SUMMER – 15 EXAMINATIONS

Subject Code: 17422 **Model Answer-Theory of Structure** **Total Pages: 01 / 43**

Important Instruction to Examiners:-

1) The answers should be examined by key words & not as word to word as given in the model answers scheme.

2) The model answers & answers written by the candidate may vary but the examiner may try to access the understanding level of the candidate.

3) The language errors such as grammatical, spelling errors should not be given more importance.

4) While assessing figures, examiners, may give credit for principle components indicated in the figure.

5) The figures drawn by candidate & model answer may vary. The examiner may give credit for any equivalent figure drawn.

5) Credit may be given step wise for numerical problems. In some cases, the assumed contact values may vary and there may be some difference in the candidate's answers and model answer.

6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding.

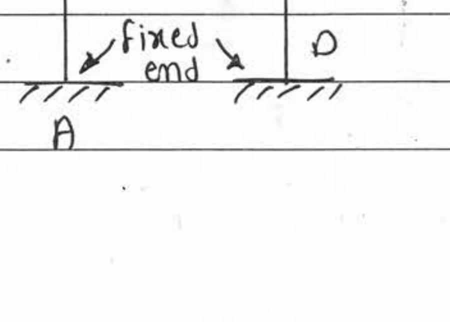
7) For programming language papers, credit may be given to any other programme based on equivalent concept.

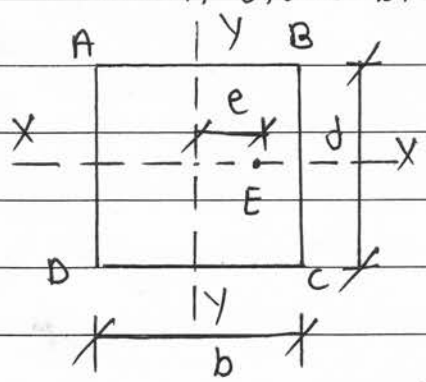
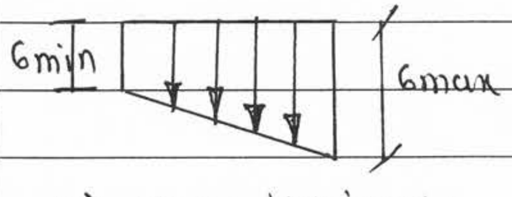
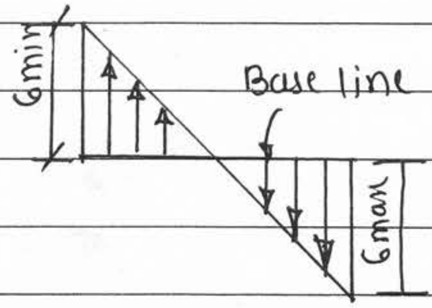
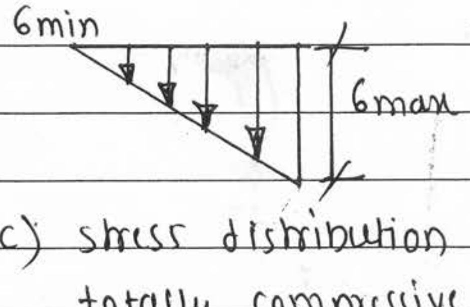
Important notes to examiner

1) **Q-4(f) In this problem the distances of point loads 96kN and 50kN from supports are not mentioned, therefore solution is prepared by assuming 1m distance from point B for 96kN load and 50kN acting at the center of span BC. students may assume any suitable distances of the point loads from the supports, examiner should give proportionate marks for the same**

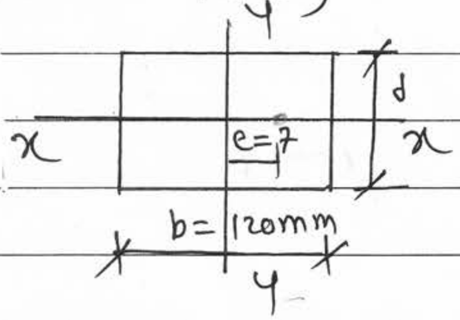
Q .NO	SOLUTION	MARKS
Q-1 (A)		
a)	i) Define eccentric load	
	A load act's away from the centroid of the section or a load whose line of action do not coincide with the axis of member is called as eccentric load.	01M
	ii) Effect of eccentric load on section.	01M
	: Whenever a body is subjected to an eccentric load, that time direct as well as bending stresses are developed in section.	
b)	Define slope and Deflection of a beam.	
	i) Deflection :-	
	when a beam is loaded, the beam is deflected from it's original position in the direction perpendicular to it's longitudinal axis. Then displacement of beam measured from it's neutral axis from unloaded condition of the beam to loaded beam is called as deflection.	01M
	ii) Slope :-	
	The angle made by a tangent at a point of a deflected beam with it's neutral surface of unloaded beam is called as slope.	01M

Q .NO	SOLUTION	MARKS
Q-1-A		
(c)	$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{M}{EI}$	02 M
	$\therefore \frac{1}{R} = EI\frac{d^2y}{dx^2}$ = Bending moment	
(d)	i> Max. deflection for simply supported beam of span 'L' carries a point load 'W' at it's center.	
	$y_{max} = y_c = -\frac{WL^3}{48EI}$	01 M
	ii> Max ~~d~~ slope for simply supported beam of span 'L' carries a point load 'W' at it's center.	
	$\theta_A = \frac{WL^2}{16EI}$ radian's	01 M
(e)	Boundary condition's for fixed end	
	At the fixed support of beam, slope and deflection both are zero	01 M (slope)
	$\theta = \frac{dy}{dx} = 0$ (slope) & $y = 0$ (Deflection)	01 M (Deflection)

Q .NO	SOLUTION	MARKS
Q-1-A		
(F)	i> Carry over factor	01 M
	The carry over factor is the ratio of moment produce at a joint to the moment applied at the other joint with out displacing it.	
	ii> Distribution factor	01 M
	The distribution factor for a member at a joint is the ratio of stiffness factor for that member and the total stiffness of all the members meeting at a joint.	
(g)	Define portal frame	
	:- A frame in which a beam rests on two column which having same length, same end condition's same moment of inertia, same modulus of elasticity and which only subjected to symmetrical loading then such portal frame is called as symmetrical portal frame.	01 M
	:- diagram  :- single bay, single storey symmetrical portal frame.	01 M for (Diagrame)

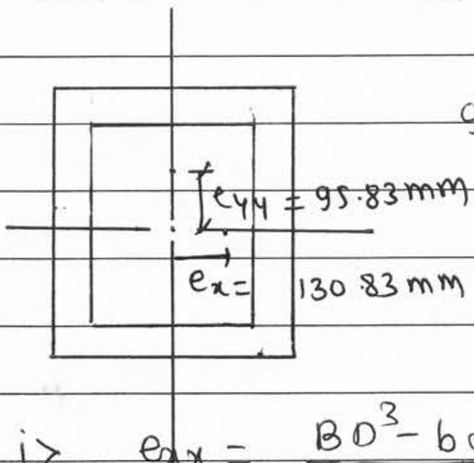
Q.NO	SOLUTION	MARKS
(h)	List of different	
	1) simple truss 5) Fink roof truss	($\frac{1}{2}$M
	2) King post truss 6) Fan roof truss	)
	3) Howe roof truss 7) warren girder	(any four)
	4) How flat roof Truss 8) pratt (Triangular) roof truss	
	9) pratt (flat) roof Truss 10) North light roof Truss	
	11) compount fink truss 12) french roof Truss	
Q-1 (B)		
a)	Resultant Stress Diagram	
		01M
	Eccentric load at E	
		01M
	b) stress distribution completly compressive $\sigma_0 > \sigma_b$ 	01M
		
	d) stress distribution partly compressive and partly tensile since $\sigma_0 < \sigma_b$	01M
	c) stress distribution totally compressive $\sigma_0 = \sigma_b$	

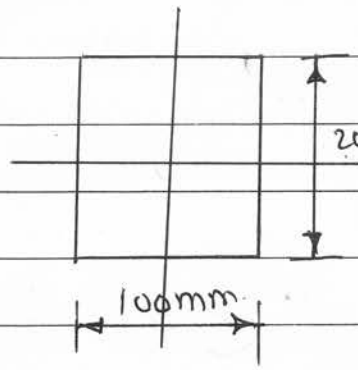
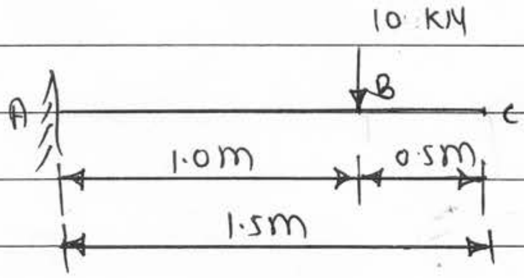
Q.NO	SOLUTION	MARKS
b)	i) Define core of section	
	the centrally located portion of a section with in the load line falls so as to produce only compressive stress is called as core of a section.	02M
	ii) State middle third rule	02M
	Middle third rule is for no-tension condition, the load must lie with in the middle third shaded area of eccentricity 2e, ie. $e \le d/6$ and $e \le b/6$	
c)	i) Assumption made in Analysis	04M
	- The joint's of truss are assumed to be pin connected and frictionless so as cannot resist's moments.	(01M for each)
	- the truss is loaded at the joints only.	write any two)
	- The truss is a perfect truss	
	- The member's of the truss are straight, uniform and they are two forces members	
	- weights of members are neglected.	
	ii) Redundant frame:	01M
	An interminate frame for which the three equation's of the static equillibrium are not sufficient to analyze for it's member forces and the external support reaction's	
	ii) condition if $m > 2j-3$ where m = member, j = joints	01M

