Que. No.	Sub. Que.	Model Answers	Marks	Total Marks

Important Instructions to the Examiners:

1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

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1)		**Attempt any TEN of the following:**		
	a)	At which point on the curve $y=3x-x^2$ the slope of the tangent is -5.		
	Ans.	$y=3x-x^2$		
		$\therefore \frac{dy}{dx}=3-2x$	½	
		But tangent is parallel to x-axis.		
		$\therefore 3-2x=-5$		
		$\therefore x=4$	½	
		$\therefore y=3(4)-(4)^2=-4$	½	
		$\therefore$ the point is $(4,\ -4)$.	½	**2**
	b)	Divide 80 into two parts such that their product is maximum.		
	Ans.	Let x, y be the numbers.		
		But x + y = 80 i. e., y = 80 - x		
		To maximize, $p=xy=x(80-x)$		
		$\therefore p=80x-x^2$		
		$\therefore \frac{dp}{dx}=80-2x$	½	
		$\therefore \frac{d^2p}{dx^2}=-2$	½	
		$For\ stationary\ values,\ \frac{dp}{dx}=0$		
		$\therefore 80-2x=0 \quad or \quad 80=2x$		
		$\therefore x=40$	½	
		$At\ x=40,\ \frac{d^2p}{dx^2}=-2<0$		
		$\therefore At\ x=40,\ p\ has\ maximum\ value.$	½	**2**
	c)	Evaluate: $\int \sin^3 x \cos x dx$		
	Ans.	$\int \sin^3 x \cos x dx$ — Put $\sin x=t$, $\therefore \cos x dx=dt$	½	
		$=\int t^3 dt$	½	
		$=\frac{t^4}{4}+c$	½	
		$=\frac{\sin^4 x}{4}+c$	½	**2**

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1)	d)	Evaluate $\int xe^{x}dx$.		
	Ans.	$\int xe^{x}dx = x\int e^{x}dx - \int\left[\left[\int e^{x}dx\right]\frac{d}{dx}(x)\cdot dx\right.$	½	
		$= xe^{x} - \int e^{x}dx$	½	
		$= xe^{x} - e^{x} + c$	½+½	**2**
		---		
	e)	Evaluate $\int \frac{1}{(x+3)(x+2)}dx$		
	Ans.	$I = \int \frac{1}{(x+3)(x+2)}dx$		
		$\frac{1}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$		
		$\therefore A = -1$	½	
		$B = 1$ ------------(*)	½	
		$\frac{1}{(x+3)(x+2)} = \frac{-1}{x+3} + \frac{1}{x+2}$		
		$\therefore I = \int\left[\frac{-1}{x+3} + \frac{1}{x+2}\right]dx$		
		$= -\log(x+3) + \log(x+2) + c$	½+½	**2**
		Note (*): There are various methods to find the values of A and B to partially factorize the given expression including direct method. Students may apply any one of the methods. Take in count all such methods.		
		OR		
		$I = \int \frac{1}{(x+3)(x+2)}dx$		
		$= \int \frac{1}{x^{2}+5x+6}dx$		
		$= \int \frac{1}{x^{2}+5x+\frac{25}{4}-\frac{25}{4}+6}dx$	½	
		$= \int \frac{1}{\left(x+\frac{5}{2}\right)^{2} - \left(\frac{1}{2}\right)^{2}}dx$	½	

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1)		$=\dfrac{1}{2\left(\frac{1}{2}\right)}\log\left(\dfrac{x+\frac{5}{2}-\frac{1}{2}}{x+\frac{5}{2}+\frac{1}{2}}\right)+c$	½	
		$=\log\left(\dfrac{x+2}{x+3}\right)+c$	½	**2**
		--		
	f)	Evaluate $\int_0^{\log_e 2} e^{2x}dx$		
	Ans.	$\int_0^{\log_e 2} e^{2x}dx=\left[\dfrac{e^{2x}}{2}\right]_0^{\log 2}$	½	
		$=\dfrac{e^{2\log 2}}{2}-\dfrac{e^0}{2}$	½	
		$=\dfrac{4}{2}-\dfrac{1}{2}$		
		$=\dfrac{3}{2}\quad or\;\; 1.5$	1	**2**
		--		
	g)	Find the area between the line $y=2x$ and $x=1$ and $x=3$.		
	Ans.	$\int_1^3 y\cdot dx=\int_1^3 2x\cdot dx$	½	
		$=\left[2\cdot\dfrac{x^2}{2}\right]_1^3 \qquad or \qquad \left[x^2\right]_1^3$	½	
		$=3^2-1$	½	
		$=8$	½	**2**
		--		
	h)	Find the order and degree of the following equation: $\dfrac{d^2y}{dx^2}+\sqrt{1+\dfrac{dy}{dx}}=0$		
	Ans.	Order = 2	1	
		For degree, $\left(\dfrac{d^2y}{dx^2}\right)^2=1+\dfrac{dy}{dx}$		
		$\therefore$ Degree = 2	1	**2**

