Model Answers

Important Instructions to examiners:

- The model answer shall be the complete solution for each and every question on the question paper.
- Numerical shall be completely solved in a step by step manner along with step marking.
- All alternative solutions shall be offered by the expert along with self-explanatory comments from the expert.
- In case of theoretical answers, the expert has to write the most acceptable answer and offer comments regarding marking scheme to the assessors.
- In should offer the most convincing figures / sketches / circuit diagrams / block diagrams / flow diagrams and offer comments for step marking to the assessors.
- In case of any missing data, the expert shall offer possible assumptions / options and the ensuing solutions along with comments to the assessors for effective assessment.
- In case of questions which are out of the scope of curricular requirement, the expert examiner shall solve the question and mention the marking scheme in the model answer. However, the experts are requested to submit their clear cut opinion about the scope of such question in the paper separately to the coordinator.
- Experts shall cross check the DTP of the final draft of the model answer prepared by them.

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		**Attempt any FIVE of the following:**		20
	a)	At what point on the curve $y=e^x$, slope is 1?		
	Ans	$y=e^x$	1	
		$\therefore \dfrac{dy}{dx}=e^x$	1	
		$\therefore e^x=1$		
		$\therefore x=0$	1	
		$\therefore y=e^0=1$	1	
		$\therefore$ Point is $(0,1)$		4
	b)	Find the radius of curvature of $y=e^x$ at $(0,1)$		
	Ans	$y=e^x$	½	
		$\therefore \dfrac{dy}{dx}=e^x$	½	
		$\therefore \dfrac{d^2y}{dx^2}=e^x$		
		at $(0,1)$	½	
		$\dfrac{dy}{dx}=e^0=1$		
		$\dfrac{d^2y}{dx^2}=e^0=1$	½	
		Radius of curvature $=\dfrac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{3/2}}{\dfrac{d^2y}{dx^2}}$		
		$=\dfrac{\left(1+(1)^2\right)^{3/2}}{1}$	1	
		$=2.828$	1	4
	c)	**Evaluate:** $\int \dfrac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}dx$		
	Ans	$\int \dfrac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}dx$		
		Put $\sqrt{x}=t$	½	
		$\therefore \dfrac{1}{2\sqrt{x}}dx=dt$		
		$\therefore \dfrac{1}{\sqrt{x}}dx=2dt$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		$= \int \sin t \; 2dt$	1	
		$= -2\cos t + c$	1	4
		$= -2\cos\sqrt{x} + c$	½	
		--		
	d)	Integrate w.r.t. x $\dfrac{\sin x}{\cos^2 x}$		
	Ans	$\int \dfrac{\sin x}{\cos^2 x}\,dx$		
		$= \int \dfrac{\sin x}{\cos x \cos x} dx$		
		$= \int \tan x \sec x\, dx$	2	4
		$= \sec x + c$	2	
		OR		
		$\int \dfrac{\sin x}{\cos^2 x}\,dx$		
		put $\cos x = t$		
		$\therefore -\sin x\,dx = dt$		
		$\therefore \sin x\,dx = -dt$	1	
		$\int \dfrac{-dt}{t^2}$	1	
		$= -\dfrac{t^{-1}}{-1} + c$	1	
		$= \dfrac{1}{t} + c$	½	
		$= \dfrac{1}{\cos x} + c$	½	
		$= \sec x + c$		4
		--		
	e)	Evaluate: $\int x e^x dx$		
	Ans	$\int x e^x dx$		
		$= x\int e^x dx - \int\left[\int e^x dx \dfrac{d}{dx} x\right]dx$	1	
		$= xe^x - \int 1.e^x dx$	1+1	
		$= xe^x - e^x + c$	1	4
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	f)	Evaluate: $\int \frac{1}{x(x+1)}dx$		
	Ans	$\int \frac{1}{x(x+1)}dx$		
		consider $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$		
		$\therefore A = 1$	1	
		$B = -1$	1	
		$\therefore \int \frac{1}{x(x+1)}dx = \int \left(\frac{1}{x} + \frac{-1}{x+1}\right)dx$		
		$= \log x - \log(x+1) + c$	1+1	4
		--		
	g)	Evaluate: $\int_0^1 \frac{1}{\sqrt{1-x^2}}dx$		
	Ans	$\int_0^1 \frac{1}{\sqrt{1-x^2}}dx$	2	
		$= \left[\sin^{-1} x\right]_0^1$	1	
		$= \left[\sin^{-1} 1\right] - \left[\sin^{-1} 0\right]$		
		$= \frac{\pi}{2}$	1	4
		--		
	h)	Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$ with X-axis		
	Ans	$A = \int_0^3 x^2 dx$	1	
		$= \left[\frac{x^3}{3}\right]_0^3$	1	
		$= \left[\frac{3^3}{3}\right] - \left[\frac{0^3}{3}\right]$	1	
		$= 9$ Sq.units	1	4
		--		
	i)	Find order and degree of the following differential equation.		