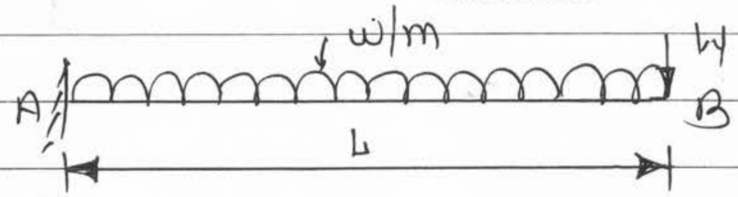
Q.NO	SOLUTION	MARKS
Q-2 (a)	$b = 120mm$	
	$d = ?$	
	$P = 140\,KN = 140 \times 10^3\,N$	
	$e = 7\,mm$	
	$\sigma_{max} = 100\,Mpa$	
	we know	
	$\sigma_{max} = \sigma_0 + \sigma_b$ --- (Tensile in nature)	½M
	$\sigma_{max} = \frac{P}{A} + \frac{M}{Z}$	½M
	$\sigma_{max} = \frac{140 \times 10^3}{120 \times d} + \frac{P.e}{\frac{db^2}{6}}$ 	½M
	$\therefore \sigma_{max} = \frac{140 \times 10^3}{120 \times d} + \frac{6P.e}{db^2}$	
	$\sigma_{max} = \frac{140 \times 10^3}{120 \times d} + \frac{6 \times 140 \times 10^3 \times 7}{d \times 120^2}$	½M
	$100 = \frac{120 \times 140 \times 10^3}{120 \times 120 \times d} + \frac{6 \times 140 \times 10^3 \times 7}{120 \times 120 \times d}$	½M
	$100 = \frac{120 \times 140 \times 10^3}{14400 d} + \frac{6 \times 140 \times 10^3 \times 7}{14400 \times d}$	
	$1.44 \times 10^6 = \frac{16.8 \times 10^6}{d} + \frac{5.88 \times 10^6}{d}$	
	$1.44 \times 10^6 = \frac{16.8 \times 10^6 + 5.88 \times 10^6}{d}$	½M
OR	$\boxed{d = 15.75\,mm}$	01M
	$\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$ $\frac{140 \times 10^3}{120 \times d}\left[1 + \frac{6 \times 7}{120}\right] = 100$	04M

$\boxed{d = 15.75\,mm}$ --- 02M for calculation & 02M for formula

Q .NO	SOLUTION	MARKS
Q-2 (b)	given	
	$h = 10\,m \quad b = 3\,m \quad d = 1.5\,m$	
	$W = 900\,KN \quad p = 1200\,N/m^2$	
	i) $\sigma_o = \frac{W}{A} = \frac{900}{3\times 1.5} = 200\,KN/m^2$	½M
	ii) projected area $= d\times h = 03\times 10 = 30\,m^2$	
	$P = C \times p \times$ projected area	
	$= 1 \times 1200 \times 3\times 10$	
	$P = 36\times 10^3\,N$	½M
	iii) Moment of P about base	
	$M = P\times h/2$	
	$= 36\times 10^3 \times 10/2$	
	$= 180\times 10^3\,N\text{-}m$	½M
	iv) section modulus about y-y-axis	
	$Z = \frac{I}{y_{max}} = \frac{bd^3/12}{d/2} = \frac{bd^2}{2}$	½M
	$Z = \frac{3\times 1.5^2}{6}$	
	$\boxed{Z = 1.125\,m^3}$	½M
	v) Bending stress (σ_b) at the base due to wind pressure	
	$\sigma_b = \pm\frac{M}{Z} = \frac{180\times 10^3}{1.125}$	
	$\boxed{\sigma_b = 160\times 10^3\,N/m^2}$	½M

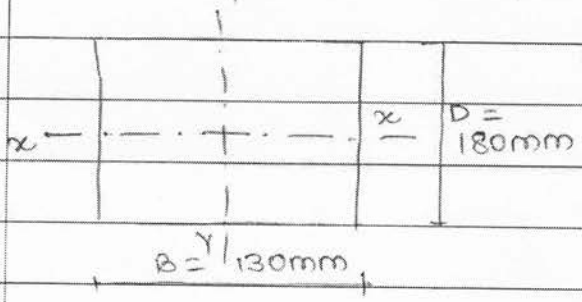
Q .NO	SOLUTION	MARKS
	vi) To find the maximum & minimum stress intensities on the base	
	$\sigma_{max} = \sigma_0 + \sigma_b = 200 \times 10^3 + 160 \times 10^3$	
	$\sigma_{max} = 360 \times 10^3\ N/m^2$	½M
	$\sigma_{min} = \sigma_0 - \sigma_b = 200 \times 10^3 - 160 \times 10^3$	
	$\sigma_{min} = 40 \times 10^3\ N/m^2$	01/2M
c)	 given : b = 250mm, d = 450mm ∴ B = 250 + 2×25 = 300mm D = 450 + 2×25 = 500mm for t = 25mm	
	i) $e_{xx} = \dfrac{BD^3 - bd^3}{6D(BD - bd)}$	01M
	$e_{xx} = \dfrac{300 \times 500^3 - 250 \times 450^3}{6 \times 500\,(300 \times 500 - 250 \times 450)}$	
	$\boxed{e_{xx} = 130.83\ mm}$	01M
	ii) $e_{yy} = \dfrac{DB^3 - db^3}{6B(BD - bd)}$	01M
	$e_{yy} = \dfrac{500 \times 300^3 - 450 \times 250^3}{6 \times 300\,(300 \times 500 - 250 \times 450)}$	
	$\boxed{e_{yy} = 95.83\ mm}$	01M

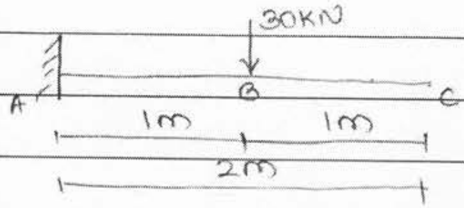
Q.NO	SOLUTION	MARKS
Q-2 (d)	given data	
	$L = 1.5\,m = 1500\,mm$ $b = 100\,mm$ $d = 200\,mm$	
	$W = 10\,KN$ $E = 90\,KN/mm^2$	
	 	½ M
	i) $I = I_{xx} = \frac{bd^3}{12} = \frac{100 \times 200^3}{12} = 66.67 \times 10^6\,mm^4$	½ M
	ii) Slope $\theta_C = \frac{WL_1^2}{2EI} = \frac{10 \times 1000^2}{2 \times 90 \times 66.67 \times 10^6} = 8.33 \times 10^{-4}$ rad	01 M
	iii) Deflection $y_B = \frac{\omega L_1^3}{3EI} + \frac{\omega L_1^2}{2EI}(L - L_1)$	01 M
	$= \frac{10 \times 1000^3}{3 \times 90 \times 66.67 \times 10^6} + \frac{10 \times 1000^2}{2 \times 90 \times 66.67 \times 10^6}(1500 - 1000)$	
	$= 0.555 + 0.4166$	
	$= 0.971\,mm$	01 M

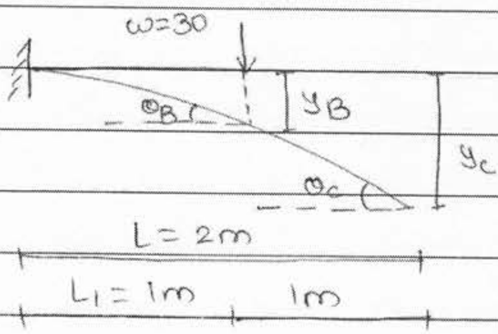
Q .NO	SOLUTION	MARKS
Q-2 (e)		
	i) Slope $\theta_B = \frac{\omega L^2}{6EI} + \frac{\omega L^2}{2EI}$ radians	02 M
	ii) Deflection $y_B = \frac{\omega L^4}{8EI} + \frac{\omega L^3}{3EI}$	02 M
Q-2 (f)	i) State effect of continuity	
	– The beam is subjected to sagging moment at mid section and hogging moment over the support due to continuity.	01 M
	– the slopes of elastic curve or deflected curve at the intermediate supports remains same	01 M
	ii) concept of zero span	
	– when the ends of the beam are fixed then an imaginary zero span is taken to the left or right of the support as the case may be and the clapeyron's theorem is applied to imaginary span it's adjacent span.	01 M
	– the area of M-diagram over imaginary span is zero	01 M

Q.NO	SOLUTION	MARKS
3-(a)	1) Boundary condition for hinged end	
	for hinged end $y = 0$,	(1m)
	$\frac{dy}{dx} \neq 0$	(1m)
	2) Boundary condition for free end	
	$y = 0$	(1M)
	$\frac{dy}{dx} \neq 0$	(1M)

3-(b)







Sol:- $L = 2m$

$L_1 = 1m$

$I_{xx} = \frac{BD^3}{12}$

$= \frac{130 \times 180^3}{12}$

$= 63.18 \times 10^6 \, mm^4$

$= 6.318 \times 10^{-5} \, m^4$

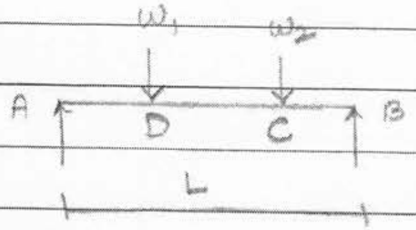
$E = 105 \frac{KN}{mm^2} = 105 \times 10^6 \frac{KN}{m^2}$

(a) Slope under point load.

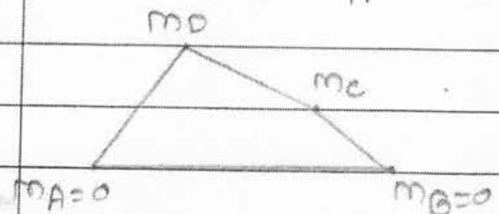
$\theta_B = \frac{wL_1^2}{2EI}$ (1M)

$= \frac{30 \times 1^2}{2 \times 105 \times 10^6 \times 6.318 \times 10^{-5}}$

Q .NO	SOLUTiON	MARKS
	$\boxed{\theta_B = 2.26 \times 10^{-3} \text{ rad}}$	(1M)
	(b) Deflection under point load.	
	$y_B = \dfrac{WL_1^3}{3EI} + \dfrac{WL_1^2}{2EI}(L - L_1)$	(1M)
	$y_B = \dfrac{30 \times 1^3}{3 \times 6.6339 \times 10^3} + \dfrac{30 \times 1^2}{2 \times 6.6339 \times 10^3}(2-1)$	
	$y_B = 3.768 \times 10^{-3}$ m	
	$y_B = 3.768$ mm.	(1M)
3 - (c)	$(W_1 > W_2)$	

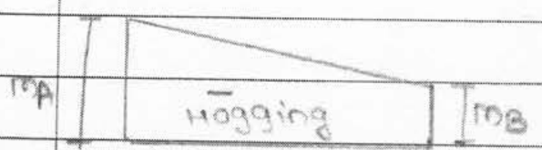


(a) simply supported beam



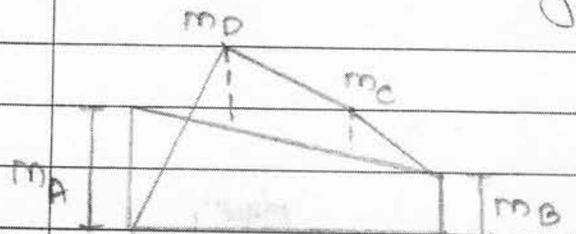
(½ M)

(b) Free Bm dig. or μ dig.



