^2+4-4t^2}dt$	
		$\therefore I=2\int_0^1\frac{1}{t^2+9}dt$	½
		$\therefore I=2\int_0^1\frac{1}{t^2+\left(3^2\right)}dt$	
		$\therefore I=\frac{2}{3}\left[\tan^{-1}\left(\frac{t}{3}\right)\right]_0^1$	1
		$\therefore I=\frac{2}{3}\left[\tan^{-1}\left(\frac{1}{3}\right)-\tan^{-1}\left(\frac{0}{3}\right)\right]$	
		$\therefore I=\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right)$	1
		--	
	f)	Evaluate: $\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}\ dx$	**04**
	Ans	Let $I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}\ dx$	
		$\therefore I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\frac{\sin x}{\cos x}}}\ dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	f)	$\therefore I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx------(1)$	
		$\therefore I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$	½
		$\therefore I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx-------(2)$	½
		add (1) and (2)	
		$\therefore I+I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx+\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dxdx$	
		$\therefore 2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,\mathrm{dx}$	½
		$\therefore 2I=\int_0^{\frac{\pi}{2}}1\,dx$	½
		$\therefore 2I=[x]_0^{\pi/2}$	½
		$\therefore 2I=\frac{\pi}{2}-0$	½
		$\therefore I=\frac{\pi}{4}$	½
		--	
4.		**Attempt any FOUR of the following:**	**16**
	a)	Evaluate $\int_1^4\frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$	**04**
	Ans	$I=\int_1^4\frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx-------(1)$	
		$I=\int_1^4\frac{\sqrt{5-(1+4-x)}}{\sqrt{1+4-x}+\sqrt{5-(1+4-x)}}\,dx$	½
		$\therefore I=\int_1^4\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	add (1) and (2)	
		$I+I=\int_1^4 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx + \int_1^4 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$	
		$\therefore 2I=\int_1^4 \frac{\sqrt{5-x}+\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\,dx$	1
		$\therefore 2I=\int_1^4 1\,dx$	½
		$\therefore 2I=\left[x\right]_1^4$	½
		$\therefore 2I=4-1$	½
		$\therefore 2I=3$	
		$I=\frac{3}{2}$	½
		--	
	b)	Evaluate: $\int \frac{x}{\left(x^2-1\right)\left(x^2+2\right)}\,dx$	**04**
	Ans	$\int \frac{x}{\left(x^2-1\right)\left(x^2+2\right)}\,dx$	
		Put $x^2=t$	
		$\therefore 2xdx=dt$	
		$\therefore xdx=\frac{dt}{2}$	½
		$\therefore \frac{1}{2}\int \frac{dt}{(t-1)(t+2)}$	
		Let $\frac{1}{(t-1)(t+2)}=\frac{A}{t-1}+\frac{B}{t+2}$	
		$1=A(t+2)+B(t-1)$	
		Put $t=-2$	
		$1=B(-3) \qquad \therefore B=-\frac{1}{3}$	1
		Put $t=1$	
		$1=A(3) \qquad \therefore A=\frac{1}{3}$	1
		$\frac{1}{(t-1)(t+2)}=\frac{\frac{1}{3}}{t-1}+\frac{-\frac{1}{3}}{t+2}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$\frac{1}{2}\int\frac{dt}{(t-1)(t+2)}=\frac{1}{2}\int\left(\frac{\frac{1}{3}}{t-1}+\frac{-\frac{1}{3}}{t+2}\right)dt$	
		$=\frac{1}{2}\left(\frac{1}{3}\log(t-1)-\frac{1}{3}\log(t+2)\right)+c$	1
		$=\frac{1}{6}\left(\log(x^2-1)-\log(x^2+2)\right)+c$	½
