



Summer 2014 Examination

Subject & Code: Applied Maths (17301)

Model Answer

Page No: 1/27

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		<p>Important Instructions to the Examiners:</p> <ol style="list-style-type: none">1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.7) For programming language papers, credit may be given to any other program based on equivalent concept.		



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	a)	Attempt any TEN of the following: Find the inclination of the tangent to the curve $y = e^{2x}$ at $(1, -3)$		
	Ans.	$y = e^{2x}$ $\therefore \frac{dy}{dx} = 2e^{2x}$ \therefore slope of tangent at $(1, -3)$ is, $m = 2e^2$ \therefore the angle of inclination is, $\theta = \tan^{-1}(m) = \tan^{-1}(2e^2)$	$\frac{1}{2}$ $\frac{1}{2}$ 1	2
	b)	Find the point on the curve $y = 2x^2 - 6x$ where the tangent is parallel to the x-axis.		
Ans.	$y = 2x^2 - 6x$ $\therefore \frac{dy}{dx} = 4x - 6$ But tangent is parallel to x-axis. $\therefore 4x - 6 = 0$ $\therefore x = \frac{3}{2}$ $\therefore y = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) = -\frac{9}{2}$ \therefore the point is $\left(\frac{3}{2}, -\frac{9}{2}\right)$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2	
c)	Evaluate $\int \sqrt{1 + \sin 2x} \cdot dx$			
Ans.	$\int \sqrt{1 + \sin 2x} \cdot dx = \int \sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} \cdot dx$ $= \int \sqrt{(\sin x + \cos x)^2} \cdot dx$ $= \int (\sin x + \cos x) \cdot dx$ $= -\cos x + \sin x + c$	$\frac{1}{2}$ $\frac{1}{2}$ 1	2	
		OR		



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1)		$\int \sqrt{1 + \sin 2x} \cdot dx = \int \sqrt{\cos^2 x + \sin^2 x + 2 \sin x \cos x} \cdot dx$ $= \int \sqrt{(\cos x + \sin x)^2} \cdot dx$ $= \int (\cos x + \sin x) \cdot dx$ $= \sin x - \cos x + c$ <p>Note: In the solution of any integration problems, if the constant c is not added, 1/2 mark may be deducted.</p>	1/2 1/2 1	2
	d)	Evaluate $\int \frac{e^x}{e^{2x} - 16} \cdot dx$		
	Ans.	$\int \frac{e^x}{e^{2x} - 16} \cdot dx$ <div style="display: inline-block; vertical-align: middle; border-left: 1px solid black; padding-left: 5px; margin-left: 10px;"> $\text{Put } e^x = t$ $\therefore e^x dx = dt$ </div> $= \int \frac{1}{t^2 - 16} \cdot dt$ $= \int \frac{1}{t^2 - 4^2} \cdot dt$ $= \frac{1}{2 \cdot 4} \log \left(\frac{t-4}{t+4} \right) + c$ $= \frac{1}{8} \log \left(\frac{e^x - 4}{e^x + 4} \right) + c$	1/2 1/2 1/2 1/2	2
		OR		
		$\int \frac{e^x}{e^{2x} - 16} \cdot dx$ <div style="display: inline-block; vertical-align: middle; border-left: 1px solid black; padding-left: 5px; margin-left: 10px;"> $\text{Put } e^x = t$ $\therefore e^x dx = dt$ </div> $= \int \frac{1}{t^2 - 16} \cdot dt$ $= \int \frac{1}{(t-4)(t+4)} \cdot dt$ $= \int \left[\frac{1/8}{t-4} - \frac{1/8}{t+4} \right] \cdot dt$ $= \frac{1}{8} \log(t-4) - \frac{1}{8} \log(t+4) + c$ $= \frac{1}{8} \log(e^x - 4) - \frac{1}{8} \log(e^x + 4) + c$	1/2 1/2 1/2 1/2	2



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1)	e)	Evaluate $\int \frac{\cos x - \sin x}{\cos x + \sin x} \cdot dx$		
	Ans.	$\int \frac{\cos x - \sin x}{\cos x + \sin x} \cdot dx$ $= \int \frac{1}{t} \cdot dt$ $= \log t + c$ $= \log (\cos x + \sin x) + c$ <p style="text-align: center;">OR</p> $\int \frac{\cos x - \sin x}{\cos x + \sin x} \cdot dx$ $= \int \frac{1 - \tan x}{1 + \tan x} \cdot dt$ $= \int \tan \left(\frac{\pi}{4} - x \right) \cdot dx$ $= \frac{\log \sec \left(\frac{\pi}{4} - x \right)}{-1} + c \quad \text{or} \quad -\log \sec \left(\frac{\pi}{4} - x \right) + c$ $\text{or} \quad \log \cos \left(\frac{\pi}{4} - x \right) + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p>	<p>2</p> <p>2</p>
	f)	Evaluate $\int \log x dx$		
	Ans.	$\int \log x dx = \int \log x \cdot 1 \cdot dx$ $= \log x \int 1 dx - \int \left(\int 1 dx \right) \frac{d}{dx} (\log x) dx$ $= \log x \cdot x - \int x \cdot \frac{1}{x} dx$ $= x \log x - \int 1 dx$ $= x \log x - x + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	<p>2</p>