(½ M)

(c) Fixed end moment dig or μ' dig

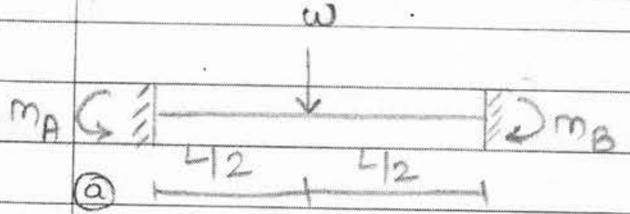


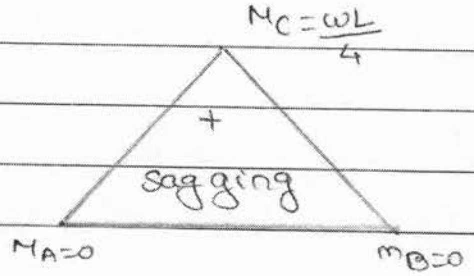
(1M)

(d) Resultant Bm dig (combined effect of free & fixed end moment)

Q.NO	SOLUTION	MARKS
	① calculate the free moment by considering the beam to be simply supported. Hence draw free BMD is positive (sagging) or is called as μ dig. BMD is plotted above the base line.	($\frac{1}{2}$M)
	② Assume the simply supported beam is only subjected to fixed end moment. M_A & M_B which are negative (hogging) Draw fixed end moment ie μ' diagram and vary M_A at A & M_B at B	($\frac{1}{2}$M)
	③ superimpose μ' diagram over μ diagram and hence resultant BMD is drawn which is final BMD for fixed beam. μ diagram and μ' diagram are opposite in nature there addition will be seen only if they are plotted on same side of base line.	($\frac{1}{2}$M)
	④ Find net BMD at D & C by interpolation	($\frac{1}{2}$M)
③-ⓓ	Fixed end moment for fixed beam carrying point load at midspan.	
	consider a fixed beam of span 'L' carrying a point load 'w' at midspan	
	1) support reaction $V_A = V_B = \frac{w}{2}$	
	2) Bending moment $M_A = 0, M_B = 0, M_C = \frac{wL}{4}$	

Q.NO	SOLUTION	MARKS

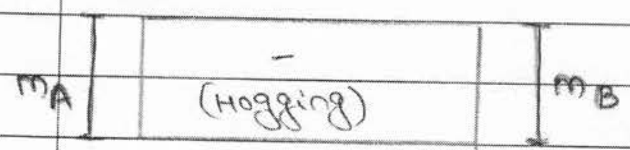




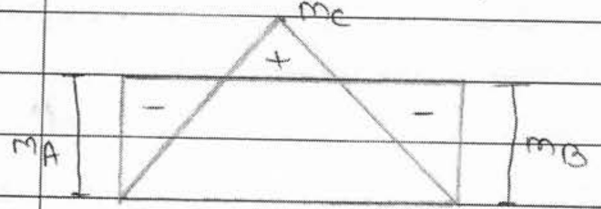
(½ M)

(b) μ diagram consider beam to be simply supported

(½ M)



(c) μ' dig. for fixed end moment



(1M)

(d) Net BMD

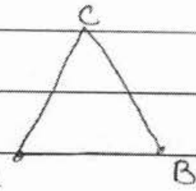
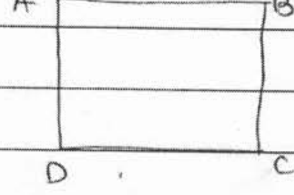
(Superimposition of μ & μ' diagram)

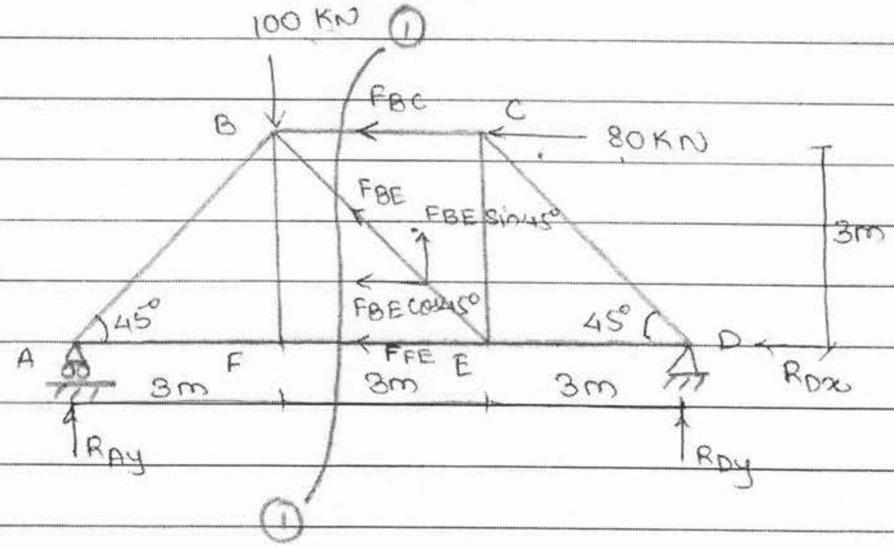
By first principle

$$a = a'$$ (1m)

$$\frac{1}{2} \times \frac{\omega L}{4} \times L = -m_A \times L$$

$$\boxed{m_A = \frac{-\omega L}{8} = m_B}$$ (1M)

Q.NO	SOLUTION	MARKS
3 - e	(a) perfect truss :- (perfect frames)	
	1) A truss which does not collapse under the loading is called as perfect truss.	$\left(\frac{1}{2}M\right)$
	$\boxed{OR}$ when the number of members in frame is exactly equal to $2J-3$ it is called as perfect frames.	
	2) A truss in which the condition $n=2j-3$ is satisfied is called as perfect frame	$\left(\frac{1}{2}M\right)$
	where n = number of member	
	J = number of joints.	
	3) consider a triangle ΔABC in this case	
	$n = 2J-3$, $n=3$, $2J-3 = 2(3)-3 = 3$	
	$\therefore$ n is equal to $(2J-3)$	
	hence it is called perfect truss. 	(1m)
	(b) Imperfect truss.	
	1) A truss which collapse when loaded is called as unstable truss.	
	OR when number of member in the frame are not equal to $(2J-3)$ then it is called imperfect frame.	(1/2M)
	2) In unstable truss condition $n=2J-3$ is not satisfied.	(1/2M)
	3) consider a frame ABCD here 	
	$n=4$, $2J-3 = 2(4)-3 = 5$	(1M)
	$\therefore n \neq 2J-3$ hence it is called imperfect truss.	

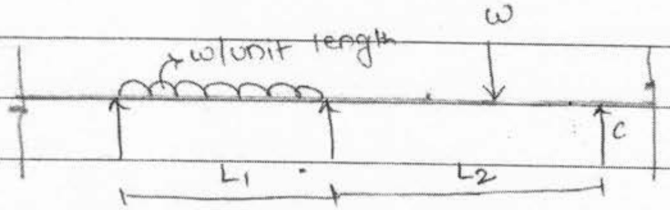
Q.NO	SOLUTION	MARKS
Q3-(f)		
	$\Sigma Fx = 0 \quad -80 - R_{Dx} = 0$	
	$R_{Dx} = -80\,KN\ (\leftarrow)$	
	$\Sigma Fy = 0 \quad R_{Ay} + R_{Dy} = 100$	
	$\Sigma M_A = 0 \quad +100\times3 - 80\times3 - R_{Dy}\times9 = 0$	
	$300 - 240 = R_{Dy}\times9$	
	$\boxed{R_{Dy} = 6.667\,KN}$	(1M)
	$R_{Ay} + R_{Dy} = 100$	
	$R_{Ay} = 100 - 6.667$	
	$\boxed{R_{Ay} = 93.333\,KN}$	
	To calculate forces in the member BC, BE, FE	
	consider section (1)-(1) which cuts the member BC, BE, EF	
	Assume F_{FE}, F_{BC}, F_{BE} to be tensile in nature	

Q .NO	SOLUTION	MARKS
	$\Sigma M_E = 0 \quad -F_{BC}\times 3 - 80\times 3 - R_{Dy}\times 3 = 0$	
	$3F_{BC} = -240 - (6.667\times 3)$	
	$3F_{BC} = -260.001$	
	$F_{BC} = -86.667\,KN$	
	$\boxed{F_{BC} = 86.667\,KN \text{ compression.}}$	(1M)
	To calculate F_{FE} take moment at B	
	$M_B = 0 \quad F_{FE}\times 3 - R_{Dy}\times 6 + R_{Dx}\times 3 = 0$	
	$3F_{FE} = 6R_{Dy} - 3R_{Dx}$	
	$3F_{FE} = (6.667\times 6) - (3\times -80)$	
	$3F_{FE} = 280.002$	
	$F_{FE} = +93.334\,KN$	
	$\boxed{F_{FE} = 93.334\,KN \text{ Tensile}}$	(1M)
	To calculate F_{BE} consider horizontal reactions.	
	$\Sigma Fx = 0 \quad -80 - R_{Dx} - F_{BE}\cos 45 - F_{BC} - F_{FE} = 0$	
	$-80 - (-80) - (-86.667) - 93.334 = F_{BE}\cos 45$	
	$-6.667 = F_{BE}\cos 45$	
	$F_{BE} = -9.428\,KN$	
	$\boxed{F_{BE} = +9.428\,KN \text{ compression.}}$	(1M)
	Force table	

SR.NO	Member	magnitude	nature
1	BC	86.667 KN	compression
2	FE	93.334 KN	Tension
3	BE	9.428 KN	compression

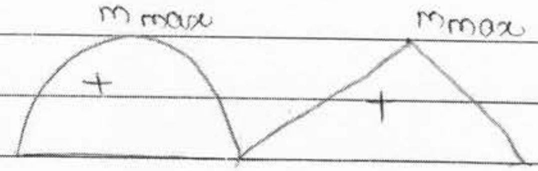
Q.NO	SOLUTION	MARKS

(4) (a) Clapeyron's Theorem :-

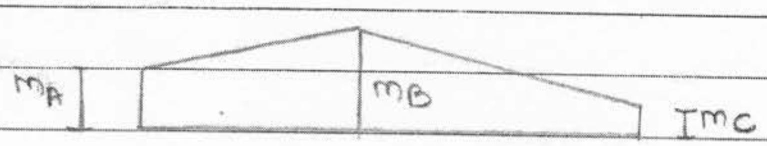


(1/2 M)

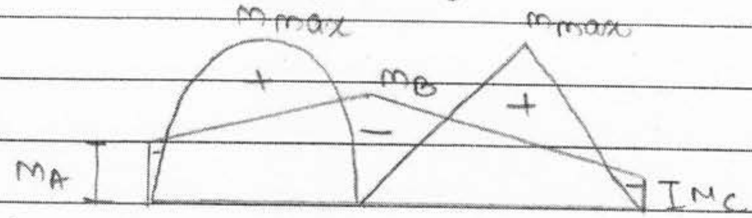
(a) Two consecutive span of continuous beam.