		--	
	c)	Find area enclosed between the parabolas $y^2=4x$ and $x^2=4y$	**04**
	Ans	$y^2=4x$ ---------(1)	
		$x^2=4y$	
		$\therefore y=\frac{x^2}{4}$	
		$\therefore \text{eq}^n.(1)\Rightarrow\left(\frac{x^2}{4}\right)^2=4x$	
		$\frac{x^4}{16}=4x$	
		$\therefore x^4=64x$	
		$\therefore x^4-64x=0$	
		$\therefore x(x^3-64)=0$	
		$\therefore x=0,4$	1
		Area $A=\int_a^b(y_1-y_2)dx$	
		$\therefore A=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx$	½
		$\therefore A=\int_0^4\left(2x^{\frac{1}{2}}-\frac{x^2}{4}\right)dx$	
		$\therefore A=\left(\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12}\right)_0^4$	1
		$\therefore A=\left(\frac{2(4)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(4)^3}{12}\right)-0$	½

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4.	c)	$\therefore A=\frac{16}{3}$ or 5.333	1
		OR	
		$y^2=4x$ ---------(1)	
		$x^2=4y$ ---------(2)	
		$\therefore x=\frac{y^2}{4}$	
		$\text{eq}^n.(2)\Rightarrow$	
		$\left(\frac{y^2}{4}\right)^2=4y$	
		$\frac{y^4}{16}=4y$	
		$\therefore y^4=64y$	
		$\therefore y^4-64y=0$	
		$\therefore y\left(y^3-64\right)=0$	
		$\therefore y=0,4$	1
		Area $A=\int_a^b\left(x_1-x_2\right)dy$	
		$\therefore A=\int_0^4\left(2\sqrt{y}-\frac{y^2}{4}\right)dy$	½
		$\therefore A=\int_0^4\left(2y^{\frac{1}{2}}-\frac{y^2}{4}\right)dy$	
		$\therefore A=\left(\frac{2y^{\frac{3}{2}}}{\frac{3}{2}}-\frac{y^3}{12}\right)_0^4$	1
		$\therefore A=\left(\frac{2(4)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(4)^3}{12}\right)-0$	½
		$\therefore A=\frac{16}{3}$ or 5.333	1
		--	
	d)	Verify that $y^2=ax^2$ is a solution of $x\left(\frac{dy}{dx}\right)^2-2y\frac{dy}{dx}+ax=0$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	$y^2 = ax^2$	
		$\therefore 2y\frac{dy}{dx} = 2ax$	
		$\therefore \frac{dy}{dx} = \frac{ax}{y}$	1
		consider	
		$L.H.S. = x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} + ax$	
		$= x\left(\frac{ax}{y}\right)^2 - 2y\frac{ax}{y} + ax$	1
		$= x\frac{a^2x^2}{y^2} - 2ax + ax$	
		$= x\frac{a^2x^2}{ax^2} - ax$	1
		$= ax - ax$	
		$= 0 = R.H.S.$	1
	e)	Solve the D.E. $x\log x\frac{dy}{dx} + y = 2\log x$	**04**
	Ans	$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x}$	½
		$\therefore P = \frac{1}{x\log x}$ and $Q = \frac{2}{x}$	
		$IF = e^{\int\frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$	1
		$\therefore y\cdot IF = \int Q\cdot IF dx + c$	
		$y\cdot\log x = \int\frac{2}{x}\log x dx + c$	1
		put $\log x = t$ $\therefore \frac{1}{x}dx = dt$	½
		$y\cdot\log x = 2\int t dt + c$	
		$y\cdot\log x = 2\cdot\frac{t^2}{2} + c$	½
		$y\cdot\log x = (\log x)^2 + c$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	Solve: $\left[4-\frac{y^2}{x^2}\right]dx+\frac{2y}{x}dy=0$	**04**
	Ans	$\left[4-\frac{y^2}{x^2}\right]dx+\frac{2y}{x}dy=0$	
		Comparing with $Mdx+Ndy=0$	
		$M=4-\frac{y^2}{x^2}$, $N=\frac{2y}{x}$	
		$\frac{\partial M}{\partial y}=-\frac{2y}{x^2}$, $\frac{\partial N}{\partial x}=-\frac{2y}{x^2}$	½+½
