SEEX1013 POWER ELECTRONICS

UNIT III DC & AC CHOPPERS 10 hrs.

DC – DC Chopper

Step down and step up choppers – One, two and four quadrant operation.

AC – AC Chopper

Single phase AC voltage controllers with R & RL load – Multistage sequence control-Single phase step up and step down cycloconverters.

DC – DC Chopper

3.1 Introduction

- Chopper is a static device.
- A variable dc voltage is obtained from a constant dc voltage source.
- Also known as dc-to-dc converter.
- Widely used for motor control.
- Also used in regenerative braking.
- Thyristor converter offers greater efficiency, faster response, lower maintenance, smaller size and smooth control.
- A chopper is a high speed on/off semiconductor switch. It connects source to load and disconnects the load from source at a fast speed.

3.2 Choppers Types

- Step Up Chopper or Boost Converter (In step up chopper output voltage is more than input voltage)
- Step Down Chopper or Buck Converter (In step down chopper output voltage is less than input voltage)
- Step Up/down or Buck -Boost converter (In step up/down chopper output voltage can be less or more than output voltage)

3.2.1 Applications of choppers

- Switched mode power supplies, including DC to DC converters.
- Speed controllers for DC motors
- Class D Electronic amplifiers
- Switched capacitor filters
- Variable-frequency drive
- D.C. motor speed control
- D.C. voltage boosting
- Battery operated electric cars
- Battery operated appliances
- Battery chargers

3.3 Principle of Step-down Chopper

- A step-down chopper with resistive load.
- The thyristor in the circuit acts as a switch.
- When thyristor is ON, supply voltage appears across the load
- When thyristor is OFF, the voltage across the load will be zero.

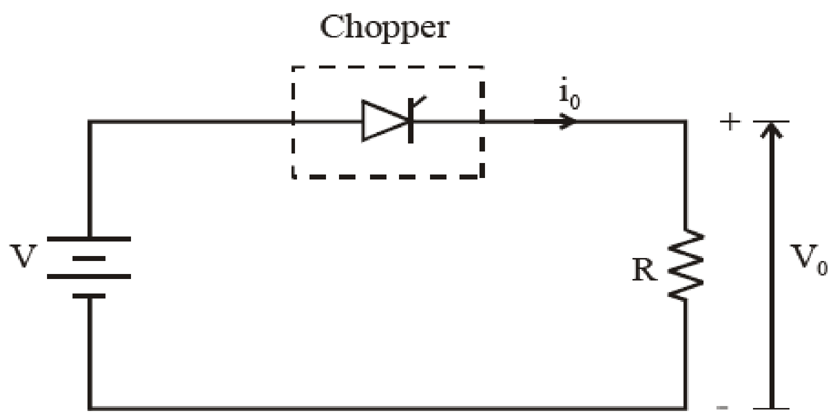


Figure 3.1 Step-down Chopper

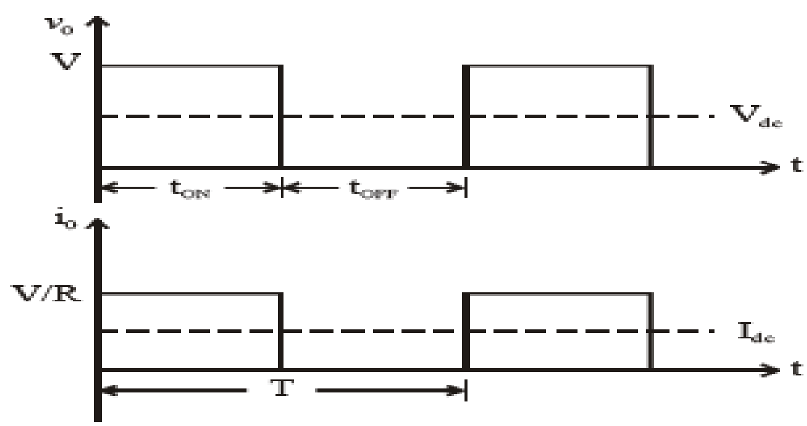


Figure 3.2 Output waveforms

V_{dc} = Average value of output or load voltage.

I_{dc} = Average value of output or load current.

t_{ON} = Time interval for which SCR conducts.

t_{OFF} = Time interval for which SCR is OFF.

$T = t_{ON} + t_{OFF}$ = Period of switching or chopping period.

$f = \frac{1}{T} =$ Freq. of chopper switching or chopping freq.

Average Output Voltage

$$V_{dc} = V\left(\frac{t_{ON}}{t_{ON} + t_{OFF}}\right)$$

$$V_{dc} = V\left(\frac{t_{ON}}{T}\right) = V.d$$

$$but \left(\frac{t_{ON}}{t}\right) = d = \text{duty cycle}$$

Average Output Current

$$I_{dc} = \frac{V_{dc}}{R}$$

$$I_{dc} = \frac{V}{R}\left(\frac{t_{ON}}{T}\right) = \frac{V}{R}d$$

RMS value of output voltage

$$V_O = \sqrt{\frac{1}{T}\int_0^{t_{ON}} v_o^2 dt}$$

But during t_{ON}, $v_o = V$

Therefore RMS output voltage

$$V_O = \sqrt{\frac{1}{T}\int_0^{t_{ON}} V^2 dt}$$

$$V_O = \sqrt{\frac{V^2}{T}t_{ON}} = \sqrt{\frac{t_{ON}}{T}}.V$$

$$V_O = \sqrt{d}.V$$

Output power $P_O = V_O I_O$

But $I_O = \frac{V_O}{R}$

$\therefore$ Output power

$$P_O = \frac{V_O^2}{R}$$

$$P_O = \frac{dV^2}{R}$$

3.4 Methods of Control

The output dc voltage can be varied by the following methods.

- Pulse width modulation control or constant frequency operation.
- Variable frequency control.

3.4.1 Pulse Width Modulation

- t_{ON} is varied keeping chopping frequency '*f*' & chopping period 'T' constant.
- Output voltage is varied by varying the ON time t_{ON}

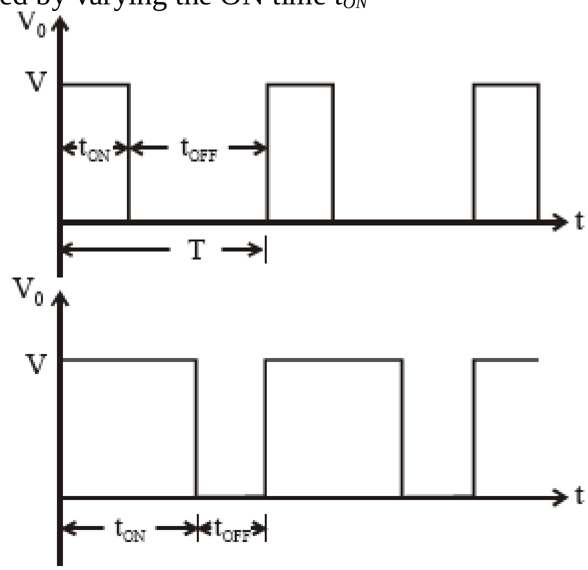


Figure 3.3 ***t_{ON} and t_{off} varied***

3.4.2 Variable Frequency Control

• Chopping frequency *'f'* is varied keeping either t_{ON} or t_{OFF} constant.