	Ans	Order = 2		
		$\frac{d^2y}{dx^2} = \sqrt{y + \left(\frac{dy}{dx}\right)^2} \quad \therefore \left(\frac{d^2y}{dx^2}\right)^2 = y + \left(\frac{dy}{dx}\right)^2$	2	
		Degree = 2	2	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	j)	Form the differential equation of the curve $y = ax^2$		
	Ans	$y = ax^2$		
		$\frac{dy}{dx} = 2ax$	1½	
		$\frac{dy}{dx} = 2\frac{y}{x^2}x$	1½	
		$x\frac{dy}{dx} - 2y = 0$	1	4
	k)	Three cards are drawn from well shuffled pack of cards .Find the probability that all of them are king.		
	Ans	$n(S) = {}^{52}C_3$	1	
		$n(A) = {}^{4}C_3$	1	
		$P(A) = \frac{n(A)}{n(S)} = \frac{4}{22100} = 0.00018$	2	4
	l)	Two coins are tossed simultaneously, find the probability of getting atleast one head.		
	Ans	S=$\{HH, HT, TH, TT\}$		
		n (S)=4	1	
		A=$\{HH, HT, TH\}$		
		n (A)=3	1	
		P(A)=$\frac{n(A)}{n(S)} = \frac{3}{4} = 0.75$	2	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		**Attempt any <u>FOUR</u> of the following:**		16
	a)	Find the equation of tangent and normal to the curve $2x^2-xy+3y^2=18$ at $(3,1)$		
	Ans	$2x^2-xy+3y^2=18$		
		$4x-\left(x\frac{dy}{dx}+y\right)+6y\frac{dy}{dx}=0$	½	
		$4x-x\frac{dy}{dx}-y+6y\frac{dy}{dx}=0$		
		$(6y-x)\frac{dy}{dx}=y-4x$		
		$\therefore \frac{dy}{dx}=\frac{y-4x}{6y-x}$	½	
		at point $(3,1)$		
		slope of tangent $=\frac{-11}{3}$	½	
		slope of normal $=\frac{3}{11}$	½	
		Equation of tangent at $(3,1)$ is		
		$y-1=\frac{-11}{3}(x-3)$	½	
		$\therefore 11x+3y-36=0$	½	
		Equation of normal at $(3,1)$ is	½	
		$y-1=\frac{3}{11}(x-3)$	½	4
		$\therefore 3x-11y+2=0$		
		--		
	b)	Show that the radius of curvature at any point on the curve $y=a\log\left(\sec\frac{x}{a}\right)$ where a is constant is $a\sec\left(\frac{x}{a}\right)$		
	Ans	$y=a\log\left(\sec\frac{x}{a}\right)$		
		$\therefore \frac{dy}{dx}=a.\frac{1}{\sec\frac{x}{a}}.\frac{d}{dx}\sec\frac{x}{a}$	1	
		$=a.\frac{1}{\sec\frac{x}{a}}.\sec\frac{x}{a}\ \tan\frac{x}{a}.\frac{1}{a}$		
		$=\tan\frac{x}{a}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$\frac{d^2y}{dx^2} = \sec^2\left(\frac{x}{a}\right)\frac{1}{a}$	1	
		$\text{Radius of curvature } \rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		
		$= \frac{\left[1+\tan^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\frac{1}{a}}$	½	
		$= \frac{\left[\sec^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\frac{1}{a}}$	½	
		$= a\sec\left(\frac{x}{a}\right)$	½	4
		--		
	c)	Find the maximum and minimum value of $x^3 - 9x^2 + 24x$		
	Ans	$\text{Let } y = x^3 - 9x^2 + 24x$		
		$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24$	½	
		$\therefore \frac{d^2y}{dx^2} = 6x - 18$	½	
		$\text{Consider } \frac{dy}{dx} = 0$		
		$3x^2 - 18x + 24 = 0$		
		$\therefore x = 2 \text{ or } x = 4$	1	
		$\text{at } x = 2$		
		$\frac{d^2y}{dx^2} = 6(2) - 18 = -6 < 0$	½	
		$\therefore y \text{ is maximum at } x = 2$		
		$y_{max} = 2^3 - 9(2)^2 + 24(2)$	½	