		$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$	
		$\therefore$ D.E. is exact	1
		Solution is	
		$\int_{y-constant} M\ dx+\int_{terms\ not\ containing\ 'x'} Ndy=c$	
		$\therefore \int_{y-constant}\left(4-\frac{y^2}{x^2}\right)dx+0=c$	1
		$\therefore 4x+\frac{y^2}{x}=c$	1
		OR	
		$\left[4-\frac{y^2}{x^2}\right]dx+\frac{2y}{x}dy=0$	
		$\left[4-\frac{y^2}{x^2}\right]+\frac{2y}{x}\frac{dy}{dx}=0$	
		Put $\frac{y}{x}=v$	½
		$\therefore y=vx$	
		$\therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$	½
		$\therefore \left(4-v^2\right)+2v\left(v+x\frac{dv}{dx}\right)=0$	½
		$\therefore 4-v^2+2v^2+2vx\frac{dv}{dx}=0$	
		$\therefore 4+v^2+2vx\frac{dv}{dx}=0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	f)	$\therefore 2vx\dfrac{dv}{dx}=-\left(4+v^2\right)$	
		$\therefore \dfrac{2v}{4+v^2}dv=-\dfrac{1}{x}dx$	½
		$\therefore \int\dfrac{2v}{4+v^2}dv=-\int\dfrac{1}{x}dx$	½
		$\log\left(4+v^2\right)=-\log x+c$	1
		$\log\left(4+\dfrac{y^2}{x^2}\right)=-\log x+c$	½
5.		**Attempt any FOUR of the following:**	**16**
	a)	Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first card is a king and the second is a queen , if the first card is (i) replaced (ii) not replaced	**04**
	Ans	(i) With replacement of first card There are 4 King cards in 52 cards $\text{P}\left(\text{first card is king}\right)=\dfrac{4}{52}=\dfrac{1}{13}$ This card is replaced and the Queen card is drawn $\therefore \text{P}\left(\text{second card is Queen}\right)=\dfrac{4}{52}=\dfrac{1}{13}$	
		$\therefore \text{P}\left(\text{first card is king}\right)\times\text{P}\left(\text{second card is Queen}\right)=\dfrac{1}{13}\times\dfrac{1}{13}=\dfrac{1}{169}$	2
		(ii) Without replacement of first card $\text{P}\left(\text{first card is king}\right)=\dfrac{4}{52}=\dfrac{1}{13}$ The card is not replaced $\therefore \text{P}\left(\text{second card is Queen}\right)=\dfrac{4}{51}$	
		$\therefore \text{P}\left(\text{first card is king}\right)\times\text{P}\left(\text{second card is Queen}\right)=\dfrac{1}{13}\times\dfrac{4}{51}=\dfrac{4}{663}$	2
	b)	If 5% of the electric bulbs manufacturing by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs. *i*) None is defective	**04**

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)	$ii)$ Five bulbs are defective (Given $e^{-5} = 0.007$)	
	Ans	Given $p = 5\% = 0.05$	
		$n = 100$	
		mean $m = np = 100 \times 0.05 \quad \therefore m = 5$	1
		$P(r) = \dfrac{e^{-m} m^r}{r!}$	
		$i)$ None is defective	
		$r = 0$	
		$P(0) = \dfrac{e^{-5}(5)^0}{0!}$	
		$P(0) = 0.007$	1½
		$ii)$ Five bulbs are defective	
		$r = 5$	
		$P(5) = \dfrac{e^{-5}(5)^5}{5!}$	
		$P(5) = 0.1823$	1½
		--	
	c)	In a certain examination 500 students appeared . Mean score is 68 and S.D. is 8. Find the number of students scoring	**04**
		$i)$ Less than 50	
		$ii)$ More than 60	
		(Given that area between : z = 0 to z = 2.25 is 0.4878 and area between : z = 0 to z = 1 is 0.3413)	
	Ans	Given	
		$\bar{x} = 68, \quad \sigma = 8 \quad N = 500$	