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1)	i)	Form a differential equation if $y^2 = 4ax$.		
	Ans.	$y^2 = 4ax$		
		$\therefore 2y\frac{dy}{dx} = 4a$	1	
		$\therefore y^2 = 2y\frac{dy}{dx}\cdot x$	½	
		$\therefore y = 2x\frac{dy}{dx}$ or $2x\frac{dy}{dx} = y$	½	
		or $2x\frac{dy}{dx} - y = 0$ or $\frac{dy}{dx} - \frac{y}{2x} = 0.$		**2**
	j)	From a pack of 52 cards one card is drawn at random. Find the probability of getting a king.		
	Ans.	$n = n(S) = 52$	½	
		$m = n(A) = 4$	½	
		$\therefore p = p(A) = \frac{n(A)}{n(S)}$		
		$= \frac{4}{52}$	½	
		$= \frac{1}{13}$ or 0.077	½	**2**
	k)	An unbiased coin is tossed 5 times. Find the probability of getting three heads.		
	Ans.	$p = \frac{1}{2} = 0.5$ $\therefore q = 1 - p = \frac{1}{2} = 0.5$	½	
		Here $n = 5$		
		$\therefore p = {}^nC_r p^r q^{n-r}$		
		$= {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$	1	
		$= \frac{5}{16}$ or 0.3125	½	**2**
		Note: Due to the use of advance non-programmable scientific calculators which is permissible in the board examination, writing directly the values of ${}^nC_r$ or ${}^nC_r p^r q^{n-r}$ is permissible. No marks to be deducted for calculating directly the value.		

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1)	*l*)	A die is thrown, find the probability of getting an odd number.		
	Ans.	$\therefore n = n(S) = 6$	½	
		$\therefore m = n(A) = 3$	½	
		$\therefore p = \frac{n(A)}{n(S)}$		
		$= \frac{3}{6}$	½	
		$= \frac{1}{2}$ or 0.5	½	**2**
2)		**Attempt any four of the following:**		
	a)	Find the equation of tangent and normal to the curve y = x(2 – x) at (2, 0).		
	Ans.	$y = x(2-x)$		
		$\therefore \frac{dy}{dx} = 2-2x$	1	
		$\therefore$ the slope of tangent at $(2, 0)$ is $m = 2-4 = -2$	½	
		$\therefore$ the equation of tangent is $y-0 = -2(x-2)$	½	
		$\therefore y = -2x+4$ or $2x+y=4$	½	
		$\therefore$ the slope of normal $= -\frac{1}{m} = \frac{1}{2}$	½	
		$\therefore$ the equation of normal is $y-0 = \frac{1}{2}(x-2)$	½	
		$\therefore 2y = x-2$ or $x-2y-2=0$ or $-x+2y+2=0$	½	**4**
	b)	Find radius of curvature of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$		
	Ans.	$x = a\cos^3\theta$		
		$\therefore \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$	½	

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2)				
		$y = a\sin^3\theta$		
		$\therefore \dfrac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$	½	
		$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$	½	
		$\& \quad \dfrac{d^2y}{dx^2} = \dfrac{d}{d\theta}\left(\dfrac{dy}{dx}\right)\times\dfrac{d\theta}{dx}$		
		$= -\sec^2\theta \times \dfrac{1}{-3a\cos^2\theta\sin\theta}$	½	
		$\therefore at\ \theta = \dfrac{\pi}{4},$		
		$\dfrac{dy}{dx} = -\tan\dfrac{\pi}{4} = -1$	½	
		$and \quad \dfrac{d^2y}{dx^2} = \sec^2\dfrac{\pi}{4}\times\dfrac{1}{3a\cos^2\dfrac{\pi}{4}\sin\dfrac{\pi}{4}}$		
		$= \dfrac{4\sqrt{2}}{3a}$	½	
		$\therefore \rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}} = \dfrac{\left[1+(-1)^2\right]^{\frac{3}{2}}}{\dfrac{4\sqrt{2}}{3a}}$	½	
		$= \dfrac{3a}{2}$	½	**4**
		---		
	c)	Find the maximum and minimum value of $y = x^3 - \dfrac{15}{2}x^2 + 18x$.		
	Ans.	$y = x^3 - \dfrac{15}{2}x^2 + 18x$		
		$\therefore \dfrac{dy}{dx} = 3x^2 - 15x + 18$	½	
		$\therefore \dfrac{d^2y}{dx^2} = 6x - 15$	½	
		$For\ stationary\ values,\ \dfrac{dy}{dx} = 0$		
		$\therefore 3x^2 - 15x + 18 = 0$		
		$\therefore x = 2,\ 3$	½+½	