		$= 20$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)		$at \quad x = 4$		
		$\frac{d^2y}{dx^2} = 6(4) - 18 = 6 > 0$	½	
		$\therefore\ y$ is minimum at $x = 4$		
		$y_{min} = 4^3 - 9(4)^2 + 24(4)$		
		$= 16$	½	4
		-----------------------------------		
	d)	Evaluate: $\int \cos^{-1} x dx$		
	Ans	$I = \int \cos^{-1} x.1dx$		
		$= \cos^{-1} x \int 1dx - \int \left(\int 1dx \frac{d}{dx} \cos^{-1} x \right) dx$	1	
		$= \left(\cos^{-1} x\right) x - \int \frac{-1}{\sqrt{1-x^2}} x dx$	1	
		$= x\cos^{-1} x + \int \frac{x}{\sqrt{1-x^2}} dx$		
		$= x\cos^{-1} x + \frac{1}{-2} \int \frac{-2x}{\sqrt{1-x^2}} dx$	1	
		$= x\cos^{-1} x - \frac{1}{2}\left(2\sqrt{1-x^2}\right) + c$	1	
		$= x\cos^{-1} x - \sqrt{1-x^2} + c$		4
		-----------------------------------		
	e)	Evaluate: $\int \frac{\left(\tan^{-1} x\right)^3}{1+x^2} dx$		
	Ans			
		$I = \int \frac{\left(\tan^{-1} x\right)^3}{1+x^2} dx$		
		Put $\tan^{-1} x = t$	½	
		$\frac{1}{1+x^2} dx = dt$	½	
		$\therefore\ I = \int t^3 dt$	1	
		$= \frac{t^4}{4} + c$	1	
		$= \frac{\left(\tan^{-1} x\right)^4}{4} + c$	1	4
		-----------------------------------		
	f)	Evaluate: $\int \frac{e^x}{\left(e^x - 1\right)\left(e^x + 1\right)} dx$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
2)	Ans	Put $e^x = t$ $e^x dx = dt$	1	
		$I = \int \frac{dt}{(t-1)(t+1)}$ $= \int \frac{dt}{t^2-1}$	1	
		$= \frac{1}{2}\log\left(\frac{t-1}{t+1}\right)+c$	1½	
		$= \frac{1}{2}\log\left(\frac{e^x-1}{e^x+1}\right)+c$	½	4
		OR		
		Put $e^x = t$ $e^x dx = dt$	1	
		Let $\frac{1}{(t-1)(t+1)} = \frac{A}{t-1}+\frac{B}{t+1}$ $1 = A(t+1)+B(t-1)$ Put $t=-1$ $1 = B(-2)$ $B = -\frac{1}{2}$	½	
		Put $t=1$ $1 = A(2)$ $A = \frac{1}{2}$	½	
		$\frac{1}{(t-1)(t+1)} = \frac{\frac{1}{2}}{t-1}+\frac{-\frac{1}{2}}{t+1}$ $\int \frac{dt}{(t-1)(t+1)} = \int \left(\frac{\frac{1}{2}}{t-1}+\frac{-\frac{1}{2}}{t+1}\right)dt$	½	
		$= \frac{1}{2}\log(t-1)-\frac{1}{2}\log(t+1)+c$ $= \frac{1}{2}\log\left(\frac{t-1}{t+1}\right)+c$	1	
		$= \frac{1}{2}\log\left(\frac{e^x-1}{e^x+1}\right)+c$	½	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		**Attempt any <u>FOUR</u> of the following:**		16
	a)	Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{4-\sin^2 x}dx$		
	Ans	Put $\sin x = t$		
		$\cos x dx = dt$ $\quad \boxed{\begin{array}{l}\text{when } x \to 0 \text{ to } \frac{\pi}{2} \\ t \to 0 \text{ to } 1\end{array}}$	1+1	
		$\therefore I = \int_0^1 \frac{1}{4-t^2}dt$		
		$I = \int_0^1 \frac{1}{(2)^2-t^2}dt$		
		$I = \left[\frac{1}{2(2)}\log\left\lvert\frac{2+t}{2-t}\right\rvert\right]_0^1$	1	
		$I = \frac{1}{4}\left[\log\left\lvert\frac{3}{1}\right\rvert - \log\left\lvert\frac{2}{2}\right\rvert\right]$	1	
		$I = \frac{1}{4}[\log 3 - \log 1]$		
		$I = \frac{1}{4}\log 3$		4
		--		
	b)	Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$		
	Ans	$I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$		
		$I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$	½	
		$I = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)dx$		