		$z = \dfrac{x - \bar{x}}{\sigma}$	
		$i)$ For $x = 50$	
		$z = \dfrac{x - \bar{x}}{\sigma} = \dfrac{50 - 68}{8} = -2.25$	½
		$p(x < 50) = A(z < -2.25)$	
		$= 0.5 - A(2.25)$	½
		$= 0.5 - 0.4878$	
		$= 0.0122$	½
		$\therefore$ No. of students $= 500 \times 0.0122 = 6.1 \approx 6$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$i)$ For $x = 60$	
		$z = \dfrac{x - \bar{x}}{\sigma} = \dfrac{60 - 68}{8} = -1$	½
		$p(x > 60) = A(z > -1)$	
		$= 0.5 + A(1)$	½
		$= 0.5 + 0.3413$	
		$= 0.8413$	½
		$\therefore$ No. of students $= 500 \times 0.8413 = 421$	½
		--	
	d)	Evaluate: $\int e^x . \sin 3x \, dx$	**04**
	Ans	$I = \int e^x \sin 3x \, dx$	
		$\therefore I = \sin 3x \int e^x dx - \int \left(\int e^x dx . \dfrac{d}{dx} \sin 3x \right) dx$	½
		$\therefore I = \sin 3x \cdot e^x - \int e^x \cos 3x . 3 \, dx$	1
		$\therefore I = \sin 3x \cdot e^x - 3 \int e^x \cos 3x \, dx$	
		$\therefore I = \sin 3x \cdot e^x - 3 \left[\cos 3x e^x - \int \left(\int e^x dx . \dfrac{d}{dx} \cos 3x \right) dx \right]$	½
		$\therefore I = \sin 3x \cdot e^x - 3 \left[\cos 3x e^x - \int e^x \left(-\sin 3x . 3 \right) dx \right]$	1
		$\therefore I = \sin 3x \cdot e^x - 3 \left[\cos 3x e^x + 3 \int e^x \sin 3x \, dx \right]$	
		$\therefore I = \sin 3x \cdot e^x - 3e^x \cos 3x - 9 \int e^x \sin 3x \, dx$	
		$\therefore I = \sin 3x \cdot e^x - 3e^x \cos 3x - 9I$	½
		$\therefore 10I = \sin 3x \cdot e^x - 3e^x \cos 3x$	
		$\therefore I = \dfrac{e^x}{10} \left(\sin 3x - 3 \cos 3x \right) + c$	½
		--	
	e)	Evaluate: $\int_0^{\frac{\pi}{2}} \sin 3x \cos 3x \, dx$	**04**
	Ans	$\therefore I = \int_0^{\frac{\pi}{2}} \sin 3x \cos 3x \, dx$	
		$\therefore I = \dfrac{1}{2} \int_0^{\frac{\pi}{2}} 2 \sin 3x \cos 3x \, dx$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	e)	$\therefore I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\sin\left(3x+3x\right)+\sin\left(3x-3x\right)\right)\,dx$	1
		$\therefore I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\sin 6x+\sin 0x\right)\,dx$ OR $\therefore I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\sin 2\left(3x\right)\,dx$	
		$\therefore I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\sin 6x dx$ $\left(\sin 2\theta=2\sin\theta\cdot\cos\theta\right)$	
		$=\frac{1}{2}\left[\frac{-\cos 6x}{6}\right]_0^{\frac{\pi}{2}}$	1
		$=-\frac{1}{2}\left[\frac{\cos 6x}{6}\right]_0^{\frac{\pi}{2}}$	
		$=-\frac{1}{2}\left[\frac{\cos 6\left(\frac{\pi}{2}\right)}{6}-\frac{\cos 0}{6}\right]$	½
		$=-\frac{1}{2}\left[\frac{\cos 3\pi}{6}-\frac{\cos 0}{6}\right]$	
		$=-\frac{1}{2}\left[-\frac{1}{6}-\frac{1}{6}\right]$	½
		$=-\frac{1}{2}\left[\frac{-1}{3}\right]$	
		$=\frac{1}{6}$	½
		--	
	f)	Solve the DE : $\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$	**04**
	Ans	$\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$	
		$\therefore \frac{dy}{dx}=e^{3x}\cdot e^{-2y}+x^2e^{-2y}$	1
		$\therefore \frac{dy}{dx}=e^{-2y}\left(e^{3x}+x^2\right)$	
		$\therefore e^{2y}dy=\left(e^{3x}+x^2\right)dx$	1