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2)		$At\ x=2,\ \frac{d^2y}{dx^2}=6(2)-15=-3<0$	½	
		$\therefore At\ x=2,\ y\ has\ \max imum\ value\ and\ it\ is$		
		$y=(2)^3-\frac{15}{2}(2)^2+18(2)=14$	½	
		$At\ x=3,\ \frac{d^2y}{dx^2}=6(3)-15=3>0$	½	
		$\therefore At\ x=3,\ y\ has\ \min imum\ value\ and\ it\ is$		
		$y=(3)^3-\frac{15}{2}(3)^2+18(3)=13.5$	½	**4**
		--		
	d)	Evaluate $\int\frac{e^x(x+1)}{\cos^2(xe^x)}dx$		
	Ans.	$\int\frac{e^x(x+1)}{\cos^2(xe^x)}dx$ $Put\ \ xe^x=t$, $\therefore\left(xe^x+e^x\cdot 1\right)dx=dt$, $\therefore e^x\left(x+1\right)dx=dt$	1	
		$=\int\frac{dt}{\cos^2 t}$	1	
		$=\int\sec^2 t dt$	½	
		$=\tan t+c$	1	
		$=\tan\left(xe^x\right)+c$	½	**4**
		--		
	e)	Evaluate $\int\frac{\sec^2 x}{3\tan^2 x-2\tan x-5}\ dx$		
	Ans.	$\int\frac{\sec^2 x}{3\tan^2 x-2\tan x-5}\ dx$ $Put\ \tan x=t$, $\therefore\sec^2 xdx=dt$	½	
		$=\int\frac{1}{3t^2-2t-5}dt$		
		$\therefore 3t^2-2t-5=3\left[t^2-\frac{2}{3}t-\frac{5}{3}\right]$		
		$=3\left[t^2-\frac{2}{3}t+\frac{1}{9}-\frac{1}{9}-\frac{5}{3}\right]$		
		$=3\left[\left(t-\frac{1}{3}\right)^2-\left(\frac{4}{3}\right)^2\right]$	1	

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2)		$\therefore I=\int \frac{1}{3\left[\left(t-\frac{1}{3}\right)^2-\left(\frac{4}{3}\right)^2\right]}dt$	½	
		$=\frac{1}{3}\int \frac{1}{\left(t-\frac{1}{3}\right)^2-\left(\frac{4}{3}\right)^2}dt$		
		$=\frac{1}{3}\cdot\frac{1}{2\left(\frac{4}{3}\right)}\cdot\log\left[\frac{t-\frac{1}{3}-\frac{4}{3}}{t-\frac{1}{3}+\frac{4}{3}}\right]+c$	1	
		$=\frac{1}{8}\cdot\log\left[\frac{t-\frac{5}{3}}{t+1}\right]+c$	½	
		$=\frac{1}{8}\cdot\log\left[\frac{3t-5}{3t+3}\right]+c$		
		$=\frac{1}{8}\cdot\log\left[\frac{3\tan x-5}{3\tan x+3}\right]+c$	½	**4**
		--		
	f)	Evaluate $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$		
	Ans.	$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ Put $\sin^{-1}x=t$ $\therefore \frac{1}{\sqrt{1-x^2}}dx=dt$ Also $x=\sin t$	1	
		$=\int \sin t\cdot t\cdot dt$	½	
		$=t\int \sin t\,dt-\int\left(\int \sin t\,dt\right)\frac{d}{dt}(t)\,dt$	½	
		$=t(-\cos t)-\int(-\cos t)\cdot 1\cdot dt$	1	
		$=-t\cos t+\int \cos t\cdot dt$		
		$=-t\cos t+\sin t+c$	½	
		$=-\sin^{-1}x\cdot\cos\left(\sin^{-1}x\right)+x+c$	½	**4**
		--		

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3)		**Attempt any four of the following:**		
	a)	Evaluate $\int_{0}^{\pi/2} \frac{dx}{\sqrt{9-4x^2}}$		
	Ans.	The given problem cannot be solved within the given limits because for the integrating the given function within the prescribed limits the function must be well defined on the given interval. For example at $x=\frac{\pi}{2}$, $\frac{1}{\sqrt{9-4x^2}}$ is a non-real number and hence the function is not defined on the interval $\left[0,\ \pi/4\right]$.	4	**4**
		---		
	b)	Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x+\cos x}dx$		
	Ans.	$I=\int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x+\cos x}dx$ Re *place* $x \to \pi/2-x$ $\therefore \sin x \to \cos x$ & $\cos x \to \sin x$	1	
		$\therefore I=\int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x+\sin x}dx$		
		$\therefore 2I=\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sin x+\cos x}dx$	1	
		$\therefore 2I=\int_{\pi/6}^{\pi/3} 1\cdot dx$	½	
		$\therefore 2I=\left[x\right]_{\pi/6}^{\pi/3}$	½	
		$=\frac{\pi}{3}-\frac{\pi}{6}$	½	
		$\therefore I=\frac{\pi}{12}$	½	**4**
		OR		
		$I=\int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x+\cos x}dx$		
		$=\int_{\pi/6}^{\pi/3} \frac{\sin\left(\pi/2-x\right)}{\sin\left(\pi/2-x\right)+\cos\left(\pi/2-x\right)}dx$	½	
		$\therefore I=\int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x+\sin x}dx$	½	