		$I = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$I=\int_0^{\frac{\pi}{4}}\log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right)dx$		
		$I=\int_0^{\frac{\pi}{4}}\log\left(\frac{2}{1+\tan x}\right)dx$	½	
		$I=\int_0^{\frac{\pi}{4}}\left[\log 2-\log\left(1+\tan x\right)\right]dx$	½	
		$I=\log 2\int_0^{\frac{\pi}{4}}dx-\int_0^{\frac{\pi}{4}}\log\left(1+\tan x\right)dx$		
		$I=\log 2\left[x\right]_0^{\frac{\pi}{4}}-I$	½	
		$2I=\log 2\left[\frac{\pi}{4}-0\right]$	½	
		$I=\frac{\pi}{8}\log 2$	½	4
		--		
	c)	Find the area of an ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ by integration		
	Ans	$\frac{x^2}{16}+\frac{y^2}{9}=1$		
		$\therefore\ y^2=\frac{9}{16}\left(16-x^2\right)$		
		$\therefore\ y=\frac{3}{4}\sqrt{16-x^2}$		
		area, $A=4\int_a^b ydx$	1	
		$A=4\left[\frac{3}{4}\int_0^4\sqrt{\left(4\right)^2-x^2}dx\right]$		
		$A=3\left[\frac{x}{2}\sqrt{\left(4\right)^2-x^2}+\frac{\left(4\right)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$	1	
		$A=3\left[8\sin^{-1}\left(1\right)-0\right]$	1	
		$A=24\frac{\pi}{2}$		
		$A=12\pi$	1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		OR $\frac{x^2}{16}+\frac{y^2}{9}=1$ $\therefore\ y^2=\frac{9}{16}\left(16-x^2\right)$ $\therefore\ y=\frac{3}{4}\sqrt{16-x^2}$		
		area, $A=\int_a^b y\,dx$	½	
		$A=\left[\frac{3}{4}\int_0^4\sqrt{(4)^2-x^2}\,dx\right]$		
		$A=\frac{3}{4}\left[\frac{x}{2}\sqrt{(4)^2-x^2}+\frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$	1	
		$A=\frac{3}{4}\left[8\sin^{-1}(1)-0\right]$	1	
		$A=\frac{3}{4}\left[8\frac{\pi}{2}\right]$ $A=3\pi$	1	
		$\therefore$ area of ellipse is $=4\times A$ $=4\times 3\pi$		
		$=12\pi$	½	4
		--		
	d)	Solve $\frac{dy}{dx}=\cos(x+y)$		
	Ans	Put $x+y=v$ $1+\frac{dy}{dx}=\frac{dv}{dx}$	1	
		$\frac{dy}{dx}=\frac{dv}{dx}-1$ $\therefore\ \frac{dv}{dx}-1=\cos v$		
		$\frac{dv}{dx}=1+\cos v$	½	
		$\frac{1}{1+\cos v}dv=dx$		
		$\int\frac{1}{1+\cos v}dv=\int dx$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$\int \frac{1}{2\cos^2\left(\frac{v}{2}\right)} dv = x + c$	½	
		$\frac{1}{2}\int \sec^2\left(\frac{v}{2}\right) dv = x + c$		
		$\frac{1}{2}\frac{\tan\left(\frac{v}{2}\right)}{\frac{1}{2}} = x + c$	1	
		$\tan\left(\frac{v}{2}\right) = x + c$		
		$\tan\left(\frac{x+y}{2}\right) = x + c$	½	4
		OR		
		$\text{Solve } \frac{dy}{dx} = \cos(x+y)$		
		$\text{Put } x + y = v$		
		$1 + \frac{dy}{dx} = \frac{dv}{dx}$	1	
		$\frac{dy}{dx} = \frac{dv}{dx} - 1$		
		$\therefore \frac{dv}{dx} - 1 = \cos v$		
		$\frac{dv}{dx} = 1 + \cos v$	½	
		$\frac{1}{1+\cos v} dv = dx$		
		$\int \frac{1}{1+\cos v} dv = \int dx$	½	
		$\text{Put } \tan\frac{v}{2} = t$		
		$dv = \frac{2dt}{1+t^2}$		
		$\cos v = \frac{1-t^2}{1+t^2}$	½	
		$\therefore \int \frac{1}{1+\frac{1-t^2}{1+t^2}} \frac{2dt}{1+t^2} = x + c$	½	
		$2\int \frac{1}{1+t^2+1-t^2} dt = x + c$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$2\int \frac{1}{2} dt = x + c$	½	