(b) μ - diagram of free BMD.



(c) μ' - diagram for support moment



(1/2 M)

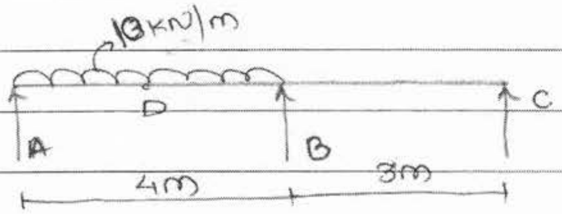
(d) Net BMD.

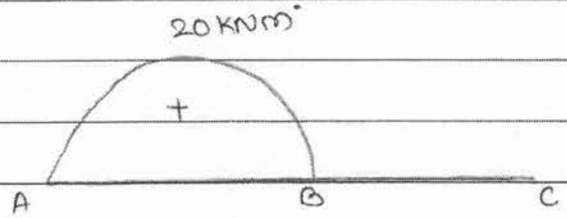
(Super impose dig. for μ' and μ dig. to get net BMD.

For any two consecutive span of continuous beam subjected to external loading and having uniform moment of inertia, the support moments

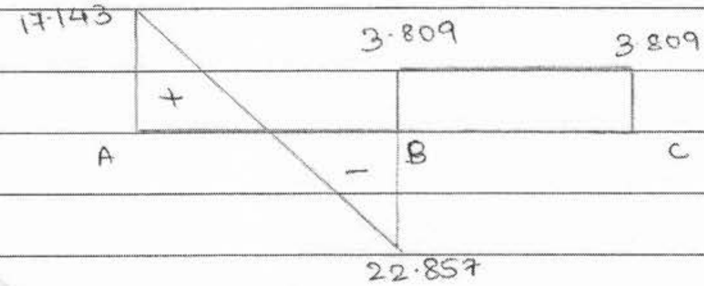
Q .NO	SOLUTION	MARKS
	M_A, M_B, M_C at supports A, B, C are given by	
	$2M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$	(2M)
	where M_A = support moment at A	
	M_B = support moment at B.	
	M_C = support moment at C	
	L_1 = length of span AB	
	L_2 = length of span BC	
	a_1 = area of 'μ' diagram or free BMD for span AB	
	a_2 = area of 'μ' diagram or free BMD for span BC.	
	$\bar{x}_1$ = centriodial distance of μ-diagram or free BMD over span AB from left end 'A'	
	$\bar{x}_2$ = centriodial distance of μ-diagram or free BMD over span BC from Right end 'C'.	
	Clapeyeons theorem for unequal M.I	
	If M.I of any two consecutive span of continuous beam is not constant or uniform then	
	$M_A\frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\frac{L_2}{I_2} = -\left(\frac{6a_1\bar{x}_1}{L_1 I_1} + \frac{6a_2\bar{x}_2}{L_2 I_2}\right)$	(1M)
	where I_1 → moment of inertia for span AB	
	I_2 → moment of inertia for span BC	

Q .NO	SOLUTION	MARKS
(4)-(b)		





(a) B.M.D or M-diagram.



(1M)

(b) SFD

sol:- $M_D = \frac{wL^2}{8} = \frac{10\times4^2}{8} = 20\ \text{kNm}$.

$M_A = 0$, $M_C = 0$

$a_1 = \frac{2}{3}\times b\times h = \frac{2}{3}\times 4\times 20 = 53.33\ m^2$ (1/2M)

$\bar{x}_1 = \frac{b}{2} = \frac{4}{2} = 2m.$

$a_2 = 0$, $\bar{x}_2 = 0$

Q .NO	SOLUTION	MARKS

Q (b) cont.

apply clapeyrons theorem for span ABC

$$m_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$$

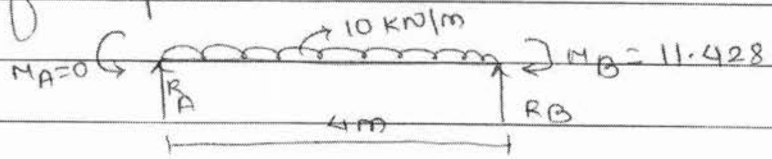
$$0 + 2M_B(4+3) + 0 = -\left(\frac{6\times 53.33\times 2}{4} + 0\right)$$

$$14\,m_B = -159.999$$

$$\boxed{m_B = -11.428 \text{ kNm}}$$ (1/2 M)

Calculation of support reaction

(a) for span AB



$\Sigma Fx = 0$, $\Sigma Fy = 0$.

$$R_A + R_B = (10\times 4)$$

$$R_A + R_B = 40$$

$\Sigma M_A = 0$ $\quad R_B \times 4 = 10\times 4\times \frac{4}{2} + 11.428$

$$R_B \times 4 = 91.428$$

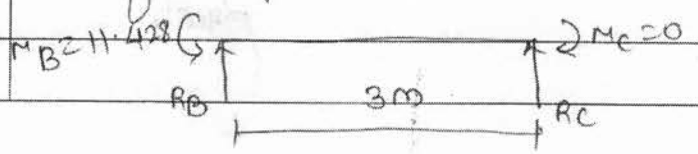
$$\boxed{R_B = 22.857 \text{ KN}}$$ (1/2 M)

$$R_A + R_B = 40$$

$$R_A = 40 - 22.857$$

$$\therefore \boxed{R_A = 17.143 \text{ KN}}$$

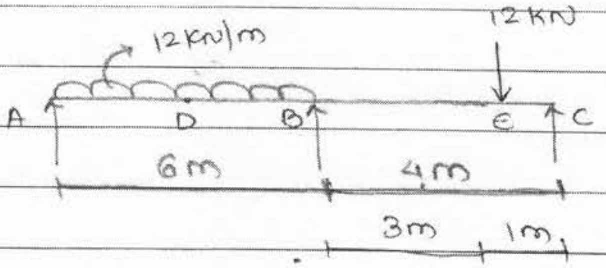
(b) for span BC

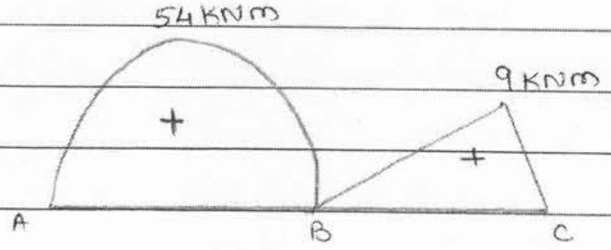


$\Sigma Fx = 0$

Q .NO	SOLUTION	MARKS
Q 4(b) (cont.)	$\Sigma F_y = 0$ $\quad R_B + R_C = 0$	
	$\Sigma M_B = 0$ $\quad -R_C \times 3 - M_B = 0$	
	$3R_C = -11.428$	
	$\boxed{R_C = -3.809\ kN\ (\uparrow)}$	
	$\therefore R_C = 3.809\ kN\ (\downarrow)$	
	$\therefore R_B + R_C = 0$	
	$\boxed{R_B = +3.809\ kN}\ \uparrow$	
	$\therefore$ $R_A = 17.143\ kN\ \uparrow$ $R_B = 22.857 + 3.809 = 26.667\ kN\ (\uparrow)$ $R_C = -3.809\ kN\ (\uparrow)$	$\left(\frac{1}{2} M\right)$
	Shear force calculation:-	
	1) Shear force at left of $A = 0$	
	2) Shear force at Right of $A = +17.143\ kN$	
	3) Shear force at left of $B = 17.143 - (10 \times 4)$ $= -22.857\ kN$	
	4) Shear force at Right of $B = -22.857 + 26.667$ $= +3.809\ kN$	(1M)
	5) Shear force at left of $C = +3.809\ kN$	
	6) Shear force at Right of $C = +3.809 - 3.809$ $= 0$	

Q.NO	SOLUTION	MARKS
4 - c		





BMD or M-diagram.