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3)		$\therefore 2I=\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sin x+\cos x}dx$	1	
		$\therefore 2I=\int_{\pi/6}^{\pi/3}1\cdot dx$	½	
		$\therefore 2I=[x]_{\pi/6}^{\pi/3}$	½	
		$=\frac{\pi}{3}-\frac{\pi}{6}$	½	
		$\therefore I=\frac{\pi}{12}$	½	**4**
		---		
	c)	Find the area bounded by two curves $y^2=x$ and $x^2=y$		
	Ans.	Given $y^2=x$ and $x^2=y$		
		$\therefore \left(x^2\right)^2=x$		
		$\therefore x=0,\quad x=1$	½ + ½	
		$A=\int_a^b\left(y_2-y_1\right)dx$		
		$=\int_0^1\left[\sqrt{x}-x^2\right]dx$	1	
		$=\left[\frac{2}{3}x^{3/2}-\frac{x^3}{3}\right]_0^1$	1	
		$=\left[\frac{2}{3}\cdot 1^{3/2}-\frac{1^3}{3}\right]-0$	½	
		$=\frac{1}{3}\quad or\quad 0.333$	½	**4**
		OR		
		$\therefore x=0,\quad x=1$	½ + ½	
		$A=\int_a^b\left(y_2-y_1\right)dx$		
		$=\int_0^1\left[x^2-\sqrt{x}\right]dx$	1	
		$=\left[\frac{x^3}{3}-\frac{2}{3}x^{3/2}\right]_0^1$	1	
		$=\left[\frac{1^3}{3}-\frac{2}{3}\cdot 1^{3/2}\right]-0$		
		$=-\frac{1}{3}\quad or\quad -0.333$	½	
		$\therefore the\ area=\frac{1}{3}\quad or\quad 0.333$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	d)	Solve $xy^2dy-\left(x^3+y^3\right)dx=0$ given y = 0 when x = 1.		
	Ans.	$xy^2dy-\left(x^3+y^3\right)dx=0$		
		$\therefore\frac{dy}{dx}=\frac{x^3+y^3}{xy^2}$		
		Put $y=vx$ $\therefore\frac{dy}{dx}=v+x\frac{dv}{dx}$	1	
		$\therefore v+x\frac{dv}{dx}=\frac{x^3+(vx)^3}{xv^2x^2}=\frac{1+v^3}{v^2}$	½	
		$\therefore x\frac{dv}{dx}=\frac{1+v^3}{v^2}-v$		
		$\therefore x\frac{dv}{dx}=\frac{1}{v^2}$	½	
		$\therefore\int v^2dv=\int\frac{1}{x}dx$	½	
		$\therefore\frac{v^3}{3}=\log x+c$		
		$\therefore\frac{y^3}{3x^3}=\log x+c$	½	
		$At\ x=1\ \&\ y=0,\ c=0$	½	
		$\therefore\frac{y^3}{3x^3}=\log x$	½	**4**
		---		
	e)	Solve the differential equation $(x+y)^2\frac{dy}{dx}=a^2$		
	Ans.	$Put\ \ x+y=v$		
		$\therefore 1+\frac{dy}{dx}=\frac{dv}{dx}$ *or* $\frac{dy}{dx}=\frac{dv}{dx}-1$	½	
		$\therefore v^2\left(\frac{dv}{dx}-1\right)=a^2$	½	
		$\therefore\frac{dv}{dx}-1=\frac{a^2}{v^2}$		
		$\therefore\frac{dv}{dx}=\frac{a^2}{v^2}+1=\frac{a^2+v^2}{v^2}$	½	
		$\therefore\left(\frac{v^2}{a^2+v^2}\right)dv=dx$	½	