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1)	g)	Evaluate $\int_{\pi/6}^{\pi/4} \cot^2 x \cdot dx$		
	Ans.	$\int_{\pi/6}^{\pi/4} \cot^2 x \cdot dx = \int_{\pi/6}^{\pi/4} (\cos ec^2 x - 1) \cdot dx$ $= [-\cot x - x]_{\pi/6}^{\pi/4}$ $= \left[-\cot \frac{\pi}{4} - \frac{\pi}{4} \right] - \left[-\cot \frac{\pi}{6} - \frac{\pi}{6} \right]$ $= -1 + \sqrt{3} - \frac{\pi}{12} \quad \text{or} \quad 0.470$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	2
		<p>Note: In case of definite integrations, the problem may be solved by without limits and then the limits would be applied, as illustrated below:</p> $\int \cot^2 x \cdot dx = \int (\cos ec^2 x - 1) \cdot dx$ $= -\cot x - x$ $\therefore \int_{\pi/6}^{\pi/4} \cot^2 x \cdot dx = [-\cot x - x]_{\pi/6}^{\pi/4}$ $= \left[-\cot \frac{\pi}{4} - \frac{\pi}{4} \right] - \left[-\cot \frac{\pi}{6} - \frac{\pi}{6} \right]$ $= -1 + \sqrt{3} - \frac{\pi}{12} \quad \text{or} \quad 0.470$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	2
	h)	Find the area enclosed by $y = 2x + x^2$ (above the x-axis) and $x = 1$ and $x = 3$.		
	Ans.	$\int_1^3 y \cdot dx = \int_1^3 (2x + x^2) \cdot dx$ $= \left[x^2 + \frac{x^3}{3} \right]_1^3$ $= \left[3^2 + \frac{3^3}{3} \right] - \left[1 + \frac{1}{3} \right]$ $= \frac{50}{3} \quad \text{or} \quad 16.667$	<p>1</p> <p>1/2</p> <p>1/2</p>	2
	i)	Find the order and degree of the following equation: $\frac{d^2 y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$		
	Ans.	Order = 2	1	



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1)		<p>For degree,</p> $\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \frac{dy}{dx}$ <p>\therefore Degree = 2</p> <p>-----</p>	1	2
	j)	<p>If a coin is tossed three times, find the probability of getting exactly two tails.</p>		
	Ans.	$p = p(\text{Tail}) = \frac{1}{2}$ $\therefore q = 1 - p = \frac{1}{2}$ $\therefore p(3) = {}^n C_r p^r q^{n-r}$ $= {}^3 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{3-2}$ $= \frac{3}{8} \text{ or } 0.375$	1/2 1/2 1/2 1/2	2
		OR		
		$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$ $n = n(S) = 8$ $A = \{HTT, THT, TTH\}$ $m = n(A) = 3$ $\therefore p = \frac{m}{n} = \frac{3}{8} \text{ or } 0.375$	1/2 1/2 1	2
		OR		
		$n = n(S) = (\text{Faces of object})^{\text{No. of repetitions}} = 2^3 = 8$ $m = n(A) = {}^3 C_2 = 3$ $\therefore p = \frac{m}{n} = \frac{3}{8} \text{ or } 0.375$	1/2 1/2 1	2
		<p>Note: Due to the use of advance non-programmable scientific calculators, writing directly the values of ${}^n C_r$ or ${}^n C_r p^r q^{n-r}$ is permissible. No marks to be deducted for calculating directly the value.</p>		



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1)	k)	Verify that $y = \cos x$ is a solution of $\frac{d^2y}{dx^2} + y = 0$.		
	Ans.	$y = \cos x$ $\therefore \frac{dy}{dx} = -\sin x$ $\therefore \frac{d^2y}{dx^2} = -\cos x$ $\therefore \frac{d^2y}{dx^2} = -y$ $\therefore \frac{d^2y}{dx^2} + y = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
		OR		
		$y = \cos x$ $\therefore \frac{dy}{dx} = -\sin x$ $\therefore \frac{d^2y}{dx^2} = -\cos x$ $\therefore \frac{d^2y}{dx^2} + y = -\cos x + \cos x = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
	l)	Two cards are drawn at random from a well shuffled pack of 52 cards. Find the probability that the two cards drawn are a king and a queen of the same unit.		
	Ans.	$n = n(S) = {}^{52}C_2 = 1326$ $m = n(\text{pair of King and Queen of same suit}) = 4$ $\therefore p = \frac{m}{n} = \frac{4}{1326}$ $= \frac{2}{663} \text{ or } 0.003$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
		<p>Note for Numerical Problems: For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus the solution is taken up to 3 decimal points only. If answer is truncated more than 3 decimal points, the final answer may vary for last decimal point. Thus $2/663$ is actually 0.00301659125188536953242835595777 but can be taken as</p>		