Sol:-

$L_1 = 6m$, $M_A = 0$

$L_2 = 4m$ $M_C = 0$

maximum Bending moment $M_D = \frac{wL^2}{8} = \frac{12 \times 6^2}{8}$

$M_D = 54$ kNm

maximum Bending moment $M_E = \frac{wab}{L} = \frac{12 \times 3 \times 1}{4}$

$M_E = 9$ kNm. (1m)

For span AB

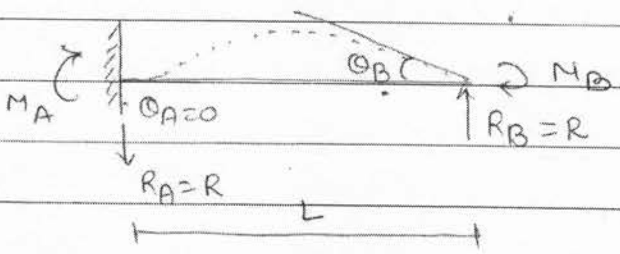
$a_1 = \frac{2}{3} \times b \times h = \frac{2}{3} \times 6 \times 54 = 216 m^2$

$\bar{x}_1 = \frac{b}{2} = \frac{6}{2} = 3m.$

Q.NO	SOLUTION	MARKS
	For span BC	
	$a_2 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 4 \times 9 = 18 m^2$	
	$\bar{x}_2 = \frac{L+b}{3} = \frac{4+1}{3} = 1.667 m.$	(1M)
	apply clapeyron's theorem for span ABC	
	$M_A L_1 + 2M_B (L_1+L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$	(1M)
	$0 + 2M_B(6+4) + 0 = -\left(\frac{6 \times 216 \times 3}{6} + \frac{6 \times 18 \times 1.667}{4}\right)$	
	$20 M_B = -(648 + 45.009)$	
	$20 M_B = -693.009$	
	$\boxed{M_B = -34.65 \text{ KNm}}$	
	Final moments :-	
	$M_A = 0$ $M_B = -34.65$ KNm $M_C = 0$	(1M)
4-d	a Define stiffness of beam The moment required at simply supported end of beam so as to produce unit rotation at that end without translation of either end is called as stiffness of beam	(2M)

Q.NO	SOLUTION	MARKS

(b) Stiffness factor for beam far end fixed and simply supported end



($\frac{1}{2}$M)

In this case $\theta_A = 0$ and slope exist at B ie. θ_B.

$$\theta_B = \frac{M_B \cdot L}{4EI}$$

($\frac{1}{2}$M)

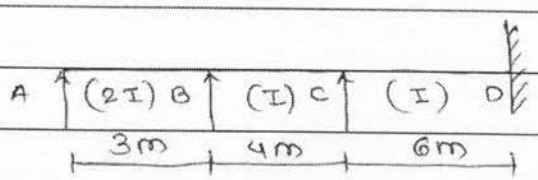
$$\therefore M_B = \frac{4EI\theta_B}{L}$$

for unit rotation $\theta_B = 1$

$$\boxed{M_B = \frac{4EI}{L}}$$

(1M)

4-(e)



Distribution factor for B & C.

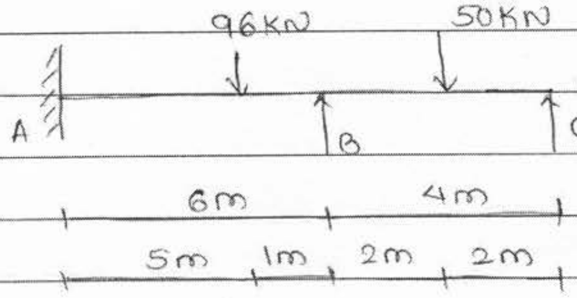
P.T.O

Q.NO	SOLUTION	MARKS

Distribution factor Table

Joint	member	Relative stiffness	Total stiffness	Distribution factor	Marks
C	CD	$\frac{4EI}{L} = \frac{4EI}{6}$ $= 0.667EI$	$0.667EI$ $+1.0EI$	$\frac{0.667EI}{1.667EI}$ $= 0.4$	(1M)
	CB	$\frac{4EI}{L} = \frac{4EI}{4}$ $= 1EI$	$= 1.667EI$	$\frac{1EI}{1.667EI}$ $= 0.6$	(1M)
B	BC	$\frac{4EI}{L} = \frac{4EI}{L}$ $= 1EI$	$1EI$ $+2EI$	$\frac{1EI}{3EI}$ $= 0.33$	(1M)
	BA	$\frac{3EI}{L} = \frac{3E(2I)}{3}$ $= 2EI$	$= 3EI$	$\frac{2EI}{3EI}$ $= 0.67$	(1M)

Q4-(f)



[Note :- Assuming 96 KN at 1m from B and 50 KN load at midspan of BC]

Note:- Q 4(F) In this Problem, the distances of Point loads 96 KN and 50 KN from any support is not mentioned, therefore solution is prepared by taking 1m distance of 96 KN load from point B & 50 KN load at centre of span BC. Student may solve by assuming any other suitable distances.

Q.NO	SOLUTION	MARKS
	Support moments	
	$m_{AB} = \frac{-wab^2}{L^2} = \frac{-96\times5\times1^2}{6^2} = -13.33$ KNm	
	$m_{BA} = +\frac{wa^2b}{L^2} = \frac{+96\times5^2\times1}{6^2} = +66.667$ KNm	(1M)
	$m_{BC} = -\frac{wl}{8} = -25$ KNm	
	$M_{CB} = +\frac{wL}{8} = +\frac{50\times4}{8} = +25$ KNm.	
	Distribution factor table.	

Joint	member	Relative Stiffness	Total Stiffness	Distribution factor
B	BA	$\frac{4EI}{L} = \frac{4EI}{6} = 0.667EI$	$0.667EI + 0.75EI = 1.417EI$	$\frac{0.667EI}{1.417EI} = 0.47$
	BC	$\frac{3EI}{L} = \frac{3EI}{4} = 0.75EI$		$\frac{0.75EI}{1.417EI} = 0.53$

(1M)

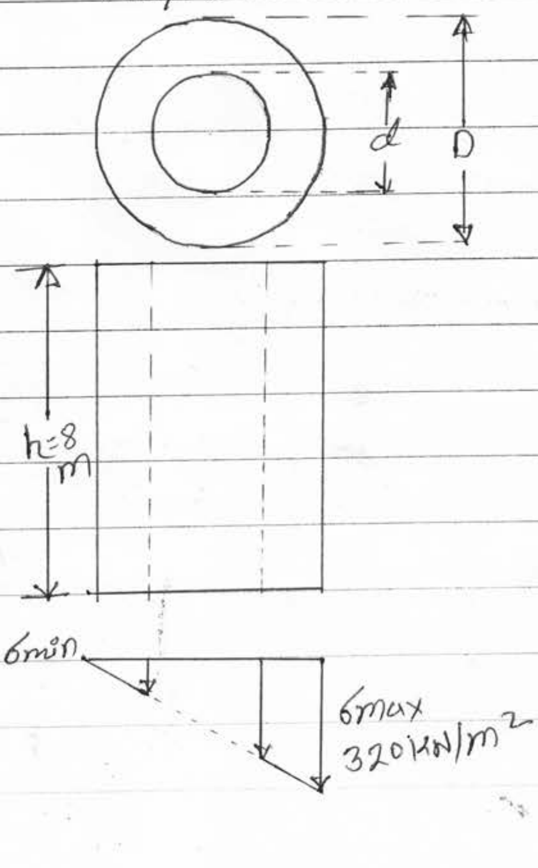
Q .NO	SOLUTION	MARKS
	Final moments.	

A		B	C
	0.47	0.53	
-13.33	66.667	-25	+25
			-25
		-12.5	
-13.33	66.667	-37.5	0
	$\boxed{-29.167}$		
	-13.71	-15.457	
-6.855			
-20.185	+52.957	-52.957	0

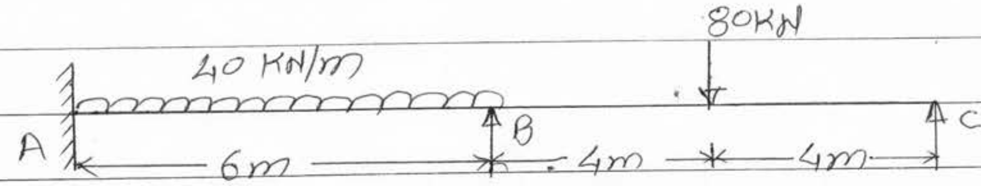
(2M)

Note:- Q4(f) In this problem the distance of point load from supports are not mentioned, thus students may assume any suitable distances for solution and examiner should give proportionate marks.

Q .NO	SOLUTION	MARKS
Q5 a)	Given, $D = 3d$; $h = 8m$; $p = 1500\ N/m^2 = 1.5\ KN/m^2$	
	$\gamma = 20\ KN/m^3$; $C_p = 0.67$	
	i) Direct stress at the base due to wt. of chimney. σ_0	
	$\sigma_0 = \frac{W}{A} = \frac{A \times h \times \gamma}{A} = h \times \gamma$	1M
	$\sigma_0 = 8 \times 20 = 160\ KN/m^2$	1M
	ii) Bending stress (σ_b)	
	$\sigma_b = \frac{M}{Z} = \frac{P \times h/2}{Z}$	$\frac{1}{2}$M
	a) Wind force $P = C_p \times p \times$ Area exposed to wind	
	$= 0.67 \times 1.5 \times D \cdot h$	
	$P = 8.04 D\ K \cdot N$	$\frac{1}{2}$M
	b) section modulus Z	
	$Z = \frac{\pi}{32D}(D^4 - d^4) = \frac{\pi}{32}\left[\frac{D^4 - (\frac{1}{3}D)^4}{D}\right]$	
	$Z = \frac{\pi}{32} \times 0.9876 D^3$	
	$Z = 96.96 \times 10^{-3} D^3$	1M
	$\therefore \sigma_b = \frac{8.04 D \times 4}{96.96 \times 10^{-3} D^3} = \frac{331.68}{D^2}$	

Q .NO	SOLUTION	MARKS
Q5 a) Contd...	iii) For No tension at the base.	
	$\sigma_0 = \sigma_b$.	1M
	$160 = \frac{331.68}{D^2}$.	
	$\therefore D = 1.4397\ m$.	1M
	iv) Internal dia (d)	
	$D = 3d$	
	$\therefore d = \frac{D}{3} = \frac{1.4397}{3} = 0.479\ m$	
	External dia $D = 1.439\ m$	
	$d = 0.479\ m$.	
	$\therefore \sigma_{max} = 2\sigma_0 = 2 \times 160 = 320\ KN/m^2$	1M
	$\therefore \sigma_{min} = 0\ KN/m^2$.	
		1M

Q .NO	SOLUTION	MARKS
Q5b		



i) Fixed end moments

$M_{AB} = M_{BA} = \pm \frac{wl^2}{12} = \pm \frac{40 \times 6^2}{12} = \pm 120\ KN{\cdot}m$ — 1M

$M_{BC} = M_{CB} = \pm \frac{WL}{8} = \pm \frac{80 \times 8}{8} = \pm 80\ KN{\cdot}m$ — 1M

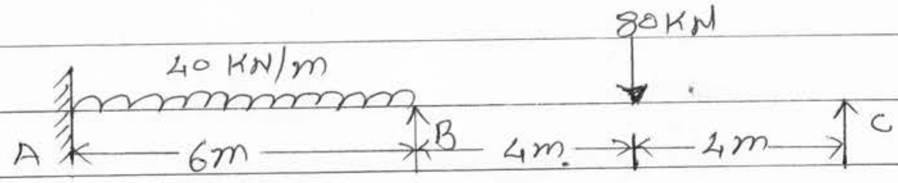
ii) Stiffness factor & Distribution factor