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3)		$\therefore \int \left(\frac{v^2}{a^2+v^2} \right) dv = \int dx$		
		$\therefore \int \left(1 - \frac{a^2}{a^2+v^2} \right) dv = \int dx$	½	
		$\therefore v - a^2 \cdot \frac{1}{a} \tan^{-1}\left(\frac{v}{a} \right) = x + c$	1	
		$\therefore x + y - a\tan^{-1}\left(\frac{x+y}{a} \right) = x + c \quad or \quad y - a\tan^{-1}\left(\frac{x+y}{a} \right) = c$	½	**4**
		---		
	f)	Solve $x\frac{dy}{dx} - y = x^2$		
	Ans.	$x\frac{dy}{dx} - y = x^2$		
		$\therefore \frac{dy}{dx} - \frac{1}{x} \cdot y = x$		
		$\therefore P = -\frac{1}{x}$ and $Q = x$		
		$\therefore IF = e^{\int pdx}$		
		$= e^{\int -\frac{1}{x}dx}$		
		$= e^{-\log x}$		
		$= \frac{1}{x}$	1	
		$\therefore y \cdot IF = \int Q \cdot IF \cdot dx + c$		
		$\therefore y \cdot \frac{1}{x} = \int x \cdot \frac{1}{x} \cdot dx + c$	1	
		$\therefore \frac{y}{x} = \int 1.dx + c$	1	
		$\therefore \frac{y}{x} = x + c$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Attempt any Four of the following:**		
	a)	Evaluate $\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$		
	Ans.	$I=\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$ Re *place* $x \to 6-x$ $\therefore 9-x \to x+3$ & $x+3 \to 9-x$	1	
		$\therefore I=\int_1^5 \frac{\sqrt[3]{x+3}}{\sqrt[3]{x+3}+\sqrt[3]{9-x}}dx$		
		$\therefore 2I=\int_1^5 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+3}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$	1	
		$\therefore 2I=\int_1^5 1\cdot dx$		
		$\therefore 2I=[x]_1^5$	½	
		$\therefore 2I=5-1$	1	
		$\therefore I=2$	½	**4**
		OR		
		$I=\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$		
		$\therefore I=\int_1^5 \frac{\sqrt[3]{9-(6-x)}}{\sqrt[3]{9-(6-x)}+\sqrt[3]{(6-x)+3}}dx$	½	
		$\therefore I=\int_1^5 \frac{\sqrt[3]{x+3}}{\sqrt[3]{x+3}+\sqrt[3]{9-x}}dx$	½	
		$\therefore 2I=\int_1^5 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+3}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$	1	
		$\therefore 2I=\int_1^5 1\cdot dx$		
		$\therefore 2I=[x]_1^5$	½	
		$\therefore 2I=5-1$	1	
		$\therefore I=2$	½	**4**