• To obtain full output voltage range, frequency has to be varied over a wide range.

• This method produces harmonics in the output and for large t_{OFF} load current may become discontinuous

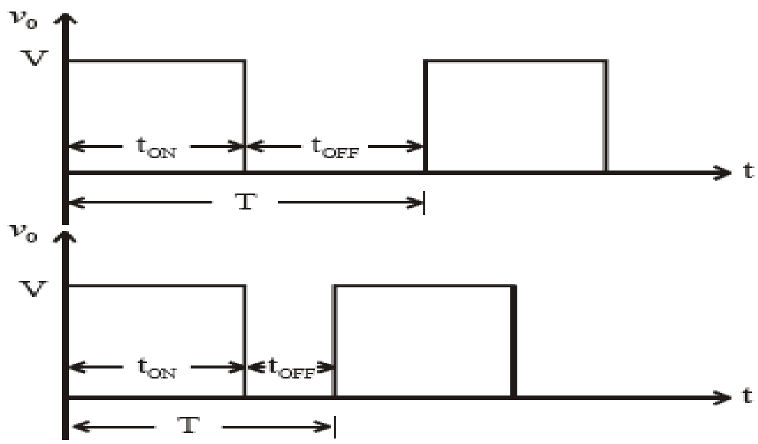


Figure 3.4 Total time period or frequency varied

3.5 Step-down Chopper with R-L Load

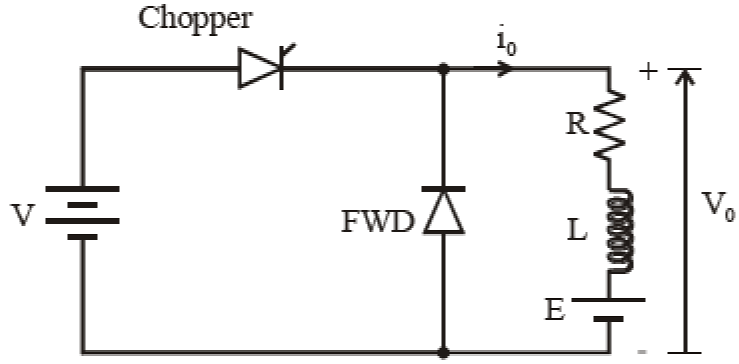


Figure 3.5 Step down chopper with R-L Load

- When chopper is ON, supply is connected across load.
- Current flows from supply to load.
- When chopper is OFF, load current continues to flow in the same direction through FWD due to energy stored in inductor *'L'*.
- Load current can be continuous or discontinuous depending on the values of *'L'* and duty cycle *'d'*
- Fo a continuous current operation, load current varies between two limits *Imax* and *Imin*
- When current becomes equal to *Imax* the chopper is turned-off and it is turned-on when current reduces to *Imin.*

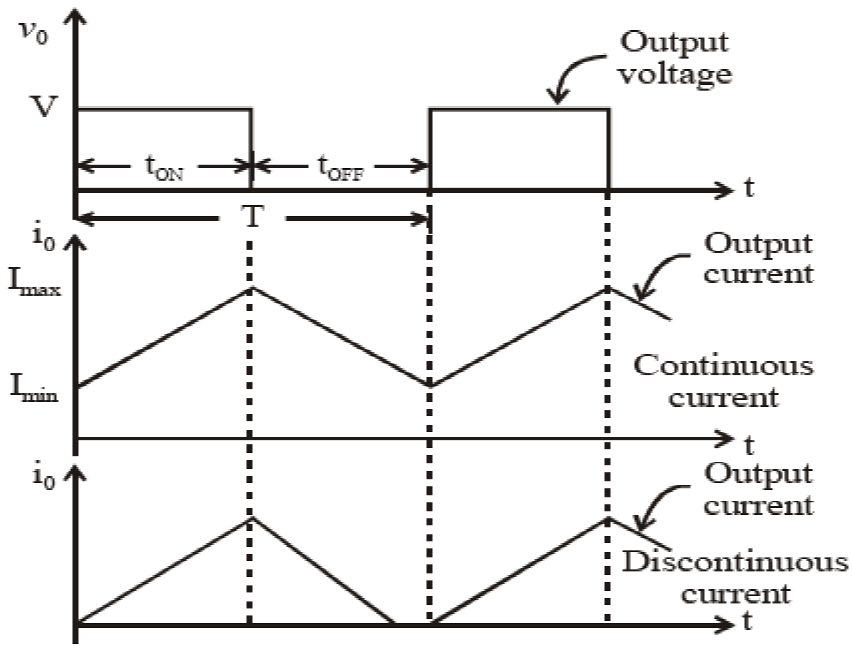
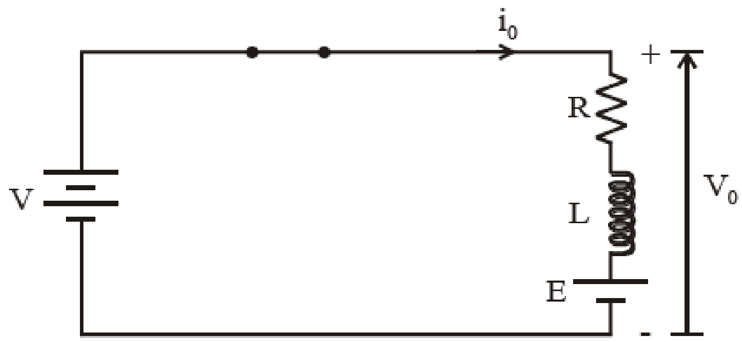


Figure 3.5 Output voltage and current waveform

Expressions for Load Current *Io* for Continuous Current Operation When Chopper is ON *(0 ≤ T ≤ Ton)*



$$V = i_O R + L\frac{di_O}{dt} + E$$

Taking Laplace Transform

$$\frac{V}{S} = RI_O(S) + L\left[S.I_O(S) - i_O(0^-)\right] + \frac{E}{S}$$

At $t = 0$, initial current $i_O(0^-) = I_{min}$

$$I_O(S) = \frac{V - E}{LS\left(S + \frac{R}{L}\right)} + \frac{I_{min}}{S + \frac{R}{L}}$$

Taking Inverse Laplace Transform

$$i_O(t) = \frac{V - E}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right] + I_{min} e^{-\left(\frac{R}{L}\right)t}$$

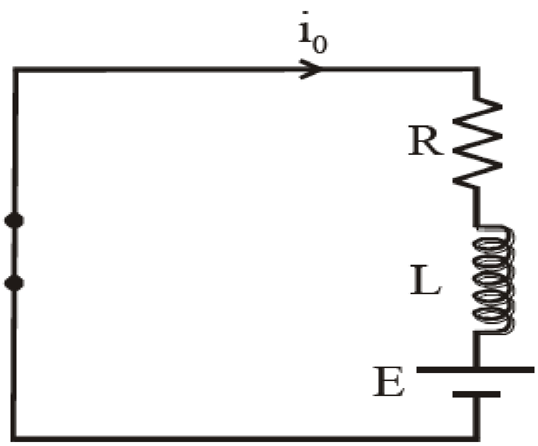
This expression is valid for $0 \le t \le t_{ON}$,

i.e., during the period chopper is ON.