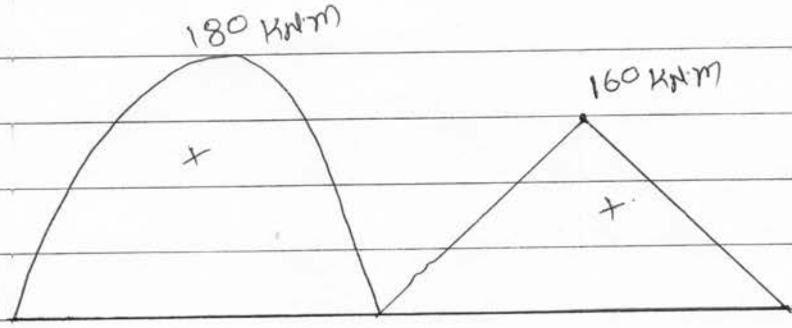
Joint	member	stiffness of member	Relative stiffness of Joint	D.F	MARKS
B	BA	$\frac{4EI}{L} = \frac{4EI}{6}$	$\frac{25}{24}EI$	0.64	1M
	BC	$\frac{3EI}{L} = \frac{3EI}{8}$		0.36	1M

iii) Moment Distribution table

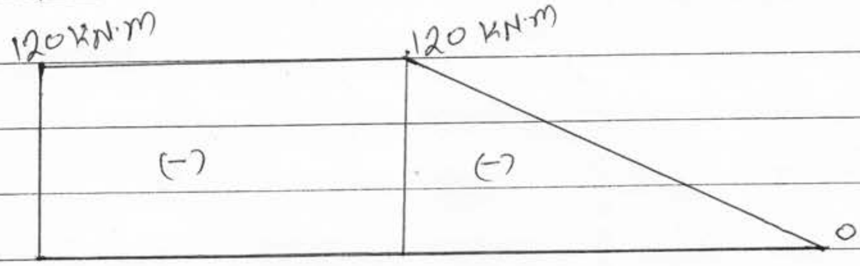
Joint	A	B		C	MARKS
member	AB	BA	BC	CB	
D.F	–	0.64	0.36	–	
F.E.M	−120	120	−80	80	1M
Release 'C' & carry over to B			−40	−80	
Initial moments	−120	120	−120	0	
Final Moments	−120	120	−120	0	1M

Q .NO	SOLUTION	MARKS
Q5b Cont...		

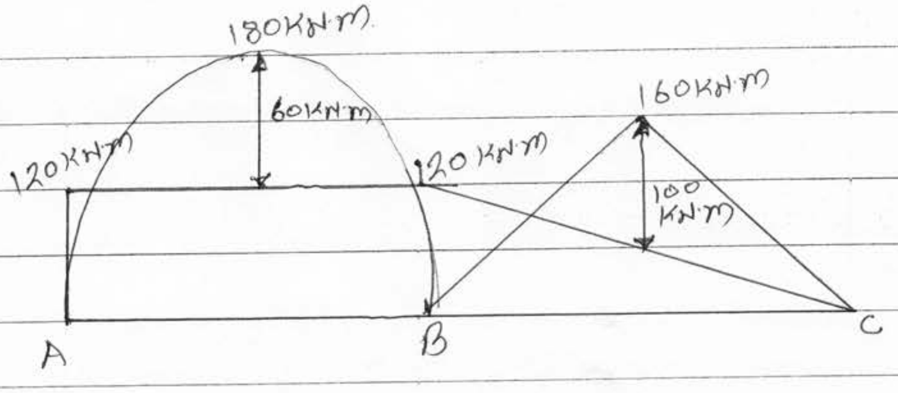




free B.M



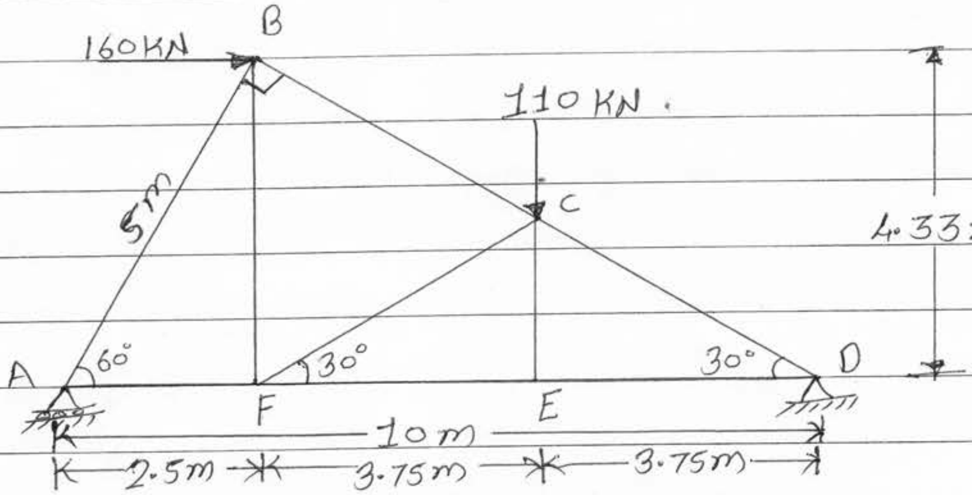
fixed B.M.



final B.M.D. — 2M

Q .NO	SOLUTION	MARKS

Q5 c)



i) From Geometry

$\angle ABD$

$\sin 30 = \frac{AB}{AD}$ $\quad \therefore AB = AD \sin 30^\circ$

$AB = 10 \sin 30 = \underline{5m}$

$\angle BAF$

$\sin 60 = \frac{BF}{AB}$ $\quad \therefore BF = AB \sin 60 = 5 \sin 60$

$\therefore BF = \underline{4.33m}$

$\cos 60 = \frac{AF}{AB}$ $\quad \therefore AF = AB \cos 60 = 5 \cos 60$

$\therefore AF = \underline{2.5m}$

1M

$\therefore EF = DE = \frac{10 - 2.5}{2} = \underline{3.75m}$

ii) Support Reactions

a) $\Sigma F_x = 0$

$160 - R_{DH} = 0 \quad \therefore R_{DH} = 160\ KN\ (\leftarrow)$

b) $\Sigma F_y = 0$

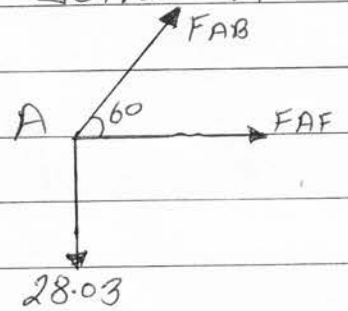
$R_A + R_{Dy} = 110$ —— (i)

c) $\Sigma M @ A = 0$

$(160 \times 4.33) + (110 \times 6.25) - 10 R_{Dy} = 0$

Q.NO	SOLUTION	MARKS
Q5C Contd...	$\therefore R_{Dy} = 138.03\ KN\ (\uparrow)$	1 M
	$\therefore R_A = 110 - 138.03 = -28.03\ KN$	(Reactions)
	$\therefore R_A = 28.03\ KN\ (\downarrow)$	

iii) Joint A



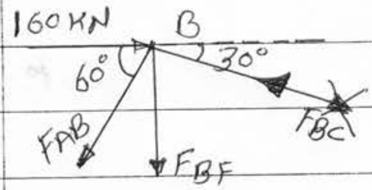
$\Sigma F_y = 0$

$-28.03 + F_{AB} \sin 60 = 0$

$F_{AB} = \frac{28.03}{\sin 60}$

$\underline{F_{AB} = 32.36\ KN}$ — 1M

Joint B



$\Sigma F_x = 0$

$160 - 32.36 \cos 60 + F_{BC} \cos 30 = 0$

$F_{BC} = \frac{-143.82}{\cos 30} = -166.06\ KN$

$\therefore F_{BC} = 166.06\ KN$ (Comp)

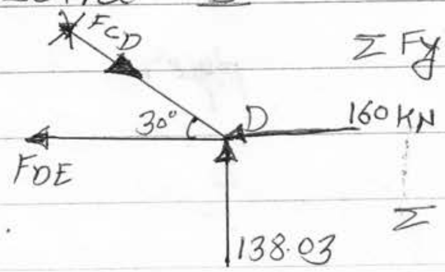
$\Sigma F_y = 0$

$-F_{BF} - 32.36 \sin 60 + 166.06 \sin 30 = 0$

$-F_{BF} = -55\ KN$

$\therefore \underline{F_{BF} = 55\ KN.}$ — 2M

Joint D



$\Sigma F_y = 0;\ +138.03 + F_{CD} \sin 30 = 0$

$F_{CD} = \frac{-138.03}{\sin 30} = -276.06\ KN$

$F_{CD} = 276.06\ KN$ (Comp)

$\Sigma F_x = 0;$
$-160 - F_{DE} + 276.06 \cos 30 = 0$

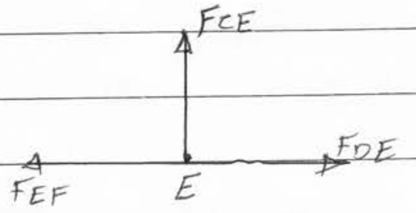
$\therefore \underline{F_{DE} = 79.07\ KN}$ (Tensile) — 2 M

Q .NO	SOLUTION	MARKS

Q5C Contd...