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4)	b)	Evaluate $\int \frac{dx}{4\cos^2 x+9\sin^2 x}$		
	Ans.	$\int \frac{dx}{4\cos^2 x+9\sin^2 x}$		
		$=\int \frac{dx/\cos^2 x}{\frac{4\cos^2 x+9\sin^2 x}{\cos^2 x}}$		
		$=\int \frac{\sec^2 xdx}{4+9\tan^2 x}$ Put $\tan x=t$, $\therefore \sec^2 xdx=dt$	½	
		$=\int \frac{dt}{4+9t^2}$	½	
		$=\int \frac{dt}{9\left(\frac{4}{9}+t^2\right)}$		
		$=\frac{1}{9}\int \frac{dt}{\left(\frac{2}{3}\right)^2+t^2}$	1	
		$=\frac{1}{9}\cdot\frac{1}{2/3}\tan^{-1}\left(\frac{t}{2/3}\right)+c$	1	
		$=\frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)+c$	1	**4**
		---		
	c)	Using integration find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$		
	Ans.	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$		
		$\therefore y^2=b^2\left(1-\frac{x^2}{a^2}\right)$		
		$\therefore y^2=\frac{b^2}{a^2}\left(a^2-x^2\right)$		
		$\therefore y=\frac{b}{a}\sqrt{a^2-x^2}$		
		$Now\ y=0\ gives\ a^2-x^2=0\ \ i.e.,\ x=a,\ -a$	1	
		$\therefore A=4\int_0^a ydx$		
		$=4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$=4\cdot\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$	1	
		$=\frac{4b}{a}\left[0+\frac{a^2}{2}\sin^{-1}(1)\right]-0$	½	
		$=\frac{4b}{a}\left[\frac{a^2}{2}\cdot\frac{\pi}{2}\right]$		
		$=\pi ab$	½	**4**
		---		
	d)	Solve $\left(2x^2+6xy-y^2\right)dx+\left(3x^2-2xy+y^2\right)dy=0$		
	Ans.	$M=2x^2+6xy-y^2$		
		$\therefore\frac{\partial M}{\partial y}=6x-2y$	1	
		$N=3x^2-2xy+y^2$		
		$\therefore\frac{\partial N}{\partial x}=6x-2y$	½	
		$\therefore$ *the equation is exact.*	½	
		$\int\limits_{y\ constant} Mdx+\int\limits_{terms\ free\ from\ x} Ndy=c$		
		$\int\left(2x^2+6xy-y^2\right)dx+\int y^2dy=c$	1	
		$\therefore 2.\frac{x^3}{3}+6y\cdot\frac{x^2}{2}-y^2x+\frac{y^3}{3}=c$	1	
		or $\frac{2}{3}x^3+3x^2y-y^2x+\frac{y^3}{3}=c$		**4**
		OR		
		$\left(2x^2+6xy-y^2\right)dx+\left(3x^2-2xy+y^2\right)dy=0$		
		$\therefore\frac{dy}{dx}=-\frac{2x^2+6xy-y^2}{3x^2-2xy+y^2}$		
		Put $y=vx\quad\therefore\frac{dy}{dx}=v+x\frac{dv}{dx}$	1	
		$\therefore v+x\frac{dv}{dx}=-\frac{2x^2+6vx^2-v^2x^2}{3x^2-2vx^2+v^2x^2}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\therefore v+x\dfrac{dv}{dx}=-\dfrac{2+6v-v^2}{3-2v+v^2}$		
		$\therefore x\dfrac{dv}{dx}=-\dfrac{2+6v-v^2}{3-2v+v^2}-v$		
		$\therefore x\dfrac{dv}{dx}=\dfrac{-\left(2+6v-v^2\right)-v\left(3-2v+v^2\right)}{3-2v+v^2}$		
		$\therefore x\dfrac{dv}{dx}=\dfrac{-2-9v+3v^2-v^3}{3-2v+v^2}$	½	
		$\therefore\left[\dfrac{3-2v+v^2}{-2-9v+3v^2-v^3}\right]dv=\dfrac{dx}{x}$		
		$\therefore\int\left[\dfrac{3-2v+v^2}{-2-9v+3v^2-v^3}\right]dv=\int\dfrac{dx}{x}$	½	
		$\therefore-\dfrac{1}{3}\int\left[\dfrac{-9+6v-3v^2}{-2-9v+3v^2-v^3}\right]dv=\int\dfrac{dx}{x}$		
		$\therefore-\dfrac{1}{3}\log\left(-2-9v+3v^2-v^3\right)=\log x+c$	1	
		$\therefore-\dfrac{1}{3}\log\left[-2-9\cdot\dfrac{y}{x}+3\left(\dfrac{y}{x}\right)^2-\left(\dfrac{y}{x}\right)^3\right]=\log x+c$	½	**4**
		--		
	e)	Solve $\left(1+x^2\right)dy-x^2ydx=0$		
	Ans.	$\left(1+x^2\right)dy-x^2ydx=0$		
		$\therefore\dfrac{1}{y}dy-\dfrac{x^2}{1+x^2}dx=0$	1	
		$\therefore\int\dfrac{1}{y}dy-\int\dfrac{x^2}{1+x^2}dx=c$	1	
		$\therefore\int\dfrac{1}{y}dy-\int\left(1-\dfrac{1}{1+x^2}\right)dx=c$	1	
		$\therefore\log y-\left(x-\tan^{-1}x\right)=c$	1	
		$or\quad\log y-x+\tan^{-1}x=c$		**4**
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	f)	Show that $y=\sin(\log x)$ is solution of differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$		
	Ans.	$y=\sin(\log x)$ $\therefore \frac{dy}{dx}=\cos(\log x)\cdot\frac{1}{x}$	1	
		$\therefore x\frac{dy}{dx}=\cos(\log x)$ $\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-\sin(\log x)\cdot\frac{1}{x}$	1	
		$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-y$	1	
		$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$	1	**4**
		--		
5)		**Attempt any Four of the following:**		
	a)	The probability that A can shoot at a target is $\frac{5}{7}$ and B can shoot at the same target is $\frac{3}{5}$. (A and B shoot independently.) Find the probability that i) The target is not shot at all. ii) The target is shot by at least one of them.		
	Ans.	$P(A)=\frac{5}{7}$ $\therefore P(A')=1-\frac{5}{7}=\frac{2}{7}$	½	
		$P(B)=\frac{3}{5}$ $\therefore P(B')=1-\frac{3}{5}=\frac{2}{5}$	½	
		$i) P(\text{target is not shot})=P(A'\&B')$ $=P(A')\cdot P(B')$ $=\frac{2}{7}\cdot\frac{2}{5}$	1	
		$=\frac{4}{35}$ *or* 0.114	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$ii) P(\text{at least 1 shoot}) = 1 - p(\text{target is not shot})$		
		$= 1 - \frac{4}{35} \quad or \quad 1 - 0.114$	1	
		$= \frac{31}{35} \quad or \quad 0.886$	½	**4**
		Note for Numerical Problems: For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus the solution is taken up to 3 decimal points only. If answer is truncated more than 3 decimal points, the final answer may vary for last decimal points. Thus 31/35 is actually 0.885714285 but can be taken as 0.886. Due to the use of advance calculators, such as modern scientific non-programmable calculators, the step 31/35 may not written by the students and then directly the answer 0.012 is written. In this case, no marks to be deducted.		