		$t = x + c$		
		$\tan\left(\frac{v}{2}\right) = x + c$	½	
		$\tan\left(\frac{x+y}{2}\right) = x + c$		4
		OR		
		Solve $\frac{dy}{dx} = \cos(x+y)$		
		Put $x + y = v$		
		$1 + \frac{dy}{dx} = \frac{dv}{dx}$		
		$\frac{dy}{dx} = \frac{dv}{dx} - 1$	1	
		$\therefore \frac{dv}{dx} - 1 = \cos v$		
		$\frac{dv}{dx} = 1 + \cos v$	½	
		$\frac{1}{1+\cos v} dv = dx$		
		$\int \frac{1}{1+\cos v} dv = \int dx$	½	
		$\int \frac{1-\cos v}{1-\cos^2 v} dv = \int dx$	½	
		$\int \frac{1-\cos v}{\sin^2 v} dv = x + c$		
		$\int \left(\frac{1}{\sin^2 v} - \frac{\cos v}{\sin^2 v}\right) dv = x + c$		
		$\int \left(\cos ec^2 v - \cot v \cos ec v\right) dv = x + c$	1	
		$-\cot v + \cos ec v = x + c$		
		$-\cot(x+y) + \cos ec(x+y) = x + c$	½	4
		--		
	e)	Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$		
	Ans	Put $y = vx$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$\frac{dy}{dx} = v + x\frac{dv}{dx}$	1	
		$\therefore v + x\frac{dv}{dx} = \frac{x^2 + (vx)^2}{x(vx)}$		
		$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{vx^2}$		
		$v + x\frac{dv}{dx} = \frac{1 + v^2}{v}$	½	
		$x\frac{dv}{dx} = \frac{1 + v^2}{v} - v$	½	
		$x\frac{dv}{dx} = \frac{1 + v^2 - v^2}{v}$		
		$x\frac{dv}{dx} = \frac{1}{v}$	½	
		$vdv = \frac{1}{x}dx$		
		$\int vdv = \int \frac{1}{x}dx$	½	
		$\frac{v^2}{2} = \log x + c$	½	
		$\frac{y^2}{2x^2} = \log x + c$	½	4
	f)	Solve $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$		
	Ans	$\frac{dy}{dx} - \frac{1}{x+1}y = e^x(x+1)$		
		$\therefore P = -\frac{1}{x+1}$ and $Q = e^x(x+1)$	1	
		$IF = e^{-\int \frac{1}{x+1}dx} = e^{-\log(x+1)} = \frac{1}{x+1}$	1	
		$\therefore yIF = \int QIF\,dx + c$		
		$y\frac{1}{x+1} = \int e^x(x+1)\frac{1}{x+1}dx + c$	1	
		$\frac{y}{x+1} = \int e^x dx + c$		
		$\frac{y}{x+1} = e^x + c$	1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Attempt any <u>FOUR</u> of the following:**		16
	a)	Evaluate $\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}dx$		
	Ans	$I = \int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}dx$ -------------------- (1)		
		$I = \int_1^5 \frac{\sqrt{9-(1+5-x)}}{\sqrt{9-(1+5-x)}+\sqrt{(1+5-x)+3}}dx$		
		$I = \int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3}+\sqrt{9-x}}dx$ -------------------- (2)	**1**	
		add (1) and (2)		
		$I + I = \int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}dx + \int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3}+\sqrt{9-x}}dx$		
		$2I = \int_1^5 \frac{\sqrt{9-x}+\sqrt{x+3}}{\sqrt{9-x}+\sqrt{x+3}}dx$	**1**	
		$2I = \int_1^5 1dx$	½	
		$2I = [x]_1^5$	½	
		$2I = 5-1$	½	4
		$I = 2$	½	
		--		
	b)	Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\cos x}$		
	Ans	Put $\tan\frac{x}{2} = t$		
		$\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2dt}{1+t^2}$ $\boxed{\begin{array}{l}\text{when } x \to 0 \text{ to } \frac{\pi}{2} \\ t \to 0 \text{ to } 1\end{array}}$	**1**	
		$\therefore I = \int_0^1 \frac{1}{4+5\left(\frac{1-t^2}{1+t^2}\right)}\frac{2dt}{1+t^2}$	½	