Joint E $\Sigma F_y = 0$

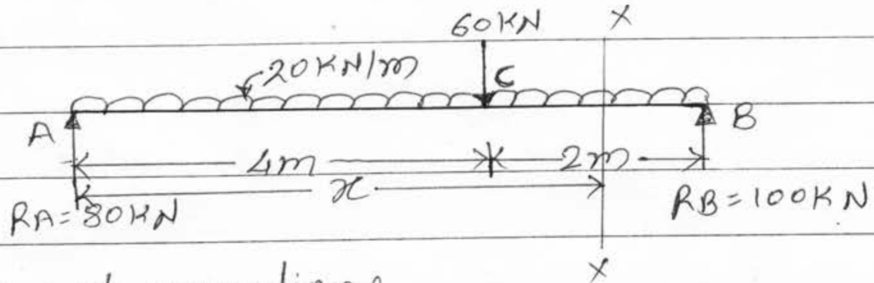
$F_{CE} = 0$ — 1M.



iv) Final member ~~m~~ forces.

member	magnitude	Nature
AB	32.36 KN	Tensile
BF	55 KN	Tensile
CE	0 KN	—
ED	79.07 KN	Tensile

Q6 a)



i) Support reactions

$\Sigma F_y = 0$; $R_A + R_B - 20\times6 - 60 = 0$

$R_A + R_B = 180$ KN

$\Sigma M@A = 0$; $(20\times6\times3) + (60\times4) - 6R_B = 0$

$\therefore R_B = 100$ KN

$\therefore R_A = 180 - 100 = 80$ KN. — 1M

ii) Consider a section x-x at a distance of x from A in portion CB.

$M_x = R_A \cdot x - 20 \cdot x \cdot \frac{x}{2} - 60(x-4)$ — 1 M

SUMMER – 15 EXAMINATION

Model Answer

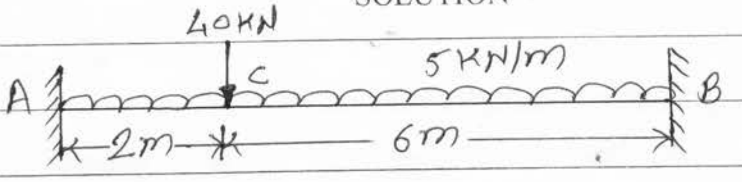
Subject Code: 17422

Q .NO	SOLUTION	MARKS
Q6 a) Conto...	But $EI\frac{d^2y}{dx^2} = M_x = 80x - 10x^2 - 60(x-4)$	
	$EI\frac{d^2y}{dx^2} = 80x - 10x^2 - 60(x-4) \cdots\cdots (A)$	
	Integrate eq^n A w.r.to x, we get	
	$EI\frac{dy}{dx} = \frac{80x^2}{2} - \frac{10x^3}{3} - 60\frac{(x-4)^2}{2} + C_1 \cdots (B)$	$\frac{1}{2}$ M
	Again integrate eq^n B w.r.to. x	
	$EI \cdot y = \frac{80x^3}{6} - \frac{10x^4}{12} - \frac{60(x-4)^3}{6} + C_1x + C_2 \cdots (C)$	$\frac{1}{2}$ M
	Apply boundary Conditions	
	a) At A, i.e, $x=0$, $y=0$ put in eq^n C	
	$EI(0) = 0 - 0 - 0 + 0 + C_2$	
	$\therefore \boxed{C_2 = 0}$	$\frac{1}{2}$ M
	b) At, B i.e, $x=6$, $y=0$ put in eq^n C	
	$EI(0) = \frac{80(6)^3}{6} - \frac{10(6)^4}{12} - \frac{60(6-4)^3}{6} + 6 \cdot C_1$	
	$0 = 2880 - 1080 - 80 + 6C_1$	
	$\therefore \boxed{C_1 = -286.67}$	$\frac{1}{2}$ M
	Substitute the value of C_1 & C_2 in eq^n B & C respectively	
	$EI\frac{dy}{dx} = \frac{80x^2}{2} - \frac{10x^3}{3} - \frac{60(x-4)^2}{2} - 286.67 \cdots$ slope eq^n	1M
	$EI \cdot y = \frac{80x^3}{6} - \frac{10x^4}{12} - \frac{60(x-4)^3}{6} - 286.67x \cdots$ Deflection eq^n	

Q .NO	SOLUTION	MARKS
Q6 a) Cont...	iii> Position of point of maximum deflection	
	At maximum Deflection slope remain zero	
	i.e. at y_{max}, $\frac{dy}{dx} = 0$	
	Assume the maximum Deflection will occure in portion AC.	
	$EI(0) = \frac{80x^2}{2} - \frac{10x^3}{3} - 286.67$	
	$0 = 40x^2 - 3.33x^3 - 286.67$	
	Solving the above eq^n for x we get	
	$x = 11.34m$ or $x = -2.44m$ or $x = 3.109m$	
	$\therefore \boxed{x = 3.109m}$ $\because$ Assume portion for maximum Deflection is correct as $x<4$.	1M
	iv> Maximum Deflection.	
	$EI \cdot y_{max} = \frac{80(3.109)^3}{6} - \frac{10(3.109)^4}{12} - 286.67 \times 3.109$	
	$EI\, y_{max} = 400.68 - 77.85 - 891.25$	
	$\therefore \boxed{y_{max} = \frac{-568.42}{EI}}$	2M

Q .NO	SOLUTION	MARKS

Q6b)



Let, $a = 2m,\ b = 6m,\ L = 8\ m$

i) Support moments

$$M_A = -\left[\frac{wL^2}{12} + \frac{Wab^2}{L^2}\right] = -\left[\frac{5\times8^2}{12} + \frac{40\times2\times6^2}{8^2}\right]$$

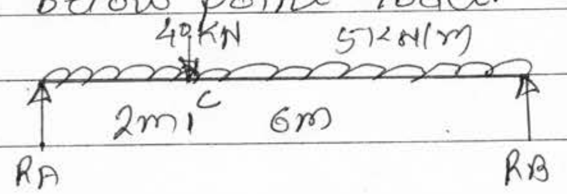
$M_A = -71.67\ KN{\cdot}m.$ — 1M

$$M_B = -\left[\frac{wL^2}{12} + \frac{Wa^2b}{L^2}\right] = -\left[\frac{5\times8^2}{12} + \frac{40\times2^2\times6}{8^2}\right]$$

$M_B = -41.67\ KN{\cdot}m$ — 1M

ii) Free B.M ordinate below point load.

a) Reactions



$\Sigma F_y = 0;\ R_A + R_B = 80\ KN.$

$\Sigma M@A = 0;$

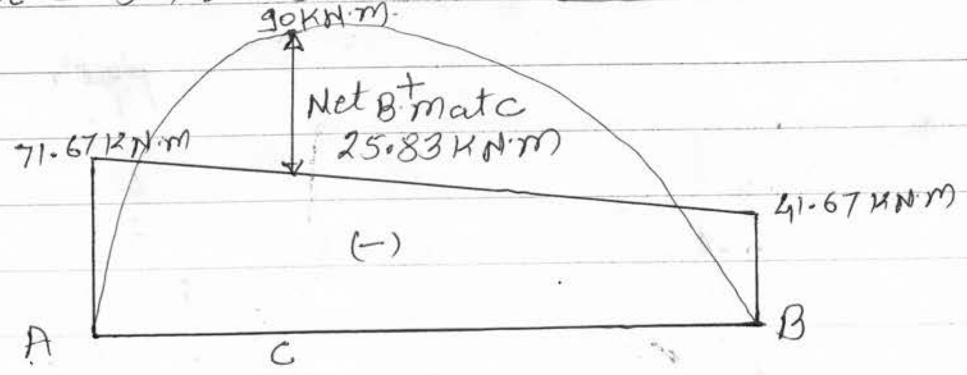
$(5\times8\times4) + (40\times2) - 8R_B = 0$

$\therefore R_B = 30\ KN$

$\therefore R_A = 50\ KN.$ — 1M

B.M at C

$M_C = 50\times2 - 5\times2\times1 = \underline{90\ KN{\cdot}m.}$



1M

Q.NO	SOLUTION	MARKS

Q6b Contd...

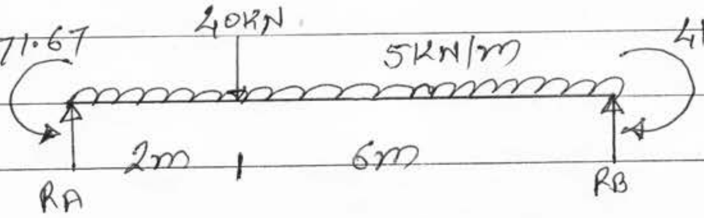
Net B.M at C

$$= 90 - \left[41.67 + \left(\frac{71.67 - 41.67}{8} \times 6\right)\right]$$

$$= 90 - 64.17$$ 1M

Net B.M at C = 25.83 KN.m

iii) Final Support Reactions to draw S.F.D.



$\Sigma F_y = 0$; $R_A + R_B = 80$ KN

$\Sigma M@A = 0$; $-71.67 + (5\times8\times4) + (40\times2) + 41.67 - 8R_B = 0$

$\therefore R_B = 26.25$ KN

$\therefore R_A = 53.75$ KN. 1M

S.F Calculations

S.F at left of A = 0

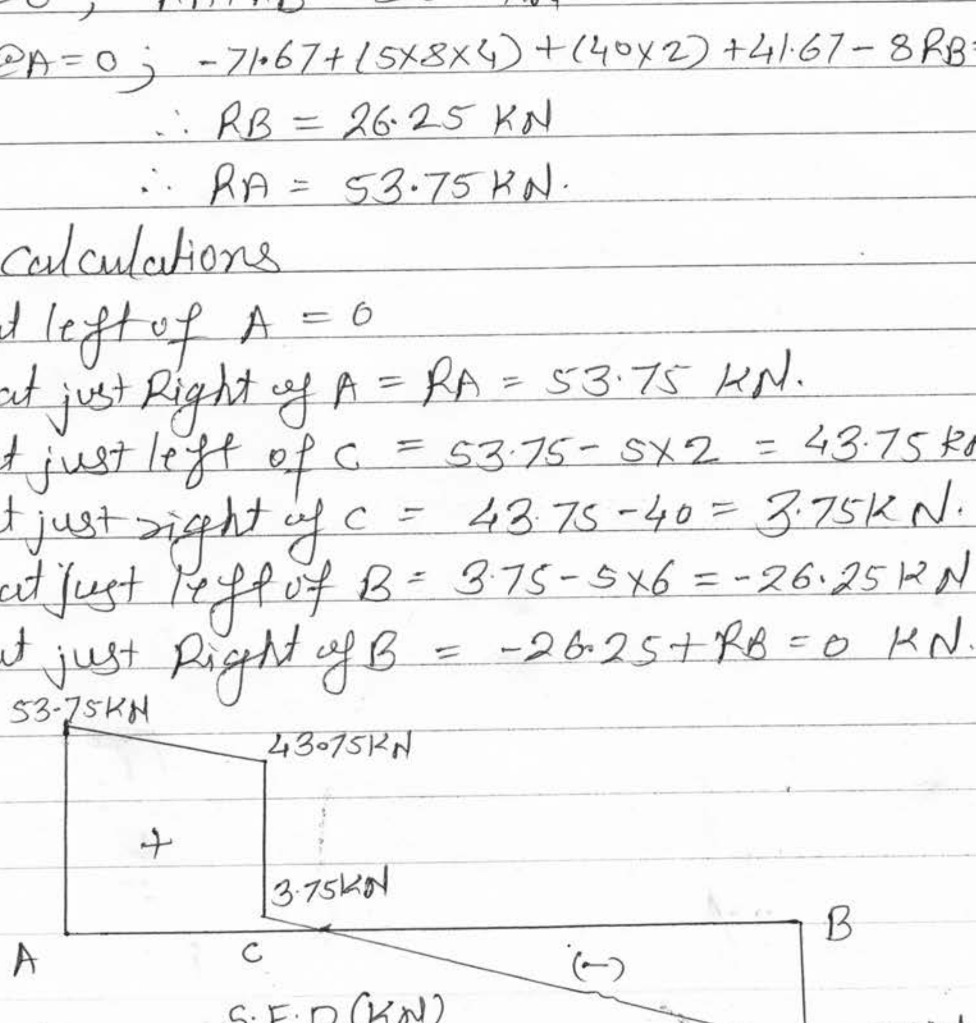
S.F at just Right of A = RA = 53.75 KN.