	b)	If 30% of the bulbs produced are defective, find the probability that out of 4 bulbs selected, a) one is defective, b) at the most two are defective.		
	Ans.	$p = \frac{30}{100} = 0.3$		
		$\therefore q = 0.7$		
		$Here \ \ n = 4$		
		$i) \ p = {}^nC_r p^r q^{n-r}$		
		$= {}^4C_1 (0.3)^1 (0.7)^3$	1	
		$= 0.412$	1	
		$ii) p = p(0) + p(1) + p(2)$		
		$= {}^4C_0 (0.3)^0 (0.7)^4 + {}^4C_1 (0.3)^1 (0.7)^3 + {}^4C_2 (0.3)^2 (0.7)^2$	1	
		$= 0.24 + 0.412 + 0.265$		
		$= 0.916$	1	**4**
		OR	OR	
		$p = 1 - [p(3) + p(4)]$		
		$= 1 - \left[{}^4C_3 (0.3)^3 (0.7)^1 + {}^4C_4 (0.3)^4 (0.7)^0 \right]$	1	
		$= 1 - [0.076 + 0.008]$		
		$= 0.916$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Note:** Due to the use of advance non-programmable scientific calculators which is permissible in the board examination, writing directly the values of ${}^{n}C_{r}$ or ${}^{n}C_{r}p^{r}q^{n-r}$ is permissible. No marks to be deducted for calculating directly the value.		
	c)	In a certain examination 500 students appeared, mean score is 68 and S. D. is 8. Assuming data is normally distributed, find the number of students scoring, a) less than 50 b) more than 60. (Given that area between Z = 0 to Z = 2.25 is 0.4878 and area between Z = 0 & Z = 1 is 0.3413.)		
	Ans.	$Given \quad \bar{x}=68 \quad \sigma=8 \quad N=500$		
		$i)\ z=\frac{x-\bar{x}}{\sigma}=\frac{50-68}{8}=-2.25$	½	
		$\therefore p(x\le 50)=p(z\le -2.25)$ $=p(2.25\le z)$ $=0.5-p(0\le z\le 2.25)$	½	
		$=0.5-0.4878$ $=0.0122$	½	
		$\therefore no.of\ students=N\cdot p$ $=500\times 0.0122$ $=6.1\ \ i.e.,\ 6$	½	
		$ii)\ z=\frac{x-\bar{x}}{\sigma}=\frac{60-68}{8}=-1$	½	
		$\therefore p(60\le x)=p(-1\le z)$ $=p(-1\le z\le 0)+p(0\le z)$	½	
		$=0.3413+0.5$ $=0.8413$	½	
		$\therefore no.of\ students=N\cdot p$ $=500\times 0.8413$ $=420.65\ \ i.e.,\ 421$	½	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	d)	Evaluate $\int_0^{\pi}\frac{dx}{5+4\cos x}$		
	Ans.	Put $\tan\frac{x}{2}=t$		
		$\therefore dx=\frac{2dt}{1+t^2}$ and $\cos x=\frac{1-t^2}{1+t^2}$	½	
		x: 0, π ; t: 0, ∞	½	
		$\therefore \int_0^{\pi}\frac{dx}{5+4\cos x}=\int_0^{\infty}\frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	½	
		$=2\int_0^{\infty}\frac{1}{t^2+9}dt$	½	
		$=2\int_0^{\infty}\frac{1}{t^2+3^2}dt$		
		$=2\times\frac{1}{3}\left[\tan^{-1}\left(\frac{t}{3}\right)\right]_0^{\infty}$	1	
		$=\frac{2}{3}\left[\tan^{-1}\infty-\tan^{-1}0\right]$	½	
		$=\frac{2}{3}\left[\frac{\pi}{2}\right]$		
		$=\frac{\pi}{3}$	½	**4**
		--		
	e)	Evaluate $\int\frac{x}{x^2+3x-4}dx$		
	Ans.	$\int\frac{x}{x^2+3x-4}dx$		
		$=\int\frac{x}{(x-1)(x+4)}dx$		
		$=\int\left[\frac{1/5}{x-1}+\frac{4/5}{x+4}\right]dx$ (Please refer next note)	2	
		$=\frac{1}{5}\log(x-1)+\frac{4}{5}\log(x+4)+c$	1+1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Note:** To find the partial fractions, traditional partial fraction method is generally used. But apart from this direct method of partial fraction is also allowed here.		
		--		
	f)	Solve $x\log x\dfrac{dy}{dx}+y=z\log x$		
	Ans.	(Considering z as constant.)		
		$x\log x\dfrac{dy}{dx}+y=z\log x$		
		$\therefore \dfrac{dy}{dx}+\dfrac{1}{x\log x}y=\dfrac{z}{x}$		
		$\therefore P=\dfrac{1}{x\log x}$ and $Q=\dfrac{z}{x}$		
		$\therefore IF=e^{\int pdx}=e^{\int\frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$	1	
		$\therefore y\cdot IF=\int Q\cdot IF\cdot dx+c$		
		$\therefore y\cdot\log x=\int\dfrac{z}{x}\cdot\log x\cdot dx+c$	1	
		Put $\log x=t$ $\therefore \dfrac{1}{x}\cdot dx=dt$	½	
		$\therefore y\cdot\log x=z\int \text{t}\cdot dt+c$	½	
		$\therefore y\cdot\log x=z\cdot\dfrac{t^2}{2}+c$	½	
		$\therefore y\cdot\log x=z\cdot\dfrac{(\log x)^2}{2}+c$	½	**4**
		--		
6)		**Attempt any Four of the following:**		
	a)	If $P(A)=\dfrac{1}{2}$, $P(B')=\dfrac{2}{3}$ and $P(A\cup B)=\dfrac{2}{3}$, find $P(A'\cap B')$ and $P\left(A/B\right)$.		
	Ans.	$P(A)=\dfrac{1}{2}$		
		$P(B)=1-\dfrac{2}{3}=\dfrac{1}{3}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$i)\ P(A' \cap B') = P(A \cup B)'$		
		$= 1 - P(A \cup B)$		
		$= 1 - \frac{2}{3}$	½	
		$= \frac{1}{3} \quad or \quad 0.333$	½	
		$ii) P(A \cap B) = P(A) + P(B) - P(A \cup B)$		
		$= \frac{1}{2} + \frac{1}{3} - \frac{2}{3}$	½	
		$= \frac{1}{6} \quad or \quad 0.167$	½	
		$\therefore P\left(A/B\right) = \frac{P(A \cap B)}{P(B)}$		
		$= \frac{1/6}{1/3}$	1	
		$= \frac{1}{2} \quad or \quad 0.5$	½	**4**
		--		
	b)	If the probability that an electric motor is defective is 0.01, what is the probability that the sample of 300 electric motors will contain exactly 5 defective motors? ($e^{-3} = 0.0498$)		
	Ans.	$p = 0.01 \qquad n = 300$		
		$\therefore m = np = 0.01 \times 300 = 3$	1	
		$p(r) = \frac{e^{-m} \cdot m^r}{r!}$		
		$\therefore p(5) = \frac{e^{-3} \cdot 3^5}{5!}$	1½	
		$= 0.101$	1½	**4**
		--		
	c)	Fit a Poisson distribution for the following observation:		