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2)	a)	<p>0.003. Due to the use of advance calculators, such as modern scientific non-programmable calculators, the step 2/663 may not be written by the students and then directly the answer 0.003 is written. In this case, no marks to be deducted.</p> <p>-----</p> <p>Attempt any FOUR of the following:</p> <p>Find the equation of the tangent and normal to the curve $4x^2 + 9y^2 = 40$ at $(1, 2)$.</p> <p>Ans.</p> $4x^2 + 9y^2 = 40$ $\therefore 8x + 18y \frac{dy}{dx} = 0$ $\therefore \frac{dy}{dx} = -\frac{8x}{18y} \quad \text{or} \quad -\frac{4x}{9y}$ <p>\therefore the slope of tangent at $(1, 2)$ is</p> $m = \frac{dy}{dx} = -\frac{8x}{18y} = -\frac{8}{36} = -\frac{2}{9}$ <p>\therefore the equation of tangent is</p> $y - 2 = -\frac{2}{9}(x - 1)$ $\therefore 9y - 18 = -2x + 2$ $\therefore 2x + 9y - 20 = 0$ <p>\therefore the slope of normal $= -\frac{1}{m} = \frac{9}{2}$</p> <p>$\therefore$ the equation of normal is</p> $y - 2 = \frac{9}{2}(x - 1)$ $\therefore 2y - 4 = 9x - 9$ $\therefore 9x - 2y - 5 = 0$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	b)	<p>Find the maximum and minimum value of $x^3 = 18x^2 + 96x$</p> <p>Note: For a given differential function only, we can find extreme values of the function. In the given problem function is not given, but polynomial equation in x is given and we never find extreme values of an equation as every equation has its own definite values known as its roots/solutions. The given one is $x^3 = 18x^2 + 96x$. Considering the given is a typographical mistake, we reconsider herein this in two different ways as follows just to provide solution of the problem.</p>		



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2)		<p>The given is $x^3 = 18x^2 + 96x$. Considering = sign as -ve sign, we have first case as $y = x^3 - 18x^2 + 96x$. And considering the given as $x^3 - 18x^2 - 96x = 0$, we consider it as $y = x^3 - 18x^2 - 96x$.</p> <p>Let $y = x^3 - 18x^2 + 96x$</p> <p>$\therefore \frac{dy}{dx} = 3x^2 - 36x + 96$</p> <p>$\therefore \frac{d^2y}{dx^2} = 6x - 36$</p> <p>For stationary values, $\frac{dy}{dx} = 0$</p> <p>$\therefore 3x^2 - 36x + 96 = 0$</p> <p>$\therefore x = 4, 8$</p> <p>At $x = 4$,</p> <p>$\frac{d^2y}{dx^2} = 6(4) - 36 = -12 < 0$</p> <p>$\therefore$ At $x = 4$, y has maximum value and it is</p> <p>$y = (4)^3 - 18(4)^2 + 96(4) = 160$</p> <p>At $x = 8$,</p> <p>$\frac{d^2y}{dx^2} = 6(8) - 36 = 12 > 0$</p> <p>$\therefore$ At $x = 8$, y has minimum value and it is</p> <p>$y = (8)^3 - 18(8)^2 + 96(8) = 128$</p> <p style="text-align: center;">OR</p> <p>Let $y = x^3 - 18x^2 - 96x$</p> <p>$\therefore \frac{dy}{dx} = 3x^2 - 36x - 96$</p> <p>$\therefore \frac{d^2y}{dx^2} = 6x - 36$</p> <p>For stationary values, $\frac{dy}{dx} = 0$</p> <p>$\therefore 3x^2 - 36x - 96 = 0$</p> <p>$\therefore x^2 - 12x - 32 = 0$</p> <p>$\therefore x = 6 - 2\sqrt{17}, 6 + 2\sqrt{17}$</p> <p>$\therefore x = -2.246, 14.246$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>	<p>4</p>



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2)		<p>At $x = -2.246$, $\frac{d^2y}{dx^2} = 6(-2.246) - 36 = -49.476 < 0$ \therefore At $x = -2.246$, y has maximum value and it is $y = (-2.246)^3 - 18(-2.246)^2 - 96(-2.246) = 113.485$</p> <p>At $x = 14.246$, $\frac{d^2y}{dx^2} = 6(14.246) - 36 = 49.476 > 0$ \therefore At $x = 14.246$, y has minimum value and it is $y = (14.246)^3 - 18(14.246)^2 - 96(14.246) = -2129.485$</p> <p>-----</p>	1/2 1/2 1/2 1/2	4
	c)	<p>Find the radius of curvature for the curve $y = 2 \sin x - \sin 2x$ at $x = \frac{\pi}{2}$.</p> <p>Ans. $y = 2 \sin x - \sin 2x$ $\therefore \frac{dy}{dx} = 2 \cos x - 2 \cos 2x$ & $\frac{d^2y}{dx^2} = -2 \sin x + 4 \sin 2x$ \therefore at $x = \frac{\pi}{2}$, $\frac{dy}{dx} = 2 \cos\left(\frac{\pi}{2}\right) - 2 \cos 2\left(\frac{\pi}{2}\right) = 2$ and $\frac{d^2y}{dx^2} = -2 \sin\left(\frac{\pi}{2}\right) + 4 \sin 2\left(\frac{\pi}{2}\right) = -2$ $\therefore \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{[1 + (2)^2]^{\frac{3}{2}}}{-2} = -5.590$</p> <p>-----</p>	1/2 1/2 1 1 1	
	d)	<p>Evaluate $\int \frac{1 + \tan^2 x}{1 - \tan^2 x} \cdot dx$</p> <p>Ans. $\int \frac{1 + \tan^2 x}{1 - \tan^2 x} \cdot dx$ $= \int \frac{\sec^2 x}{1 - \tan^2 x} \cdot dx$ $\left \begin{array}{l} \text{Put } \tan x = t \\ \therefore \sec^2 x dx = dt \end{array} \right.$</p>	1	