		$\therefore \int e^{2y}dy=\int\left(e^{3x}+x^2\right)dx$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	f)	$\therefore \frac{e^{2y}}{2}=\frac{e^{3x}}{3}+\frac{x^3}{3}+c$	1
		--	
6.		**Attempt any FOUR of the following:**	**16**
	a)	Find the equation of tangent and normal to the curve $y=t-\frac{1}{t}$ and $x=\frac{1}{t}$, when $t=2$	**04**
	Ans	$x=\frac{1}{t}$, $y=t-\frac{1}{t}$	
		when $t=2$, $x=\frac{1}{2}$, $y=\frac{3}{2}$	½
		point is $\left(\frac{1}{2},\frac{3}{2}\right)$	
		$\frac{dx}{dt}=-\frac{1}{t^2}$, $\frac{dy}{dt}=1+\frac{1}{t^2}=\frac{t^2+1}{t^2}$	½+½
		$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\left(t^2+1\right)/t^2}{-1/t^2}$	
		$\frac{dy}{dx}=-\left(t^2+1\right)$	½
		$\therefore$ slope of tangent at t = 2 , $\frac{dy}{dx}=m=-5$	½
		$\therefore$ the equation of tangent is	
		$\therefore y-\frac{3}{2}=-5\left(x-\frac{1}{2}\right)$	½
		$\therefore 2y-3=-10x+5$	
		$\therefore 10x+2y-8=0$	
		$\therefore 5x+y-4=0$	
		$\therefore$ slope of normal is $=\frac{-1}{m}=\frac{-1}{-5}=\frac{1}{5}$	½
		$\therefore$ the equation of normal is	
		$\therefore y-\frac{3}{2}=\frac{1}{5}\left(x-\frac{1}{2}\right)$	½
		$\therefore 5y-\frac{15}{2}=1\left(x-\frac{1}{2}\right)$	
		$\therefore x-5y+7=0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length = y,breadth = x	
		Perimeter is $2x+2y=36$	
		$\therefore x+y=18$	½
		$y=18-x$	
		Area is $A=xy$	
		$\therefore A=x(18-x)$	
		$\therefore A=18x-x^2$	1
		$\therefore \frac{dA}{dx}=18-2x$	½
		$\therefore \frac{d^2A}{dx^2}=-2$	½
		Consider $\frac{dA}{dx}=0$	
		$\therefore 18-2x=0$	½
		$\therefore x=9$	
		$\therefore at\ x=9$	
		$\frac{d^2A}{dx^2}=-2<0$	
		$\therefore A$ is maximum when $x=9$	½
		$\therefore$ breadth $x=9$	
		length $y=9$	½
		--	
	c)	Two six face unbaised dice are thrown .Find the probability that the sum of the numbers shown is 7 or product is 12.	**04**
	Ans	$S=\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$	
		$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$	
		$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$	
		$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$	
		$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$	
		$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	$n(s)=36$	1
		sum is 7 or product is 12	
		$\therefore A=\{(1,6)(2,5)(3,4)(4,3)(2,6)(5,2)(6,1)(6,2)\}$	
		$\therefore n(A)=8$	1
		$p(A)=\dfrac{n(A)}{n(S)}$	
		$p(A)=\dfrac{8}{36}$ or 0.222	2
		--	
	d)	If $P(A)=\dfrac{1}{2}, P(B)=\dfrac{1}{3}, P(A\cap B)=\dfrac{7}{12}$, find $P(A'\cap B')$	**04**
	Ans	$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
		$P(A\cup B)=\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{7}{12}$	2
		$\therefore P(A\cup B)=\dfrac{3}{12}=\dfrac{1}{4}$	½
		$\therefore P(A'\cap B')=P(A\cup B)'=1-P(A\cup B)$	
		$\therefore P(A'\cap B')=1-\dfrac{1}{4}$	1
		$\therefore P(A'\cap B')=\dfrac{3}{4}$	½
		--	
	e)	In 200 sets of tosses of 5 fair coins in how many ways you can expect	**04**
		i) at least two heads.	