At the instant the chopper is turned off,

Load current $i_o = I_{MAX}$

When Chopper is OFF



When Chopper is OFF $\quad 0 \le t \le t_{OFF}$

$$0 = Ri_O + L\frac{di_O}{dt} + E$$

Talking Laplace transform

$$0 = RI_O\ S\ + L\left[SI_O\ S\ - i_O\ 0^-\ \right] + \frac{E}{S}$$

$$\therefore I_O\ S\ = \frac{I_{\max}}{S + \frac{R}{L}} - \frac{E}{LS\left(S + \frac{R}{L}\right)}$$

Taking Inverse Laplace Transform

$$i_O\ t\ = I_{\max} e^{-\frac{R}{L}t} - \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

The expression is valid for $0 \le t \le t_{OFF}$,

i.e., during the period chopper is OFF

At the instant the chopper is turned ON or at the end of the off period, the load current is

$$i_O\ t_{OFF}\ = I_{\min}$$

3.6 Principle of Step-up Chopper

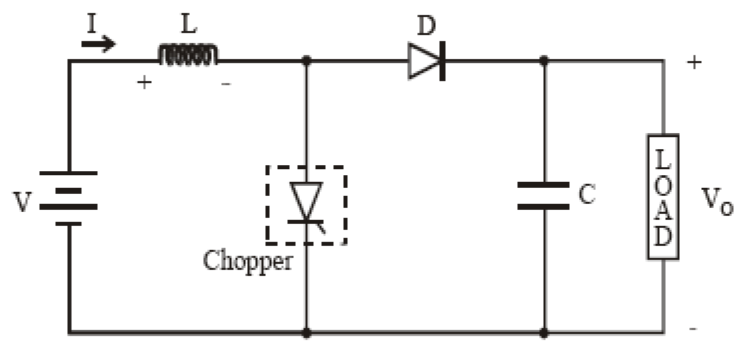


Figure 3.6

- Step-up chopper is used to obtain a load voltage higher than the input voltage *V*.
- The values of *L* and *C* are chosen depending upon the requirement of output voltage and current.
- When the chopper is *ON,* the inductor *L* is connected across the supply.
- The inductor current *'I'* rises and the inductor stores energy during the *ON* time of the chopper, t_{ON}.
- When the chopper is off, the inductor current *I* is forced to flow through the diode *D* and load for a period, t_{OFF}.
- The current tends to decrease resulting in reversing the polarity of induced EMF in *L*.
- Therefore voltage across load is given by,

$$V_O = V + L\frac{dI}{dt} \quad i.e., \quad V_O > V$$

- A large capacitor 'C' connected across the load, will provide a continuous output voltage .
- Diode *D* prevents any current flow from capacitor to the source.
- Step up choppers are used for regenerative braking of dc motors.

(i) Expression For Output Voltage

Assume the average inductor current to be I during ON and OFF time of Chopper.

When Chopper is ON

Voltage across inductor $L = V$

Therefore energy stored in inductor

$$= V.I.t_{ON}$$

Where $t_{ON} = ON$ period of chopper.

When Chopper is OFF

(energy is supplied by inductor to load)

Voltage across $L = V_O - V$

Energy supplied by inductor $L = \ V_O - V \ \ It_{OFF}$

where $t_{OFF} = OFF$ period of Chopper.

Neglecting losses, energy stored in inductor L = energy supplied by inductor L

$$\therefore \quad VIt_{ON} = \ V_O - V \ \ It_{OFF}$$

$$V_O = \frac{V \ \ t_{ON} + t_{OFF}}{t_{OFF}}$$

$$V_O = V\left(\frac{T}{T - t_{ON}}\right)$$

Where

T = Chopping period or period of switching.

$$T = t_{ON} + t_{OFF}$$

$$V_O = V\left(\frac{1}{1 - \frac{t_{ON}}{T}}\right)$$

$$\therefore \quad V_O = V\left(\frac{1}{1 - d}\right)$$

Where $d = \frac{t_{ON}}{T} =$ duty cyle

Problem

1. A Chopper circuit is operating on TRC at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.

Solution:

$V = 460$ V, $V_{dc} = 350$ V, f = 2 kHz

Chopping period $T = \frac{1}{f}$

$$T = \frac{1}{2\times10^{-3}} = 0.5\ m\text{sec}$$

Output voltage $V_{dc} = \left(\frac{t_{ON}}{T}\right)V$

Conduction period of thyristor

$$t_{ON} = \frac{T \times V_{dc}}{V}$$

$$t_{ON} = \frac{0.5\times10^{-3}\times350}{460}$$

$$t_{ON} = 0.38 \text{ msec}$$

Problem

2. Input to the step up chopper is 200 V. The output required is 600 V. If the conducting time of thyristor is 200 μsec. Compute

- Chopping frequency,
- If the pulse width is halved for constant frequency of operation, find the new output voltage.

Solution:

$$V = 200\ V,\ \ t_{ON} = 200\mu s,\ \ V_{dc} = 600V$$

$$V_{dc} = V\left(\frac{T}{T - t_{ON}}\right)$$

$$600 = 200\left(\frac{T}{T - 200\times10^{-6}}\right)$$

Solving for T

$$T = 300\mu s$$

Chopping frequency

$$f = \frac{1}{T}$$

$$f = \frac{1}{300\times10^{-6}} = 3.33KHz$$

Pulse width is halved

$$\therefore \qquad t_{ON} = \frac{200\times10^{-6}}{2} = 100\mu s$$

Frequency is constant

$$\therefore \qquad f = 3.33KHz$$

$$T = \frac{1}{f} = 300\mu s$$

$$\therefore \text{Output voltage} = V\left(\frac{T}{T-t_{ON}}\right)$$

$$= 200\left(\frac{300\times10^{-6}}{300-100\ \ 10^{-6}}\right) = 300 \text{ Volts}$$

Problem

3. A dc chopper has a resistive load of 20Ω and input voltage VS = 220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.

Solution:

$$V_S = 220V,\ R = 20\Omega,\ \ f\ = \ 10\ kHz$$

$$d = \frac{t_{ON}}{T} = 0.80$$

V_{ch} = Voltage drop across chopper = 1.5 volts

Average output voltage

$$V_{dc} = \left(\frac{t_{ON}}{T}\right)\ V_S - V_{ch}$$

$$V_{dc} = 0.80\ \ 220 - 1.5\ \ = 174.8 \text{ Volts}$$

Chopper ON time, $t_{ON} = dT$

Chopping period, $T = \frac{1}{f}$

$$T = \frac{1}{10 \times 10^3} = 0.1 \times 10^{-3} \text{ secs} = 100 \ \mu\text{secs}$$

Chopper ON time,

$$t_{ON} = dT$$

$$t_{ON} = 0.80 \times 0.1 \times 10^{-3}$$

$$t_{ON} = 0.08 \times 10^{-3} = 80 \ \mu\text{secs}$$

Problem

4. In a dc chopper, the average load current is 30 Amps, chopping frequency is 250 Hz, supply voltage is 110 volts. Calculate the ON and OFF periods of the chopper if the load resistance is 2 ohms.