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2)		$= \int \frac{1}{1-t^2} \cdot dt$ $= \frac{1}{2} \log \left(\frac{1-t}{1+t} \right) + c$ $= \frac{1}{2} \log \left(\frac{1-\tan x}{1+\tan x} \right) + c$ <p style="text-align: center;">OR</p> $\int \frac{1+\tan^2 x}{1-\tan^2 x} \cdot dx = \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x} \cdot dx$ $= \int \frac{1}{\cos 2x} \cdot dx$ $= \int \sec 2x \cdot dx$ $= \frac{\log(\sec 2x + \tan 2x)}{2} + c$	1 1 1 1 1 1	4 4
	e)	Evaluate $\int \frac{(x-1)e^x}{x^2 \sin^2(e^x/x)} dx$		
	Ans.	$\int \frac{(x-1)e^x}{x^2 \sin^2(e^x/x)} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px 0;"> $\text{Put } \frac{e^x}{x} = t$ $\therefore \frac{xe^x - e^x \cdot 1}{x^2} dx = dt$ $\therefore \frac{(x-1)e^x}{x^2} dx = dt$ </div> $= \int \frac{1}{\sin^2(e^x/x)} \cdot \frac{(x-1)e^x}{x^2} dx$ $= \int \frac{1}{\sin^2 t} dt$ $= \int \csc^2 t dt$ $= -\cot t + c$ $= -\cot \left(\frac{e^x}{x} \right) + c$	1 1 1 1	4



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2)	f)	Evaluate $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} \cdot dx$			
	Ans.	$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} \cdot dx$ $= \int t^2 \cdot 2dt$ $= 2 \cdot \frac{t^3}{3} + c$ $= 2 \cdot \frac{(1+\sqrt{x})^3}{3} + c$	1 1 1 1	4	
		OR			
		$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} \cdot dx$ $= \int (1+t)^2 \cdot 2dt$ $= 2 \cdot \frac{(1+t)^3}{3} + c$ $= 2 \cdot \frac{(1+\sqrt{x})^3}{3} + c$	1 1 1 1		4
		OR			
		$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} \cdot dx$ $= \int \frac{1+2\sqrt{x}+x}{\sqrt{x}} \cdot dx$ $= \int \left(\frac{1}{\sqrt{x}} + 2 + \sqrt{x} \right) \cdot dx$ $= 2\sqrt{x} + 2x + \frac{2}{3}x^{3/2} + c$	1 1 1+1	4	



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3)		<p>Attempt any FOUR of the following:</p> <p>a) Evaluate $\int \frac{dx}{4\cos^2x + 9\sin^2x}$</p> <p>Ans. $\int \frac{dx}{4\cos^2x + 9\sin^2x}$ $= \int \frac{dx/\cos^2x}{\frac{4\cos^2x + 9\sin^2x}{\cos^2x}}$ $= \int \frac{\sec^2x dx}{4 + 9\tan^2x} \quad \left \begin{array}{l} \text{Put } \tan x = t \\ \therefore \sec^2x dx = dt \end{array} \right.$ $= \int \frac{dt}{4 + 9t^2}$ $= \int \frac{dt}{9\left(\frac{4}{9} + t^2\right)}$ $= \frac{1}{9} \int \frac{dt}{\left(\frac{2}{3}\right)^2 + t^2}$ $= \frac{1}{9} \cdot \frac{1}{\frac{2}{3}} \tan^{-1}\left(\frac{t}{\frac{2}{3}}\right) + c$ $= \frac{1}{6} \tan^{-1}\left(\frac{3\tan x}{2}\right) + c$</p> <hr style="border-top: 1px dashed black;"/> <p>b) Evaluate $\int \sin(\log x) \cdot dx$</p> <p>Ans. $I = \int \sin(\log x) \cdot dx \quad \left \begin{array}{l} \text{Put } \log x = t \\ \therefore x = e^t \\ \therefore dx = e^t dt \end{array} \right.$ $= \int e^t \sin t \cdot dt$ $= \sin t \int e^t dt - \int \left(\int e^t dt\right) \frac{d}{dt}(\sin t) dt$ $= \sin t \cdot e^t - \int e^t \cdot \cos t dt$</p>	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	<p>4</p>