		ii) At the most two heads.	
	Ans	$p=\dfrac{1}{2}$, $q=\dfrac{1}{2}$	
		$n=5$	
		$p(r)={}^{n}C_{r}\,p^{r}q^{n-r}$	
		i) at least two heads	
		$P(r)=1-[p(0)+p(1)]$	
		$=1-\left[{}^{5}C_{0}\left(\dfrac{1}{2}\right)^{0}\left(\dfrac{1}{2}\right)^{5-0}+{}^{5}C_{1}\left(\dfrac{1}{2}\right)^{1}\left(\dfrac{1}{2}\right)^{5-1}\right]=1-0.1875$	1
		$=\dfrac{13}{16}$ or 0.8125	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	e)	No. of ways $= 200\times 0.8125 = 162.5 \approx 163$	½
		$ii)$ At the most two heads	
		$P(r) = p(0)+p(1)+p(2)$	
		$P(r) = p(0)+p(1)+p(2)$	
		$= 0.1875 + {}^5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{5-2}$	1
		$= 0.5$	½
		No. of ways $= 200\times 0.5 = 100$	½
		--	
	f)	A problem is given to a three students Ram , Shyam and Amit, whose chances of solving it are $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$ respectively. If they attempt to solve a problem independently , find the probability is solved by at least one of them.	**04**
	Ans	Given Probability of Ram is $p(A)=\frac{1}{2}$ $\therefore p(A')=1-\frac{1}{2}=\frac{1}{2}$	½
		Probability of Shyam is $p(B)=\frac{1}{3}$ $\therefore p(B')=1-\frac{1}{3}=\frac{2}{3}$	½
		Probability of Amit is $p(C)=\frac{1}{4}$ $\therefore p(C')=1-\frac{1}{4}=\frac{3}{4}$	½
		Problem is not solved by each of them	
		$p(A'\cap B'\cap C') = p(A')\times p(B')\times p(C')$	
		$=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$	½
		$=\frac{1}{4}$ or $= 0.25$	½
		Problem is solved by at least one of them	
		$=1-p(A'\cap B'\cap C')$	
		$=1-\frac{1}{4}$	1
		$=\frac{3}{4}$ or $= 0.75$	½
		OR	
		Given Probability of Ram is $p(A)=\frac{1}{2}$ $\therefore p(A')=1-\frac{1}{2}=\frac{1}{2}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	f)	Probability of Shyam is $p(B)=\frac{1}{3}$ $\therefore p(B')=1-\frac{1}{3}=\frac{2}{3}$	½
		Probability of Amit is $p(C)=\frac{1}{4}$ $\therefore p(C')=1-\frac{1}{4}=\frac{3}{4}$	½
		$\therefore$ $p(\text{Problem is solved by at least one of them})$	
		$=p(A\cup B\cup C)$	½
		$=1-p(A\cup B\cup C)'$	½
		$=1-p(A'\cap B'\cap C')$	½
		$=1-\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}\right)$	½
		$=1-\frac{1}{4}$	
		$=\frac{3}{4}$ or 0.75	½
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		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
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