Solution:

$$I_{dc} = 30 \ Amps, \ f = 250 \ Hz, \ V = 110 \ V, \ R = 2\Omega$$

Chopping period, $T = \frac{1}{f} = \frac{1}{250} = 4 \times 10^{-3} = 4$ msecs

$$I_{dc} = \frac{V_{dc}}{R} \ \& \ V_{dc} = dV$$

$$\therefore \quad I_{dc} = \frac{dV}{R}$$

$$d = \frac{I_{dc} R}{V} = \frac{30 \times 2}{110} = 0.545$$

Chopper ON period,

$$t_{ON} = dT = 0.545 \times 4 \times 10^{-3} = 2.18 \text{ msecs}$$

Chopper OFF period,

$$t_{OFF} = T - t_{ON}$$

$$t_{OFF} = 4 \times 10^{-3} - 2.18 \times 10^{-3}$$

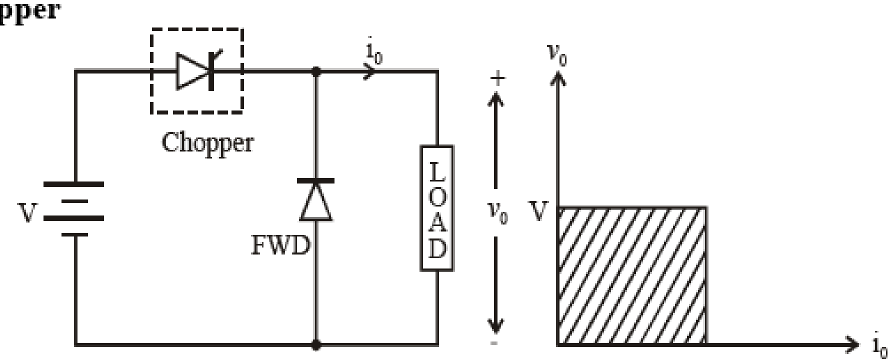
$$t_{OFF} = 1.82 \times 10^{-3} = 1.82 \text{ msec}$$

3.7 Classification of Choppers

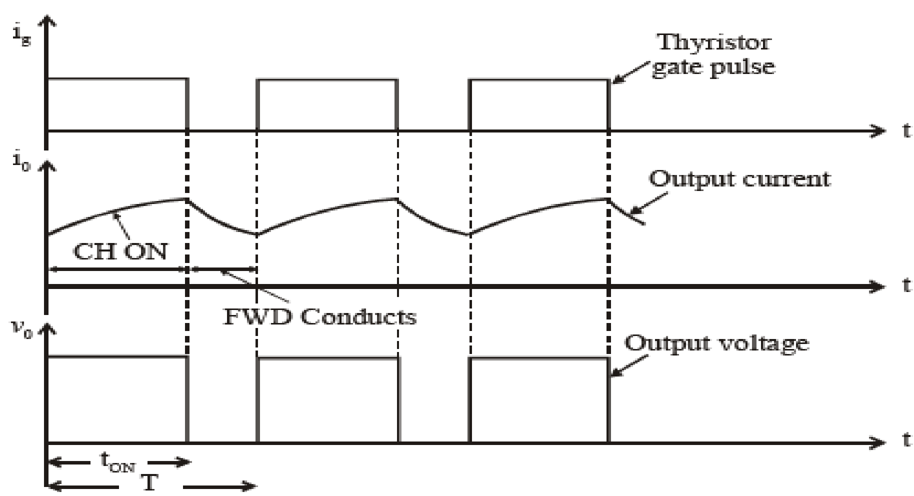
Choppers are classified as

- Class A Chopper
- Class B Chopper
- Class C Chopper
- Class D Chopper
- Class E Chopper

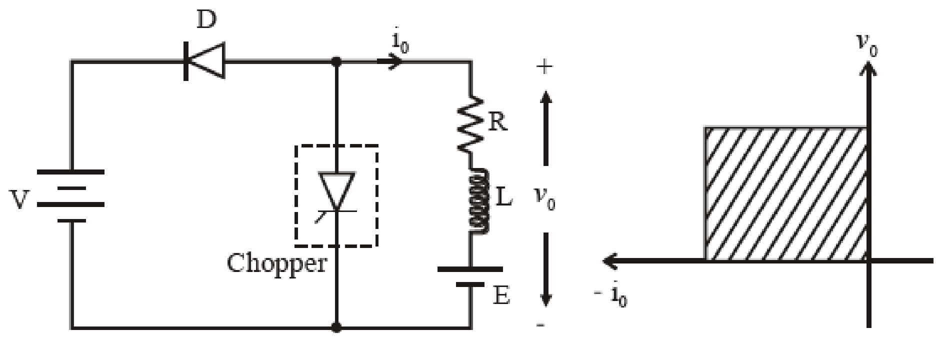
1. Class A Chopper



- When chopper is *ON,* supply voltage *V* is connected across the load.
- When chopper is OFF, *v*O = 0 and the load current continues to flow in the same direction through the FWD.
- The average values of output voltage and current are always positive.
- *Class A Chopper* is a first quadrant chopper .
- *Class A Chopper* is a step-down chopper in which power always flows form source to load.
- It is used to control the speed of dc motor.
- The output current equations obtained in step down chopper with *R-L* load can be used to study the performance of *Class A Chopper.*