x	20	30	40	50	60	70
f	8	12	30	10	6	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	Ans.			

x	f	xy
20	8	160
30	12	360
40	30	1200
50	10	500
60	6	360
70	4	280
	70	**2860**

(1)

$\therefore mean\ \ m = \frac{2860}{70} = 40.857$ (2)

$\therefore p = \frac{e^{-m}m^r}{r!}$

$= \frac{e^{-40.857}(40.857)^r}{r!}$ (1) **4**

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d) A metal wire 36 m long is bent to form a rectangle. Find its dimensions when its area is maximum.

Ans. Let x and y be the sides of rectangle.

$\therefore 2x + 2y = 36 \quad or \quad x + y = 18$

$\therefore y = 18 - x$ (½)

$But\ area\ A = xy = x(18 - x) = 18x - x^2$ (1)

$\therefore \frac{dA}{dx} = 18 - 2x$ (½)

$\therefore \frac{d^2A}{dx^2} = -2$ (½)

$For\ stationary\ values,\ \frac{dA}{dx} = 0$

$\therefore 18 - 2x = 0$

$\therefore x = 9$ (½)

$At\ x = 9,\ \frac{d^2A}{dx^2} = -2 < 0$ (½)

$\therefore At\ x = 9,\ A\ has\ \max imum\ value$

$and\ the\ other\ side\ is$

$y = 18 - x = 9$ (½) **4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	e)	Find the equation of tangent to the curve $x=\frac{1}{t},\ \ y=1-\frac{1}{t}$, when t=2.		
	Ans.	$x=\frac{1}{t},\ \ y=1-\frac{1}{t}$ $\therefore y=1-x$	1	
		$\therefore \frac{dy}{dx}=-1$ $\therefore at\, t=2,\ x=0.5\ \ and\ \ y=0.5$	½ + ½	
		$and\ \ slope\ \ m=-1$ $\therefore the\, equation\, is,$ $y-b=m\left(x-a\right)$ $\therefore y-0.5=-1\left(x-0.5\right)$	1	
		$\therefore y-0.5=-x+0.5$ $\therefore x+y-1=0$	1	**4**
		OR		
		$x=\frac{1}{t},\ \ y=1-\frac{1}{t}$ $\therefore \frac{dx}{dt}=-\frac{1}{t^2}\ \ \ and\ \ \ \frac{dy}{dt}=\frac{1}{t^2}$	½	
		$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{1/t^2}{-1/t^2}$ $\therefore \frac{dy}{dx}=-1$	½	
		$\therefore at\, t=2,\ x=0.5\ \ and\ \ y=0.5$	½ + ½	
		$and\ \ slope\ \ m=-1$ $\therefore the\, equation\, is,$ $y-b=m\left(x-a\right)$ $\therefore y-0.5=-1\left(x-0.5\right)$	1	
		$\therefore y-0.5=-x+0.5$ $\therefore x+y-1=0$	1	**4**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	f)	Find the area between the parabola $y=4-x^2$ and the x-axis.		
	Ans.	$y=4-x^2$ *and* $x-axis$ *i.e.,* $y=0$		
		$\therefore 4-x^2=0$		
		$\therefore x=-2,\ 2$	½ + ½	
		$\therefore A=\int_a^b y\,dx$		
		$=\int_{-2}^{2}\left(4-x^2\right)dx$	½	
		$=\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$	1	
		$=\left[2^3-\frac{2^3}{3}\right]-\left[(-2)^3-\frac{(-2)^3}{3}\right]$	½	
		$=\frac{32}{3}$ *or* 10.667	1	**4**

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. **In such case, FIRST SEE whether the method falls within the scope of the curriculum**, and THEN ONLY give appropriate marks in accordance with the scheme of marking.