S.F at just left of C = 53.75 − 5×2 = 43.75 KN 1M

S.F at just right of C = 43.75 − 40 = 3.75 KN.

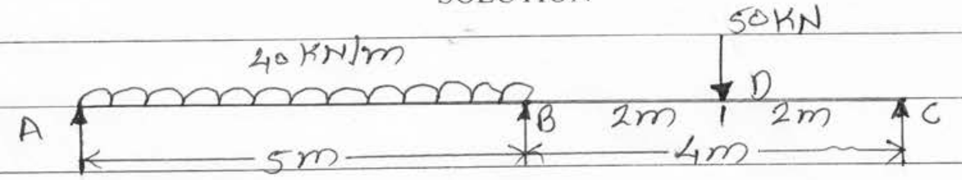
S.F at just left of B = 3.75 − 5×6 = −26.25 KN

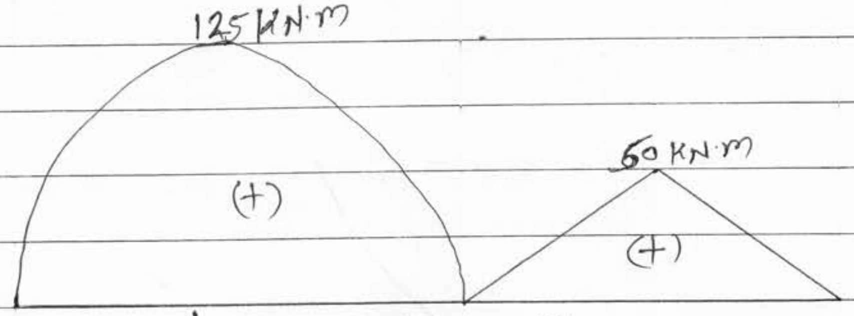
S.F at just Right of B = −26.25 + RB = 0 KN.



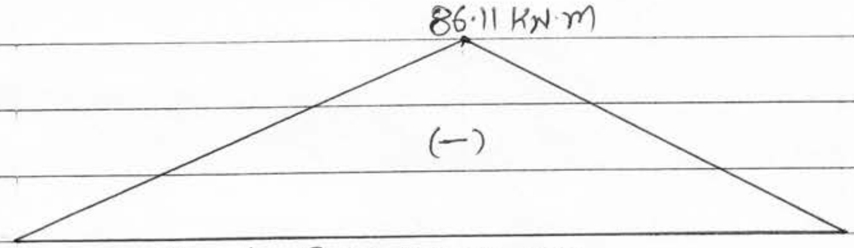
S.F.D (KN) 1M

Q .NO	SOLUTION	MARKS
Q6 c)		

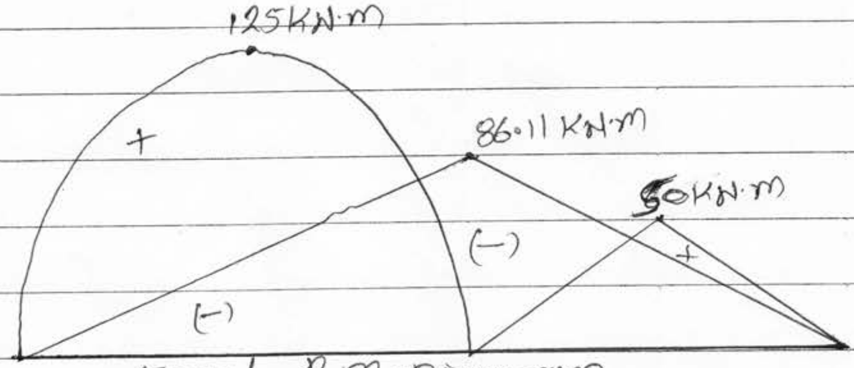




Free B.M. Diagram

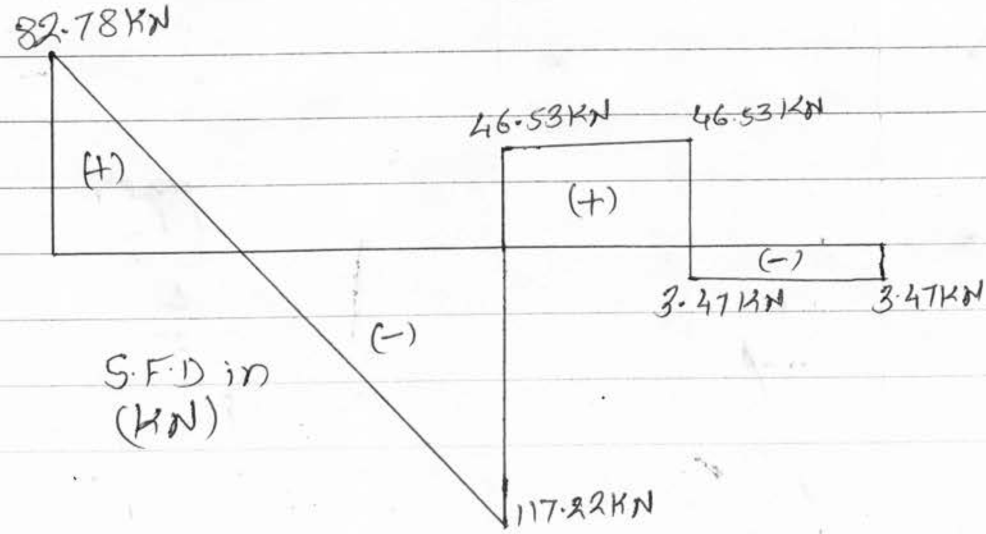


fixed B.M. Diagram.



1M

Final B.M. Diagram.



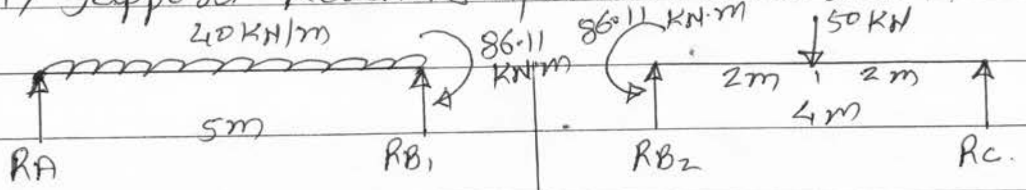
1M

Q .NO	SOLUTION	MARKS
Q6C Cont...	i) Assume the beam is a series of s.s.beam & draw free B.M. diagram.	
	span AB, max. B.M $= \frac{wl^2}{8} = \frac{40\times5^2}{8} = 125$ KN.m	
	span BC, max. B.M $= \frac{WL}{4} = \frac{50\times4}{4} = 50$ KN.m	1M
	ii) Apply clapeyron's theorem to span ABC.	
	$M_A(L_1) + 2M_B(L_1+L_2) + M_C(L_2) = -\left[\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right]$	1M
	Known moments	
	$M_A = M_C = 0$ .... exterior simple support	
	$a_1 = \frac{2}{3}\times5\times125$ $a_2 = \frac{1}{2}\times4\times50$	
	$a_1 = 416.67$ $a_2 = 100$	1M
	$\bar{x}_1 = \frac{5}{2} = 2.5$ $\bar{x}_2 = 4/2 = 2m.$	
	$L_1 = 5m$ $L_2 = 4m.$	
	$2M_B(5+4) = -\left[\frac{6\times416.67\times2.5}{5} + \frac{6\times100\times2}{4}\right]$	
	$18M_B = -1550.01$	
	$\therefore \boxed{M_B = 86.11 \text{ KN.m}}$ (Hogging)	1M

Q.NO	SOLUTION	MARKS

Q6C Contd...

iii> Support Reactions of a Continuous beam.



$\Sigma Fy = 0$, $R_A + R_{B_1} = 200$ KN

$\Sigma M@A = 0$

$(40 \times 5 \times 2.5) + 86.11 - 5R_{B_1} = 0$

$\therefore R_{B_1} = 117.22$ KN

$\therefore R_A = 82.78$ KN

$\Sigma Fy = 0$; $R_{B_2} + R_C = 50$ KN

$\Sigma M@B = 0$

$-86.11 + 50 \times 2 - 4R_C = 0$

$13.89 = 4R_C$

$R_C = 3.47$ KN

$\therefore R_{B_2} = 46.53$ KN

$\therefore R_A = 82.78 = 82.78$ KN

$R_B = R_{B_1} + R_{B_2} = 163.75$ KN — 1 M

$R_C = 3.47 = 3.47$ KN

iv> S.F. Calculations

S.F at just left of A = 0

S.F at just right of A = 82.78 KN (RA)

S.F at just left of B = 82.78 − 40×5 = −117.22 KN

S.F at just right of B = −117.22 + RB = 46.53 KN — 1 M

S.F at just left of D = 46.53 KN

S.F at just right of D = 46.53 − 50 = −3.47 KN

S.F at just left of C = −3.47 KN

S.F at just right of C = −3.47 + RC = 0 KN.