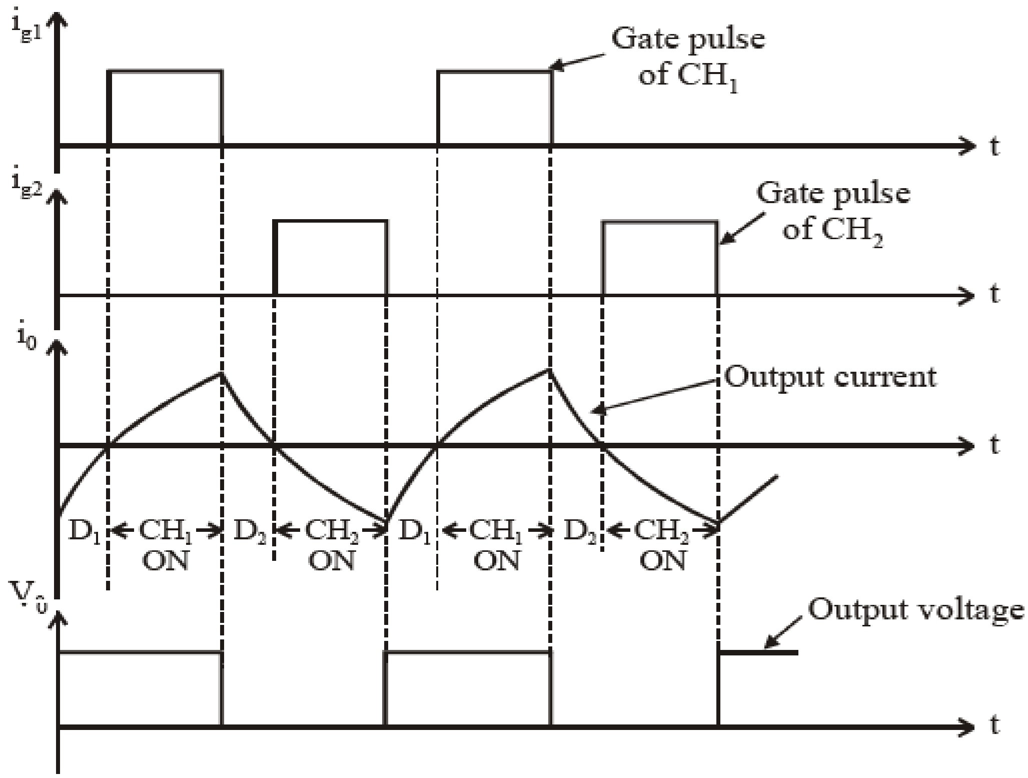


2. Class B Chopper

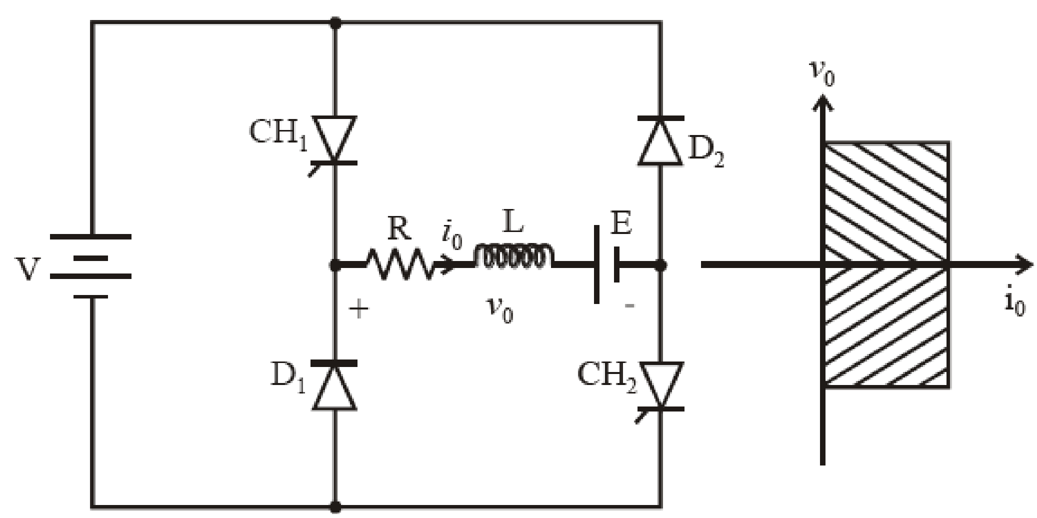


- When chopper is ON, *E* drives a current through *L* and R in a direction opposite to that shown in figure.
- During the ON period of the chopper, the inductance *L* stores energy.
- When Chopper is OFF, diode *D* conducts, and part of the energy stored in inductor *L* is returned to the supply.
- Average output voltage is positive.
- Average output current is negative.
- Therefore *Class B Chopper* operates in second quadrant.
- In this chopper, power flows from load to source.
- *Class B Chopper* is used for regenerative braking of dc motor.
- *Class B Chopper* is a step-up chopper.

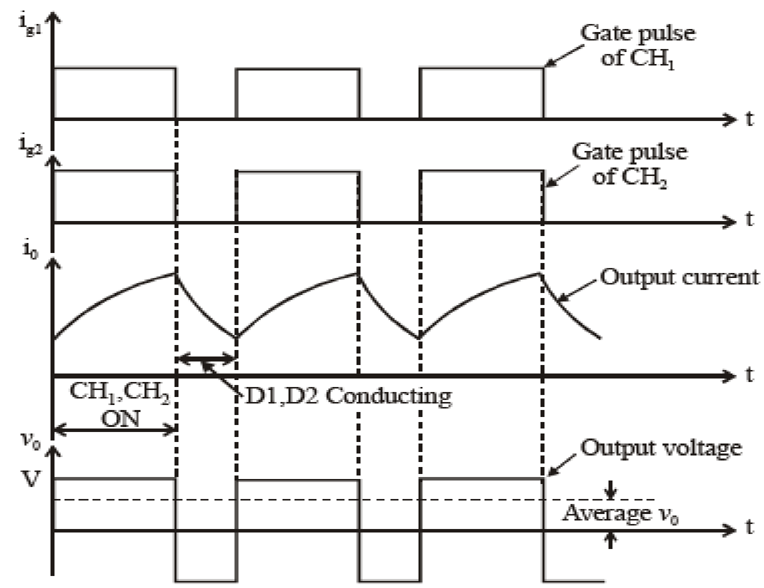
- The output voltage is zero.
- On turning OFF *CH2* , the energy stored in the inductance drives current through diode *D1* and the supply
- Output voltage is *V*, the input current becomes negative and power flows from load to source.
- Average output voltage is positive
- Average output current can take both positive and negative values.
- Choppers *CH1 & CH2* should not be turned ON simultaneously as it would result in short circuiting the supply.
- *Class C Chopper* can be used both for dc motor control and regenerative braking of dc motor.
- *Class C Chopper* can be used as a step-up or step-down chopper.

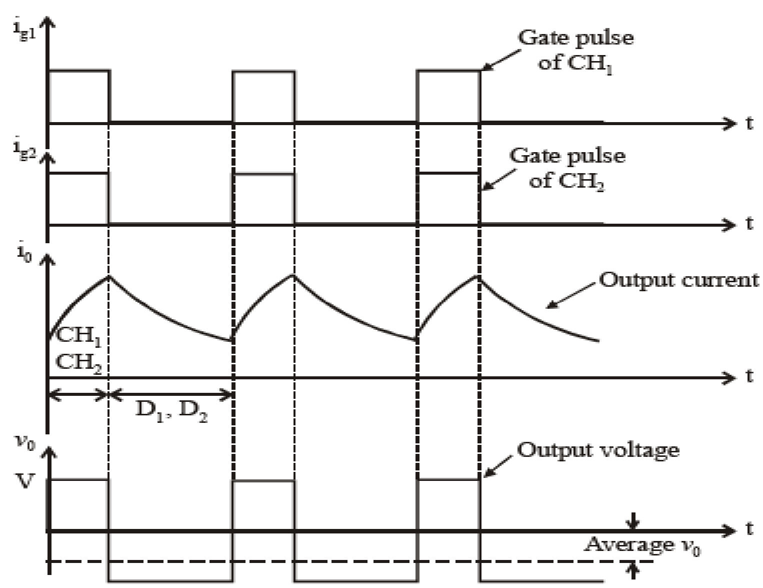


4. Class D Chopper

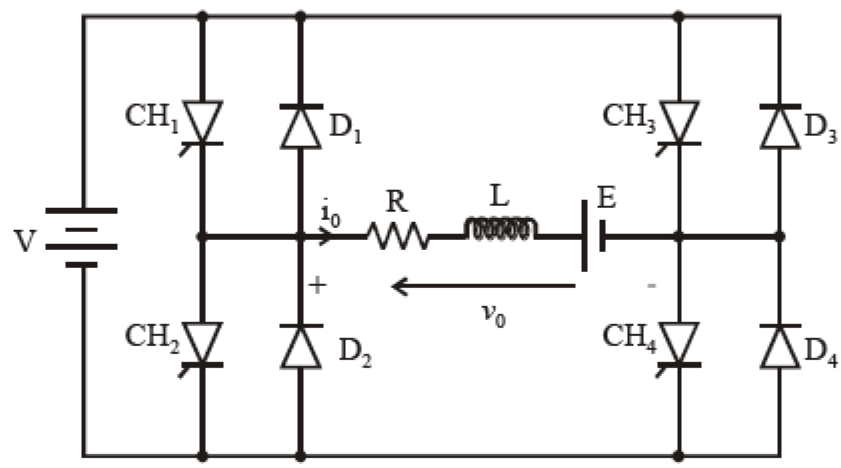


- Class D is a two quadrant chopper.
- When both *CH1* and *CH2* are triggered simultaneously, the output voltage $vO = V$ and output current flows through the load.
- When *CH1* and *CH2* are turned OFF, the load current continues to flow in the same direction through load, *D1* and *D2* , due to the energy stored in the inductor L.
- Output voltage $vO = - V$.
- Average load voltage is positive if chopper ON time is more than the OFF time
- Average output voltage becomes negative if $tON < tOFF$.
- Hence the direction of load current is always positive but load voltage can be positive or negative.