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3)		$= \sin t \cdot e^t - \left[\cos t \int e^t dt - \int \left(\int e^t dt \right) \frac{d}{dt} (\cos t) dt \right]$ $= \sin t \cdot e^t - \left[\cos t \cdot e^t - \int e^t \cdot (-\sin t) \cdot dt \right]$ $= \sin t \cdot e^t - \left[\cos t \cdot e^t + \int e^t \sin t dt \right]$ $= \sin t \cdot e^t - \left[\cos t \cdot e^t + I \right]$ $= \sin t \cdot e^t - \cos t \cdot e^t - I$ $\therefore I + I = \sin t \cdot e^t - \cos t \cdot e^t$ $\therefore 2I = e^t (\sin t - \cos t)$ $\therefore I = \frac{e^t}{2} [\sin t - \cos t] = \frac{e^{\log x}}{2} [\sin(\log x) - \cos(\log x)] + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4
	c)	<p>Evaluate $\int \frac{\log x}{x(2+\log x)(3+\log x)} dx$</p> <p>Ans. $\int \frac{\log x}{x(2+\log x)(3+\log x)} dx$ Put $\log x = t$ $\therefore \frac{1}{x} dx = dt$</p> $= \int \frac{t}{(2+t)(3+t)} dt$ $= \int \left[\frac{-2}{2+t} + \frac{3}{3+t} \right] dt$ $= -2 \log(2+t) + 3 \log(3+t) + c$ $= -2 \log(2+\log x) + 3 \log(3+\log x) + c$ <p>Note: Direct method of partial fraction is allowed.</p>	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p>	
	d)	<p>Evaluate $\int_0^1 x \tan^{-1} x dx$</p> <p>Ans. $\int_0^1 x \tan^{-1} x dx$</p> $= \left[\tan^{-1} x \int x dx - \int \left(\int x dx \right) \frac{d}{dx} (\tan^{-1} x) dx \right]_0^1$ $= \left[\frac{x^2}{2} \tan^{-1} x - \int \frac{x^2}{2} \cdot \frac{1}{x^2+1} \cdot dx \right]_0^1$	<p>1/2</p> <p>1</p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks						
3)		$= \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} \cdot dx \right]_0^1$ $= \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{(x^2+1)-1}{x^2+1} \cdot dx \right]_0^1$ $= \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{x^2+1} \right) \cdot dx \right]_0^1$ $= \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) \right]_0^1$ $= \left[\frac{1}{2} \tan^{-1} 1 - \frac{1}{2} (1 - \tan^{-1} 1) \right] - \left[0 - \frac{1}{2} (0 - \tan^{-1} 0) \right]$ $= \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{2} \left(1 - \frac{\pi}{4} \right)$ $= \frac{\pi}{4} - \frac{1}{2} \quad \text{or} \quad 0.285$ <p>Note: This is example is of “integration by parts”, which can be first solved without limits and then the limits can be applied.</p> <hr style="border-top: 1px dashed black;"/> <p>e) Evaluate $\int_0^{\pi} \frac{dx}{5+4 \cos x}$</p> <p>Ans. Put $\tan \frac{x}{2} = t$</p> $\therefore dx = \frac{2dt}{1+t^2} \quad \text{and} \quad \cos x = \frac{1-t^2}{1+t^2}$ <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>π</td> </tr> <tr> <td>t</td> <td>0</td> <td>∞</td> </tr> </table> $\therefore \int_0^{\pi} \frac{dx}{5+4 \cos x} = \int_0^{\infty} \frac{1}{5+4 \left(\frac{1-t^2}{1+t^2} \right)} \cdot \frac{2dt}{1+t^2}$ $= 2 \int_0^{\infty} \frac{1}{t^2+9} dt$ $= 2 \int_0^{\infty} \frac{1}{t^2+3^2} dt$	x	0	π	t	0	∞	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	4
x	0	π								
t	0	∞								



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$= 2 \times \frac{1}{3} \left[\tan^{-1} \left(\frac{t}{3} \right) \right]_0^{\infty}$ $= \frac{2}{3} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$ $= \frac{2}{3} \left[\frac{\pi}{2} \right]$ $= \frac{\pi}{3}$	1 1/2 1/2	4
	f)	Obtain the differential equation of $y = A \cos(\log x) + B \sin(\log x)$		
	Ans.	$y = A \cos(\log x) + B \sin(\log x)$ $\therefore \frac{dy}{dx} = -A \sin(\log x) \cdot \frac{1}{x} + B \cos(\log x) \cdot \frac{1}{x}$ $\therefore x \frac{dy}{dx} = -A \sin(\log x) + B \cos(\log x)$ $\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -A \cos(\log x) \cdot \frac{1}{x} - B \sin(\log x) \cdot \frac{1}{x}$ $\therefore x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -A \cos(\log x) - B \sin(\log x)$ $\therefore x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -[A \cos(\log x) + B \sin(\log x)]$ $= -y$ $\therefore x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$	1 1 1 1	4
4)		Attempt any FOUR of the followings:		
	a)	Evaluate $\int_0^{\pi/2} \frac{1}{1 + \tan x} dx$		
	Ans.	$I = \int_0^{\pi/2} \frac{1}{1 + \tan x} dx$ $= \int_0^{\pi/2} \frac{1}{1 + \frac{\sin x}{\cos x}} dx$		