		$I = 2\int_0^1 \frac{1}{4(1+t^2)+5(1-t^2)}dt$		
		$I = 2\int_0^1 \frac{1}{4+4t^2+5-5t^2}dt$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$I = 2\int_0^1 \frac{1}{9-t^2}dt$	½	
		$I = 2\int_0^1 \frac{1}{(3)^2-t^2}dt$		
		$I = 2\left[\frac{1}{2(3)}\log\left\|\frac{3+t}{3-t}\right\|\right]_0^1$	**1**	
		$I = \frac{1}{3}\left[\log\left\|\frac{4}{2}\right\| - \log\left\|\frac{3}{3}\right\|\right]$	½	
		$I = \frac{1}{3}\left[\log\|2\| - \log\|1\|\right]$		
		$I = \frac{1}{3}\log\|2\|$	½	4
		---		
	c)	Find the area between the parabola $y^2 = 4x$ and the line $y = 2x+3$		
	Ans	As in the given problem Curves are Not intersecting thus finding the area between the given two curves is not possible. So while assessing this question Assessor NOT TO REWARD MARKS for this Question in either case whether candidate attempted the question or not.	Do Not Assess	**Do Not Assess**

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	d)	Solve $\frac{dy}{dx} = e^{2x+y} + x^2 e^y$		
	Ans	$\frac{dy}{dx} = \left(e^{2x} + x^2\right)e^y$		
		$e^{-y}dy = \left(e^{2x} + x^2\right)dx$	1	
		$\int e^{-y}dy = \int\left(e^{2x} + x^2\right)dx$	1	
		$\frac{e^{-y}}{-1} = \frac{e^{2x}}{2} + \frac{x^3}{3} + c$	2	4
	e)	Solve $\left(2x + 3\cos y\right)dx + \left(2y - 3x\sin y\right)dy = 0$		
	Ans	$M = 2x + 3\cos y$, $N = 2y - 3x\sin y$		
		$\frac{\partial M}{\partial y} = -3\sin y$,	1	
		$\frac{\partial N}{\partial x} = -3\sin y$	1	
		$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$		
		$\therefore$ equation is an exact D.E.		
		$\int\limits_{y-\text{constant}} M\,dx + \int\limits_{\substack{\text{terms free}\\ \text{from } x}} N\,dy = c$		
		$\therefore \int\limits_{y-\text{constant}} \left(2x + 3\cos y\right)dx + \int 2y\,dy = c$	1	
		$x^2 + 3x\cos y + y^2 = c$	1	4
	f)	Show that $y = A\sin mx + B\cos mx$ is a solution of differential equation $\frac{d^2y}{dx^2} + m^2y = 0$		
	Ans	$y = A\sin mx + B\cos mx$		
		$\frac{dy}{dx} = mA\cos mx - mB\sin mx$	1	
		$\frac{d^2y}{dx^2} = -m^2A\sin mx - m^2B\cos mx$	1	
		$\frac{d^2y}{dx^2} = -m^2\left(A\sin mx + B\cos mx\right)$	1	
		$\frac{d^2y}{dx^2} = -m^2y$		
		$\frac{d^2y}{dx^2} + m^2y = 0$	1	4
		OR		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$y = A\sin mx + B\cos mx$		
		$\frac{dy}{dx} = mA\cos mx - mB\sin mx$	**1**	
		$\frac{d^2y}{dx^2} = -m^2A\sin mx - m^2B\cos mx$	**1**	
		$\text{L.H.S.} = \frac{d^2y}{dx^2} + m^2y$		
		$= -m^2A\sin mx - m^2B\cos mx + m^2\left(A\sin mx + B\cos mx\right)$	**1**	
		$= -m^2A\sin mx - m^2B\cos mx + m^2A\sin mx + m^2B\cos mx$		
		$= 0 = R.H.S.$	**1**	4
		--		
5)		**Attempt any <u>FOUR</u> of the following:**		16
	a)	A problem is given to three students X,Y,Z whose chances of solving are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ respectively. Find the probability that:		
		i) The probem is solved by each of them.		
		ii) The problem is not solved by any of them.		