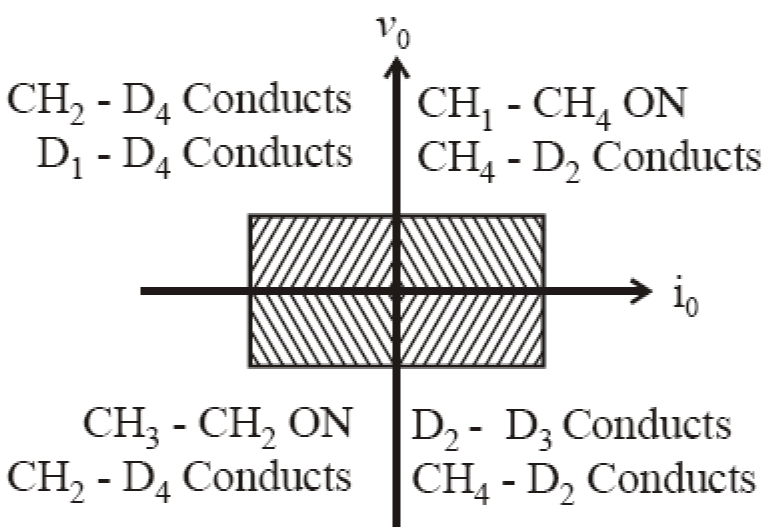


5. Class E Chopper



Four Quadrant Operation

• Class E is a four quadrant chopper
• When *CH1* and *CH4* are triggered, output current iO flows in positive direction through *CH1* and *CH4*, and with output voltage *vO* = *V*.
• This gives the first quadrant operation.
• When both *CH1* and *CH4* are OFF, the energy stored in the inductor L drives *iO* through D2 and *D3* in the same direction, but output voltage *vO* = *-V*.
• Therefore the chopper operates in the fourth quadrant.

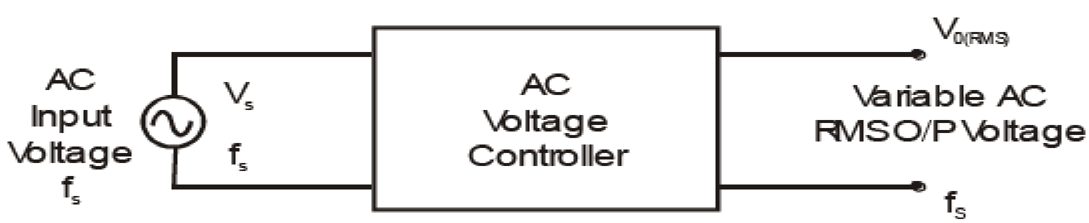


- When *CH2* and *CH3* are triggered, the load current i_O *flows* in opposite direction & output voltage $v_O = -V$.
- Since both i_O and v_O are negative, the chopper operates in third quadrant.
- When both *CH2* and *CH3* are OFF, the load current i_O continues to flow in the same direction *D1* and *D4* and the output voltage $v_O = V$.
- Therefore the chopper operates in second quadrant as v_O is positive but i_O is negative.

3.8 AC – AC CHOPPERS

AC voltage controllers (ac line voltage controllers) are employed to vary the RMS value of the alternating voltage applied to a load circuit by introducing Thyristors between the load and a constant voltage ac source. The RMS value of alternating voltage applied to a load circuit is controlled by controlling the triggering angle of the Thyristors in the ac voltage controller circuits.

In brief, an ac voltage controller is a type of thyristor power converter which is used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output. The RMS value of the ac output voltage and the ac power flow to the load is controlled by varying (adjusting) the trigger angle 'α'



There are two different types of thyristor control used in practice to control the ac power flow,

- On-Off control
- Phase control

These are the two ac output voltage control techniques. In On-Off control technique Thyristors are used as switches to connect the load circuit to the ac supply (source) for a few cycles of the input ac supply and then to disconnect it for few input cycles. The Thyristors thus act as a high speed contactor (or high speed ac switch).

3.8.1 Phase Control

In phase control the Thyristors are used as switches to connect the load circuit to the input ac supply, for a part of every input cycle. That is the ac supply voltage is chopped using Thyristors during a part of each input cycle. The thyristor switch is turned on for a part of every half cycle, so that input supply voltage appears across the load and then turned off during the remaining part of input half cycle to disconnect the ac supply from the load.

By controlling the phase angle or the trigger angle ‘’ (delay angle), the output RMS voltage across the load can be controlled.The trigger delay angle ‘’ is defined as the phase angle (the value of t) at which the thyristor turns on and the load current begins to flow.

Thyristor ac voltage controllers use ac line commutation or ac phase commutation. Thyristors in ac voltage controllers are line commutated (phase commutated) since the input supply is ac. When the input ac voltage reverses and becomes negative during the negative half cycle the current flowing through the conducting thyristor decreases and falls to zero. Thus the ON thyristor naturally turns off, when the device current falls to zero.

Phase control Thyristors which are relatively inexpensive, converter grade Thyristors which are slower than fast switching inverter grade Thyristors are normally used.For applications upto 400Hz, if Triacs are available to meet the voltage and current ratings of a particular application, Triacs are more commonly used.

Due to ac line commutation or natural commutation, there is no need of extra commutation circuitry or components and the circuits for ac voltage controllers are very simple.Due to the nature of the output waveforms, the analysis, derivations of expressions for performance parameters are not simple, especially for the phase controlled ac voltage controllers with RL load. But however most of the practical loads are of the RL type and hence RL load should be considered in the analysis and design of ac voltage controller circuits.

3.8.2 Principle of On-Off Control Technique (Integral Cycle Control)

The basic principle of on-off control technique is explained with reference to a single phase full wave ac voltage controller circuit shown below. The thyristor switches T_1 and T_2 are turned on by applying appropriate gate trigger pulses to connect the input ac supply to the load for ‘n’ number of input cycles during the time interval t on. The thyristor switches T_1 and T_2 are turned off by blocking the gate trigger pulses for ‘m’ number of input cycles during the time interval t_{OFF}. The ac controller ON time ton usually consists of an integral number of input cycles.