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 5px 0;"> Replace $x \rightarrow \pi/2 - x$ $\therefore \sin x \rightarrow \cos x$ & $\cos x \rightarrow \sin x$ </div> $\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ <hr/> $\therefore 2I = \int_0^{\pi/2} \frac{\cos x + \sin x}{\cos x + \sin x} dx$ $\therefore 2I = \int_0^{\pi/2} 1 \cdot dx$ $\therefore 2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$ <p style="text-align: center;">OR</p> $I = \int_0^{\pi/2} \frac{1}{1 + \tan x} dx$ $= \int_0^{\pi/2} \frac{1}{1 + \frac{\sin x}{\cos x}} dx$ $I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$ $= \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx$ $\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ <hr/> $\therefore 2I = \int_0^{\pi/2} \frac{\cos x + \sin x}{\cos x + \sin x} dx$ $\therefore 2I = \int_0^{\pi/2} 1 \cdot dx$ $\therefore 2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$	<p>1</p> <p>1</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>1</p> <p>4</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>1</p> <p>4</p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	b)	Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$		
	Ans.	$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;"> Replace $x \rightarrow 7-x$ $\therefore x \rightarrow 7-x$ & $7-x \rightarrow x$ </div> $\therefore I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$ $\therefore 2I = \int_2^5 \frac{\sqrt{x} + \sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$ $\therefore 2I = \int_2^5 1 \cdot dx$ $\therefore 2I = [x]_2^5$ $\therefore 2I = 5 - 2 = 3$ $\therefore I = \frac{3}{2}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1	4
		OR		
		$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx$ $\therefore I = \int_2^5 \frac{\sqrt{5+2-x}}{\sqrt{7-(5+2-x)} + \sqrt{5+2-x}} dx$ $\therefore I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$ $\therefore 2I = \int_2^5 \frac{\sqrt{x} + \sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$ $\therefore 2I = \int_2^5 1 \cdot dx$ $\therefore 2I = [x]_2^5$ $\therefore 2I = 5 - 2 = 3$ $\therefore I = \frac{3}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	c)	Evaluate $\int_0^1 x^2 \sqrt{1-x} \cdot dx$		
	Ans.	$I = \int_0^1 x^2 \sqrt{1-x} \cdot dx$ $\therefore I = \int_0^1 (1-x)^2 \sqrt{1-(1-x)} \cdot dx$ $= \int_0^1 (1-2x+x^2) \sqrt{x} \cdot dx$ $= \int_0^1 (\sqrt{x} - 2x^{3/2} + x^{5/2}) \cdot dx$ $= \left[\frac{2}{3} x^{3/2} - 2 \frac{x^{5/2}}{5/2} + \frac{x^{7/2}}{7/2} \right]_0^1$ $= \left[\frac{2}{3} x^{3/2} - \frac{4}{5} x^{5/2} + \frac{2}{7} x^{7/2} \right]_0^1$ $= \left[\frac{2}{3} 1^{3/2} - \frac{4}{5} 1^{5/2} + \frac{2}{7} 1^{7/2} \right] - [0-0+0]$ $= \frac{16}{105} \quad \text{or} \quad 0.152$	<p>1/2</p> <p>1</p> <p>1</p> <p>1</p> <p>1/2</p>	4
d)	Prove that the area of circle $x^2 + y^2 = a^2$ is πa^2 sq. units.			
	Ans.	$x^2 + y^2 = a^2$ $\therefore y^2 = a^2 - x^2$ $\therefore y = \sqrt{a^2 - x^2}$ <p>At $y=0$, $a^2 - x^2 = 0$</p> $\therefore x = -a, a$ $\therefore A = 4 \int_a^b y dx$ $= 4 \int_0^a \sqrt{a^2 - x^2} dx$ $= 4 \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]_0^a$ $= 4 \left[0 + \frac{a^2}{2} \sin^{-1}(1) \right] - \left[0 + \frac{a^2}{2} \sin^{-1}(0) \right]$ $= 4 \left[\frac{a^2}{2} \cdot \frac{\pi}{2} \right]$ $= \pi a^2$	<p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		<p style="text-align: center;">OR</p> $\begin{aligned} \therefore A &= 2 \int_a^b y dx \\ &= 2 \int_{-a}^a \sqrt{a^2 - x^2} dx \\ &= 2 \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]_{-a}^a \\ &= 2 \left[0 + \frac{a^2}{2} \sin^{-1}(1) \right] - \left[0 + \frac{a^2}{2} \sin^{-1}(-1) \right] \\ &= 2 \left[\frac{a^2}{2} \cdot \frac{\pi}{2} + \frac{a^2}{2} \cdot \frac{\pi}{2} \right] \\ &= \pi a^2 \end{aligned}$ <hr style="border-top: 1px dashed black;"/>	<p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1/2</p>	4
	c)	<p>Find the area between the parabola $y = 4x - x^2$ and the x-axis.</p> <p>Ans. $y = 4x - x^2$ and x-axis i.e., $y = 0$</p> $\begin{aligned} \therefore 4x - x^2 &= 0 \\ \therefore x &= 0, 4 \\ \therefore A &= \int_0^4 y dx \\ &= \int_0^4 (4x - x^2) dx \\ &= \left[4 \cdot \frac{x^2}{2} - \frac{x^3}{3} \right]_0^4 \\ &= \left[2x^2 - \frac{x^3}{3} \right]_0^4 \\ &= \left[2 \cdot 4^2 - \frac{4^3}{3} \right] - [0 - 0] \\ &= \frac{32}{3} \text{ or } 10.667 \end{aligned}$ <hr style="border-top: 1px dashed black;"/>	<p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1/2</p> <p style="text-align: center;">1</p>	4
	f)	<p>Find the area bounded by $y^2 = 2x$ and $x - y = 4$</p> <p>Ans. $y^2 = 2x$ and $x - y = 4$</p> $\begin{aligned} \therefore (x - 4)^2 &= 2x \\ \therefore x^2 - 10x + 16 &= 0 \\ \therefore x &= 2, 8 \end{aligned}$	1	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\therefore A = \int_a^b (y_2 - y_1) dx$ $= \int_2^8 (x - 4 - \sqrt{2x}) dx$ $= \left[\frac{x^2}{2} - 4x - \sqrt{2} \cdot \frac{2}{3} x^{3/2} \right]_2^8$ $= \left[\frac{8^2}{2} - 4(8) - \sqrt{2} \cdot \frac{2}{3} \cdot 8^{3/2} \right] - \left[\frac{2^2}{2} - 4(2) - \sqrt{2} \cdot \frac{2}{3} \cdot 2^{3/2} \right]$ $= \frac{38}{3} \text{ or } 12.667$ <p style="text-align: center;">OR</p> $\therefore A = \int_a^b (y_1 - y_2) dx$ $= \int_2^8 (\sqrt{2x} - (x - 4)) dx$ $= \int_2^8 (\sqrt{2x} - x + 4) dx$ $= \left[\sqrt{2} \cdot \frac{2}{3} x^{3/2} - \frac{x^2}{2} + 4x \right]_2^8$ $= \left[\sqrt{2} \cdot \frac{2}{3} \cdot 8^{3/2} - \frac{8^2}{2} + 4(8) \right] - \left[\sqrt{2} \cdot \frac{2}{3} \cdot 2^{3/2} - \frac{2^2}{2} + 4(2) \right]$ $= -\frac{38}{3} \text{ or } -12.667$ $\therefore A = \frac{38}{3} \text{ or } 12.667 \quad (\because \text{area is always +ve})$	<p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>	4
5)		<p>Attempt any FOUR of the followings:</p> <p>a) Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$</p> <p>Ans. $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$</p> $\therefore \frac{dy}{dx} = e^{3x} \cdot e^{-2y} + x^2 e^{-2y}$ $= (e^{3x} + x^2) e^{-2y}$		