	Ans	$P(X) = \frac{1}{2} \qquad \therefore P(X') = 1 - \frac{1}{2} = \frac{1}{2}$		
		$P(Y) = \frac{1}{3} \qquad \therefore P(Y') = 1 - \frac{1}{3} = \frac{2}{3}$		
		$P(Z) = \frac{1}{4} \qquad \therefore P(Z') = 1 - \frac{1}{4} = \frac{3}{4}$	**1**	
		i)Problem is solved by each of them is:		
		$=P(X \cap Y \cap Z)$		
		$= P(X) \times P(Y) \times P(Z)$		
		$= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$		
		$= \frac{1}{24} \quad or \quad 0.0417$	**1½**	
		ii)Problem is not solved by any of them is:		
		$P(X' \cap Y' \cap Z') = P(X') \times P(Y') \times P(Z')$		
		$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$		
		$= \frac{1}{4} \quad or \quad 0.25$	**1½**	
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	b)	If 30% of the bulbs produced are defective , find the probability that out of 4 bulbs selected:		
		i) One is defective		
		ii) At the most two are defective		
	Ans	$p = 30\% = 0.3$		
		$q = 1 - p = 0.7$		
		$n = 4$	1	
		Binomial Distribution is:		
		$p(r) = {}^nC_r p^r q^{n-r}$		
		i) One is defective, $r = 1$		
		$p(1) = {}^4C_1 (0.3)^1 (0.7)^{4-1}$		
		$p(1) = 0.4116$	1½	
		ii) At the most two are defective		
		$= p(0) + p(1) + p(2)$		
		$= {}^4C_0 (0.3)^0 (0.7)^{4-0} + 0.4116 + {}^4C_2 (0.3)^2 (0.7)^{4-2}$		
		$= 0.2401 + 0.4116 + 0.2646$	1½	4
		$= 0.9163$		
		--		
	c)	Using Poisson distribution , find the probability that the ace of spade will be drawn from a pack of well shuffled cards at least once in 104 consecutive trials.		
	Ans	$Given \quad n = 104$		
		$p = \frac{1}{52}$	1	
		$m = np$		
		$= 104 \times \frac{1}{52} = 2$	1	
		r = atleast one		
		$= 1, 2, 3 \ldots$		
		$\therefore \ p(r) = \frac{e^{-m} m^r}{r!}$		
		$p(r) = 1 - p(0)$		
		$= 1 - \frac{e^{-2} 2^0}{0!}$	1	
		$= 0.8646$	1	4
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	d)	Evaluate $\int \frac{dx}{2+3\cos x}$		
	Ans	Put $\tan\frac{x}{2}=t$		
		$\cos x=\frac{1-t^2}{1+t^2}\quad , dx=\frac{2dt}{1+t^2}$	1	
		$\therefore I=\int\frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)}\frac{2dt}{1+t^2}$	½	
		$I=2\int\frac{1}{2\left(1+t^2\right)+3\left(1-t^2\right)}dt$		
		$I=2\int\frac{1}{2+2t^2+3-3t^2}dt$		
		$I=2\int\frac{1}{5-t^2}dt$	½	
		$I=2\int\frac{1}{\left(\sqrt{5}\right)^2-\left(t\right)^2}dt$	½	
		$I=\frac{2}{2\sqrt{5}}\log\left\|\frac{\sqrt{5}+t}{\sqrt{5}-t}\right\|+c$	1	
		$I=\frac{1}{\sqrt{5}}\log\left\|\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}\right\|+c$	½	4
		--		
	e)	Evaluate $\int_0^1 x\tan^{-1}x\,dx$		
	Ans	$=\left[\tan^{-1}x\int x\,dx-\int\left(\int x\,dx\frac{d}{dx}\tan^{-1}x\right)dx\right]_0^1$	½	
		$=\left[\tan^{-1}x\ \frac{x^2}{2}-\int\frac{x^2}{2}\frac{1}{1+x^2}dx\right]_0^1$	1	
		$=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx\right]_0^1$		
		$=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx\right]_0^1$	½	
		$=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)\right]_0^1$	1	
		$=\left[\frac{1}{2}\tan^{-1}(1)-\frac{1}{2}\left(1-\tan^{-1}1\right)\right]-0$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$= \frac{1}{2}\frac{\pi}{4} - \frac{1}{2} + \frac{1}{2}\frac{\pi}{4}$		
		$= \frac{\pi}{4} - \frac{1}{2} \quad \text{or} \quad \frac{1}{2}\left(\frac{\pi}{2} - 1\right)$	½	4
	f)	Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$		
	Ans	Put $\frac{y}{x} = v \quad \therefore \; y = vx$		
		$\frac{dy}{dx} = v + x\frac{dv}{dx}$	1	
		$\therefore \; v + x\frac{dv}{dx} = v + \sin v$	½	
		$x\frac{dv}{dx} = \sin v$	½	
		$\frac{1}{\sin v}dv = \frac{1}{x}dx$		
		$\int \operatorname{cosec} v\,dv = \int \frac{1}{x}dx$	½	
		$\log\left\lvert \operatorname{cosec} v - \cot v \right\rvert = \log\lvert x\rvert + c$	½+½	
		$\log\left\lvert \operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x} \right\rvert = \log\lvert x\rvert + c$	½	4
6)		**Attempt any <u>FOUR</u> of the following:**		**16**
	a)	A bag contains 20 tickets numbered from 1 to 20. One ticket is drawn at random. Find the probability that it is numbered with multiple of 3 or 5.		