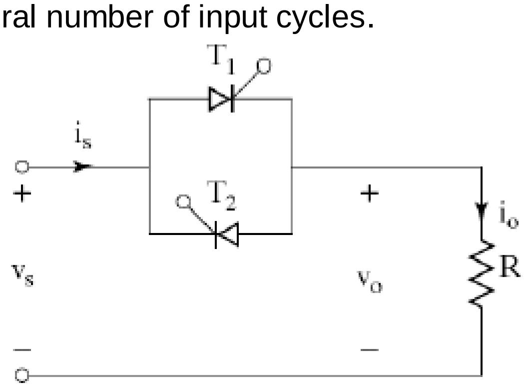


Fig 3.7 Single phase full wave AC voltage controller

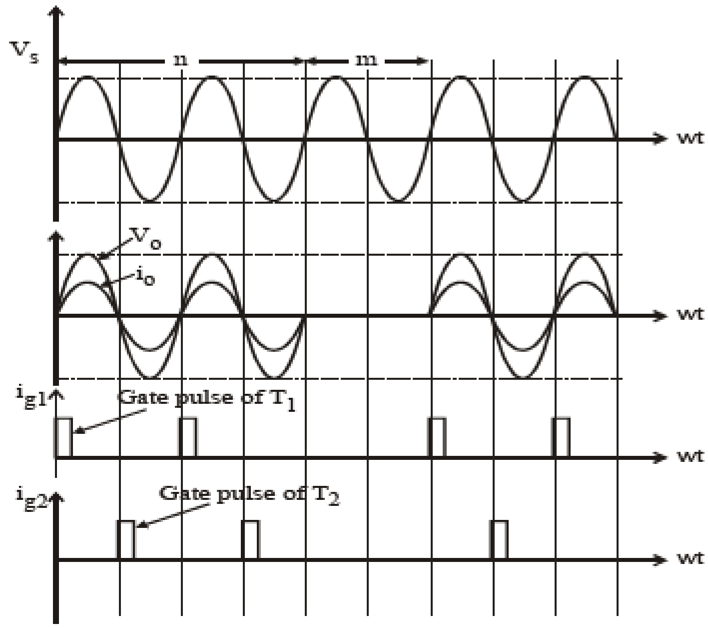


Fig 3.8 *Waveforms*

Referring to the waveforms of ON-OFF control technique in the above diagram,

$n =$ Two input cycles. Thyristors are turned ON during t_{ON} for two input cycles.

$m =$ One input cycle. Thyristors are turned OFF during t_{OFF} for one input cycle

For a sine wave input supply voltage,

$$v_s = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t$$

$$V_S = \text{RMS value of input ac supply} = \frac{V_m}{\sqrt{2}} = \text{RMS phase supply voltage.}$$

If the input ac supply is connected to load for 'n' number of input cycles and disconnected for 'm' number of input cycles, then

$$t_{ON} = n \times T, \qquad t_{OFF} = m \times T$$

Where $T = \frac{1}{f}$ = input cycle time (time period) and

f = input supply frequency.

t_{ON} = controller on time $= n \times T$.

t_{OFF} = controller off time $= m \times T$.

T_O = Output time period = $t_{ON} + t_{OFF} = nT + mT$.

We can show that,

Output RMS voltage $V_{O\ RMS} = V_{i\ RMS}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$

Where $V_{i\ RMS}$ is the RMS input supply voltage = V_S .

(i) To derive an expression for the rms value of output voltage, for on-off control method.

Output RMS voltage $V_{O\ RMS} = \sqrt{\frac{1}{\omega T_O}\int_{\omega t=0}^{\omega t_{ON}} V_m^2 Sin^2\omega t.d\ \ \omega t}$

$$V_{O\ RMS} = \sqrt{\frac{V_m^2}{\omega T_O}\int_0^{\omega t_{ON}} Sin^2\omega t.d\ \ \omega t}$$

Substituting for $Sin^2\theta = \frac{1-Cos2\theta}{2}$

$$V_{O\ RMS} = \sqrt{\frac{V_m^2}{\omega T_O}\int_0^{\omega t_{ON}} \left[\frac{1-Cos2\omega t}{2}\right] d\ \ \omega t}$$

$$V_{O\ RMS} = \sqrt{\frac{V_m^2}{2\omega T_O}\left[\int_0^{\omega t_{ON}} d\ \ \omega t\ \ - \int_0^{\omega t_{ON}} Cos2\omega t.d\ \ \omega t\right]}$$

$$V_{O\ RMS} = \sqrt{\frac{V_m^2}{2\omega T_O}\left[\ \omega t \Big/_0^{\omega t_{ON}} - \frac{Sin2\omega t}{2}\Big/_0^{\omega t_{ON}}\right]}$$

$$V_{O\ RMS} = \sqrt{\frac{V_m^2}{2\omega T_O}\left[\ \omega t_{ON} - 0\ \ - \frac{\sin 2\omega t_{ON} - \sin 0}{2}\right]}$$

Now t_{ON} = an integral number of input cycles; Hence

$t_{ON} = T, 2T, 3T, 4T, 5T, ...$ & $\omega t_{ON} = 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, ...$

$$V_{O\ RMS} = V_{i\ RMS}\sqrt{k} = V_S\sqrt{k}$$

Where $V_S = V_{i\ RMS}$ = RMS value of input supply voltage.

Where $V_S = V_{i\ RMS}$ = RMS value of input supply voltage.

Where T is the input supply time period (T = input cycle time period). Thus we note that $\sin 2\omega t_{ON} = 0$

$$V_{O\ RMS} = \sqrt{\frac{V_m^{\ 2}\,\omega\, t_{ON}}{2\,\omega\, T_O}} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{t_{ON}}{T_O}}$$

$$V_{O\ RMS} = V_{i\ RMS}\sqrt{\frac{t_{ON}}{T_O}} = V_S\sqrt{\frac{t_{ON}}{T_O}}$$

Where $V_{i\ RMS} = \frac{V_m}{\sqrt{2}} = V_S =$ RMS value of input supply voltage;

$$\frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON}+t_{OFF}} = \frac{nT}{nT+mT} = \frac{n}{n+m} = k = \text{duty cycle (d).}$$

$$V_{O\ RMS} = V_S\sqrt{\frac{n}{m+n}} = V_S\sqrt{k}$$

Performance Parameters of Ac Voltage Controllers

- **RMS Output (Load) Voltage**

$$V_{O\ RMS} = \left[\frac{n}{2\pi\,(n+m)}\int_0^{2\pi} V_m^{\ 2}\sin^2\omega t . d(\omega t)\right]^{1/2}$$

$$V_{O\ RMS} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{n}{m+n}} = V_{i\ RMS}\sqrt{k} = V_S\sqrt{k}$$

Where $V_S = V_{i\ RMS}$ = RMS value of input supply voltage.

- **Duty Cycle**

$$k = \frac{t_{ON}}{T_O} = \frac{t_{ON}}{t_{ON}+t_{OFF}} = \frac{nT}{(m+n)\,T}$$

Where, $k = \frac{n}{m+n} =$ duty cycle (d).