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$\therefore e^{2y} dy = (e^{3x} + x^2) dx$ $\therefore \int e^{2y} \cdot dy = \int (e^{3x} + x^2) \cdot dx$ $\therefore \frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$	1 1 1+1	4
	b)	Solve $\frac{dy}{dx} = \cos(x+y)$		
	Ans.	$\frac{dy}{dx} = \cos(x+y)$ Put $x+y = v$ $\therefore 1 + \frac{dy}{dx} = \frac{dv}{dx}$ $\therefore \frac{dv}{dx} - 1 = \cos v$ $\therefore \frac{dv}{dx} = 1 + \cos v$ $\therefore \frac{dv}{1 + \cos v} = dx$ $\therefore \int \frac{dv}{1 + \cos v} = \int dx$ $\therefore \int \frac{dv}{2 \cos^2\left(\frac{v}{2}\right)} = \int dx$ $\therefore \frac{1}{2} \int \sec^2\left(\frac{v}{2}\right) dv = \int dx$ $\therefore \frac{1}{2} \cdot \frac{\tan\left(\frac{v}{2}\right)}{\frac{1}{2}} = x + c$ $\therefore \tan\left(\frac{x+y}{2}\right) = x + c$	1 1/2 1/2 1/2 1 1/2	4
	c)	Solve $(x^3 + y^3) \frac{dy}{dx} = x^2 y$		
	Ans.	$\therefore \frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$		



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$\text{Put } y = vx \quad \therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\therefore v + x \frac{dv}{dx} = \frac{x^2 vx}{x^3 + (vx)^3} = \frac{v}{1+v^3}$ $\therefore x \frac{dv}{dx} = \frac{v}{1+v^3} - v$ $\therefore x \frac{dv}{dx} = -\frac{v^4}{1+v^3}$ $\therefore \frac{1+v^3}{v^4} dv = -\frac{1}{x} dx$ $\therefore \int \left(\frac{1}{v^4} + \frac{1}{v} \right) dv = -\int \frac{1}{x} dx$ $\therefore \frac{v^{-3}}{-3} + \log v = -\log x + c$ $\therefore \frac{1}{-3v^3} + \log v = -\log x + c$ $\therefore \frac{x^3}{-3y^3} + \log \left(\frac{y}{x} \right) = -\log x + c$	1 1/2 1/2 1/2 1 1/2	4
	d)	Solve $(4x^3y^2 + y \cos xy)dx + (2x^4y + x \cos xy)dy = 0$		
	Ans.	$(4x^3y^2 + y \cos xy)dx + (2x^4y + x \cos xy)dy = 0$ $M = 4x^3y^2 + y \cos xy$ $\therefore \frac{\partial M}{\partial y} = 8x^3y - y \sin xy \cdot x + \cos xy$ $N = 2x^4y + x \cos xy$ $\therefore \frac{\partial N}{\partial x} = 8x^3y - x \sin xy \cdot y + \cos xy$ <p>\therefore the equation is exact.</p> $\int_{y \text{ constant}} M dx + \int_{\text{terms free from } x} N dy = c$ $\int (4x^3y^2 + y \cos xy) dx + \int 0 dy = c$ $\therefore 4 \cdot \frac{x^4}{4} \cdot y^2 + y \cdot \frac{\sin xy}{y} = c$ <p>or $x^4y^2 + \sin xy = c$</p>	1 1/2 1/2 1 1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	e)	Solve $(1+x^2)\frac{dy}{dx} + y = e \tan^{-1} x$		
	Ans.	$(1+x^2)\frac{dy}{dx} + y = e \tan^{-1} x$ $\therefore \frac{dy}{dx} + \frac{1}{1+x^2} \cdot y = \frac{e \tan^{-1} x}{1+x^2}$ $\therefore P = \frac{1}{1+x^2} \quad \text{and} \quad Q = \frac{e \tan^{-1} x}{1+x^2}$ $\therefore IF = e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1} x}$ $\therefore y \cdot IF = \int Q \cdot IF \cdot dx + c$ $\therefore y \cdot e^{\tan^{-1} x} = \int \frac{e \tan^{-1} x}{1+x^2} \cdot e^{\tan^{-1} x} \cdot dx + c$ <p>Put $\tan^{-1} x = t \quad \therefore \frac{1}{1+x^2} \cdot dx = dt$</p> $\therefore y \cdot e^{\tan^{-1} x} = \int e t e^t \cdot dt + c$ $\therefore y \cdot e^{\tan^{-1} x} = e(te^t - e^t) + c$ $\therefore y \cdot e^{\tan^{-1} x} = e(\tan^{-1} x e^{\tan^{-1} x} - e^{\tan^{-1} x}) + c$ <hr/>	1 1 1/2 1 1/2	4
	f)	If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction. (Given $e^2 = 7.4$)		
	Ans.	$n = 2000, \quad p = 0.001$ $m = np = 2$ $\therefore p = \frac{e^{-m} m^r}{r!}$ $p(\text{more than } 2) = 1 - p(\text{maximum } 2)$ $= 1 - [p(0) + p(1) + p(2)]$ $= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} \right]$ $= 1 - [0.1353 + 0.2706 + 0.2706]$ $= 0.3235$ <hr/>	1 1 1 1	4