	Ans	$S = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$		
		$\therefore \; n(S) = 20$	1	
		number multiple of 3 or 5		
		$A = \{3,5,6,9,10,12,15,18,20\}$	1½	
		$\therefore \; n(A) = 9$		
		$p(A) = \frac{n(A)}{n(S)}$		
		$= \frac{9}{20} \quad or \quad 0.45$	1½	4
		OR		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$S = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$ $\therefore n(S) = 20$ number multiple of 3 or 5 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $n(A \cup B) = 6 + 4 - 1$ $\therefore n(A \cup B) = 9$ $p(A \cup B) = \dfrac{n(A \cup B)}{n(S)}$ $= \dfrac{9}{20}$ or 0.45	1 1 1 1	4
	b) Ans	A firm produces articles of which 0.1% are defective, out of 500 articles. If wholesaler purchases 100 such cases , how many can be expected to have one defective? Given: $e^{-0.5} = 0.6065$ $p = 0.1\% = 0.001$ $n = 500$ mean $m = np = 500 \times 0.001$ $m = 0.5$ Poisson Distribution is $P(r) = \dfrac{e^{-m} m^r}{r!}$ One is defective $r = 1$ $P(1) = \dfrac{e^{-0.5}(0.5)^1}{1!}$ $P(1) = 0.30325$ $No.$ of cases $= 100 \times 0.30325$ $= 30.325 \approx 30$	½ 1 1 ½ 1	4
	c) Ans	I.Q.'s are normally distributed with mean 100 and standard deviatio15. Find the probability that a randomly selected person has: i) An I.Q.more than 130 ii) An I.Q.between 85 and 115 $[z = 2, Area = 0.4772, z = 1, Area = 0.3413]$ Given $\bar{x} = 100$, $\sigma = 15$ Standard normal variate, $Z = \dfrac{x - \bar{x}}{\sigma}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		i) For $x=130$, $Z=\frac{130-100}{15}=2$	½	
		$p=(\text{area more than }2)=0.5-A(2)$ $=0.5-0.4772$	1	
		$=0.0228$	½	
		ii) For $x=85$, $Z=\frac{85-100}{15}=-1$		
		For $x=115$, $Z=\frac{115-100}{15}=1$	½	
		$p(\text{I.Q.between 85 and 115})=A(-1)+A(1)$ $=0.3413+0.3413$	1	
		$=0.6826$	½	4
		--		
	d) Ans	Divide 80 into two parts such that their product is maximum. consider x and y be the two parts $\therefore\ x+y=80$ $y=80-x$ product is, $P=xy$ $P=x(80-x)$ $P=80x-x^2$	1	
		$\frac{dP}{dx}=80-2x$	½	
		$\frac{d^2P}{dx^2}=-2$	½	
		For maximum value $\frac{dP}{dx}=0$ $\therefore\ 80-2x=0$ $x=40$	1	
		At $x=40$, $\frac{d^2P}{dx^2}=-2$, *i.e.* Product is maximum	½	
		$\therefore\ x=40, y=40$	½	4
		--		
	e)	The equation of the tangent at the point $(2,3)$ on the curve $y=ax^3+b$ is $y=4x-5$. Find the values of a and b		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	Ans	The equation of the tangent is, $y = 4x - 5$ $\therefore slope = m = 4$	½	
		$y = ax^3 + b$ $\frac{dy}{dx} = 3ax^2$	1	
		at $(2,3)$ $\frac{dy}{dx} = 12a$	½	
		$m = 12a$ $\therefore 4 = 12a$ $a = \frac{1}{3}$	1	
		$y = ax^3 + b$ $\therefore 3 = \left(\frac{1}{3}\right)(2)^3 + b$ $b = \frac{1}{3}$	1	
		$a = \frac{1}{3}, b = \frac{1}{3}$		4
	f) Ans	Find the area of circle $x^2 + y^2 = 16$ by integration $x^2 + y^2 = 16$ $\therefore y = \sqrt{16 - x^2}$ area, $A = 4\int_a^b y\,dx$	1	
		$A = 4\left[\int_0^4 \sqrt{(4)^2 - x^2}\,dx\right]$ $A = 4\left[\frac{x}{2}\sqrt{(4)^2 - x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$	1	
		$A = 4\left[8\sin^{-1}(1) - 0\right]$	1	
		$A = 4\left[8\frac{\pi}{2}\right]$ $A = 16\pi$	1	4

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
		Important Note In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking. --		