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		<p>Attempt any FOUR of the followings:</p> <p>a) If $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$, $P(B/A) = \frac{2}{3}$, find $P(A/B)$ and $P(A \cup B)$.</p> <p>Ans. The values of the data provided in this example are incorrect in accordance with the theory of probability. Theoretically speaking $A \cap B \subseteq B$. Therefore, $P(A \cap B) \leq P(B)$ Further note that, $P(A \cap B) = P(A) \cdot P(B/A)$ Consequently, $P(A) \cdot P(B/A) \leq P(B)$ Thus the given values must satisfy this relation. But here in this example $P(A) \cdot P(B/A) = \frac{3}{5} \cdot \frac{2}{3} = \frac{2}{5}$ and $P(B) = \frac{1}{5}$ $\therefore P(A) \cdot P(B/A) \not\leq P(B)$</p> <p>-----</p> <p>b) If two dice are rolled simultaneously, find the probability that the total is 6 or 10.</p> <p>Ans. $n = n(S) = 6^2 = 36$ $A = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (4, 6), (5, 5), (6, 4)\}$ $m = n(A) = 8$ $\therefore p = \frac{m}{n} = \frac{8}{36}$ $= \frac{2}{9}$ or 0.222</p> <p>-----</p> <p>c) If 2% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs i) 3 are defective ii) at least two are defective.</p> <p>Ans. $p = 2\% = 0.02$, $n = 100$ $m = np = 2$ $\therefore p = \frac{e^{-m} m^r}{r!}$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>4</p>	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$i) p(3) = \frac{e^{-2} 2^3}{3!} = 0.1804 \quad (\text{or also } 0.180)$ $ii) p(\text{at least } 2) = 1 - p(\text{maximum } 1)$ $= 1 - [p(0) + p(1)]$ $= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} \right]$ $= 1 - [0.1353 + 0.2707]$ $= 0.2706 \quad (\text{or also } 0.271)$ <p>Note: Please refer note stated in Q. 1 (l).</p>	1 1 1 1	4
	d)	<p>The probability that a man aged 65 will live to 75 is 0.65. What is the probability that out of 10 men which are now 65, 7 will live to 75?</p> <p>Ans. $p = 0.65$ $\therefore q = 1 - p = 0.35$ $\therefore p(3) = {}^n C_r p^r q^{n-r}$ $= {}^{10} C_7 (0.65)^7 (0.35)^{10-7}$ $= 0.252$</p>	1 2 1	4
	e)	<p>A problem is given to three students A, B, C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the change (chance) that the problem is solved?</p> <p>Ans. $P(A) = \frac{1}{2} \quad \therefore P(A') = 1 - P(A) = \frac{1}{2}$ $P(B) = \frac{3}{4} \quad \therefore P(B') = 1 - P(B) = \frac{1}{4}$ $P(C) = \frac{1}{4} \quad \therefore P(C') = 1 - P(C) = \frac{3}{4}$ $\therefore p = p(\text{the problem is solved})$ $= 1 - p(\text{the problem is not solved by all A, B, C})$ $= 1 - p(A' \cap B' \cap C')$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	



Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$= 1 - \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}$ $= \frac{29}{32} \quad \text{or} \quad 0.906$	1 ½	4
	f)	<p>A metal wire 36 cm long is bent to form a rectangle. Find its dimensions, when its area is maximum.</p>		
	Ans.	<p>Let x and y be the sides of rectangle. $\therefore 2x + 2y = 36$ or $x + y = 18$ $\therefore y = 18 - x$ <i>But area</i> $A = xy = x(18 - x) = 18x - x^2$ $\therefore \frac{dA}{dx} = 18 - 2x$ $\therefore \frac{d^2p}{dx^2} = -2$ <i>For stationary values,</i> $\frac{dp}{dx} = 0$ $\therefore 18 - 2x = 0$ $\therefore x = 9$ <i>At</i> $x = 9$, $\frac{d^2p}{dx^2} = -2 < 0$ \therefore <i>At</i> $x = 9$, <i>A has maximum value</i> <i>and the other side is</i> $y = 18 - x = 9$</p>	½ 1 ½ ½ ½ ½	4
		<p style="text-align: center;">Important Note</p> <p>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, FIRST SEE whether the method falls within the scope of the curriculum, and THEN ONLY give appropriate marks in accordance with the scheme of marking.</p>		