- **RMS Load Current**

$$I_{O\ RMS} = \frac{V_{O\ RMS}}{Z} = \frac{V_{O\ RMS}}{R_L}; \quad \text{For a resistive load } Z = R_L.$$

- **Output AC (Load) Power**

$$P_O = I_{O\ RMS}^2 \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{VA} = \frac{\text{output load power}}{\text{input supply volt amperes}} = \frac{P_O}{V_S I_S}$$

$$PF = \frac{I_{O\ RMS}^2 \times R_L}{V_{i\ RMS} \times I_{in\ RMS}}; \qquad I_S = I_{in\ RMS} = \text{RMS input supply current.}$$

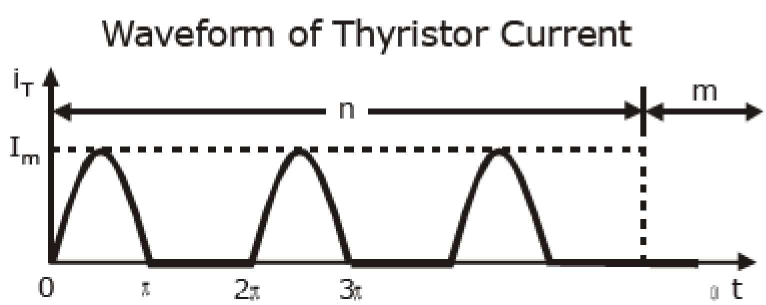
The input supply current is same as the load current $I_{in} = I_O = I_L$

Hence, RMS supply current = RMS load current; $I_{in\ RMS} = I_{O\ RMS}$.

$$PF = \frac{I_{O\ RMS}^2 \times R_L}{V_{i\ RMS} \times I_{in\ RMS}} = \frac{V_{O\ RMS}}{V_{i\ RMS}} = \frac{V_{i\ RMS}\sqrt{k}}{V_{i\ RMS}} = \sqrt{k}$$

$$PF = \sqrt{k} = \sqrt{\frac{n}{m+n}}$$

- **The Average Current of Thyristor $I_{T\ Avg}$**



$$I_{T\ Avg} = \frac{n}{2\pi(m+n)} \int_0^{\pi} I_m \sin\omega t . d(\omega t)$$

$$I_{T\ Avg} = \frac{nI_m}{2\pi \;\; m+n}\int_0^{\pi}\sin\omega t.d \;\; \omega t$$

$$I_{T\ Avg} = \frac{nI_m}{2\pi \;\; m+n}\left[-\cos\omega t\Big/_0^{\pi}\right]$$

$$I_{T\ Avg} = \frac{nI_m}{2\pi \;\; m+n} \;\; -\cos\pi + \cos 0$$

$$I_{T\ Avg} = \frac{nI_m}{2\pi \;\; m+n}\left[- \;\; -1 \;\; +1\right]$$

$$I_{T\ Avg} = \frac{n}{2\pi \;\; m+n} \;\; 2I_m$$

$$I_{T\ Avg} = \frac{I_m n}{\pi \;\; m+n} = \frac{k.I_m}{\pi}$$

$$k = \text{duty cycle} = \frac{t_{ON}}{t_{ON}+t_{OFF}} = \frac{n}{n+m}$$

$$I_{T\ Avg} = \frac{I_m n}{\pi \;\; m+n} = \frac{k.I_m}{\pi},$$

Where $I_m = \frac{V_m}{R_L}$ = maximum or peak thyristor current.

- **RMS Current of Thyristor** $I_{T\ RMS}$

$$I_{T\ RMS} = \left[\frac{n}{2\pi \;\; n+m}\int_0^{\pi} I_m^2 \sin^2\omega t.d \;\; \omega t \;\;\right]^{1/2}$$

$$I_{T\ RMS} = \left[\frac{nI_m^2}{4\pi\ n+m} \left\{ \omega t \Big/_0^{\pi} - \left(\frac{\sin 2\omega t}{2} \right) \Big/_0^{\pi} \right\} \right]^{1/2}$$

$$I_{T\ RMS} = \left[\frac{nI_m^2}{4\pi\ n+m} \left\{ \pi - 0\ - \left(\frac{\sin 2\pi - \sin 0}{2} \right) \right\} \right]^{1/2}$$

$$I_{T\ RMS} = \left[\frac{nI_m^2}{4\pi\ n+m}\ \pi - 0 - 0 \right]^{1/2}$$

$$I_{T\ RMS} = \left[\frac{nI_m^2 \pi}{4\pi\ n+m} \right]^{1/2} = \left[\frac{nI_m^2}{4\ n+m} \right]^{1/2}$$

$$I_{T\ RMS} = \frac{I_m}{2} \sqrt{\frac{n}{m+n}} = \frac{I_m}{2} \sqrt{k}$$

$$I_{T\ RMS} = \frac{I_m}{2} \sqrt{k}$$

Problem

1. A single phase full wave ac voltage controller working on ON-OFF control technique has supply voltage of 230V, RMS 50Hz, load = 50Ω. The controller is ON for 30 cycles and off for 40 cycles. Calculate
    - ON & OFF time intervals.
    - RMS output voltage.
    - Input P.F.
    - Average and RMS thyristor currents.

$V_{in\ RMS} = 230V$, $V_m = \sqrt{2} \times 230V = 325.269\,\text{V}$, $V_m = 325.269V$,

$T = \frac{1}{f} = \frac{1}{50Hz} = 0.02\sec$, $T = 20ms$.

$n =$ number of input cycles during which controller is ON; $n = 30$.

$m =$ number of input cycles during which controller is OFF; $m = 40$.

$$t_{ON} = n \times T = 30 \times 20ms = 600ms = 0.6\sec$$

$$t_{ON} = n \times T = 0.6\sec = \text{controller ON time.}$$

$$t_{OFF} = m \times T = 40 \times 20ms = 800ms = 0.8\sec$$

$t_{OFF} = m \times T = 0.8\sec$ = controller OFF time.

Duty cycle $k = \frac{n}{m+n} = \frac{30}{40+30} = 0.4285$

RMS output voltage

$$V_{O\ RMS} = V_{i\ RMS} \times \sqrt{\frac{n}{m+n}}$$

$$V_{O\ RMS} = 230V \times \sqrt{\frac{30}{30+40}} = 230\sqrt{\frac{3}{7}}$$

$$V_{O\ RMS} = 230V\sqrt{0.42857} = 230 \times 0.65465$$

$$V_{O\ RMS} = 150.570V$$

$$I_{O\ RMS} = \frac{V_{O\ RMS}}{Z} = \frac{V_{O\ RMS}}{R_L} = \frac{150.570V}{50\Omega} = 3.0114A$$

$$P_O = I^2_{O\ RMS} \times R_L = 3.0114^2 \times 50 = 453.426498W$$

Input Power Factor $P.F = \sqrt{k}$

$$PF = \sqrt{\frac{n}{m+n}} = \sqrt{\frac{30}{70}} = \sqrt{0.4285}$$

$$PF = 0.654653$$

Average Thyristor Current Rating

$$I_{T\ Avg} = \frac{I_m}{\pi} \times \left(\frac{n}{m+n} \right) = \frac{k \times I_m}{\pi}$$

where $$I_m = \frac{V_m}{R_L} = \frac{\sqrt{2} \times 230}{50} = \frac{325.269}{50}$$

$I_m = 6.505382A$ = Peak (maximum) thyristor current.

$$I_{T\ Avg} = \frac{6.505382}{\pi} \times \left(\frac{3}{7} \right)$$

$$I_{T\ Avg} = 0.88745A$$

RMS Current Rating of Thyristor

$t_{OFF} = m \times T = 0.8\sec$ = controller OFF time.

Duty cycle $k = \dfrac{n}{m+n} = \dfrac{30}{40+30} = 0.4285$

RMS output voltage

$$V_{O\ RMS} = V_{i\ RMS} \times \sqrt{\frac{n}{m+n}}$$

$$V_{O\ RMS} = 230V \times \sqrt{\frac{30}{30+40}} = 230\sqrt{\frac{3}{7}}$$

$$V_{O\ RMS} = 230V\sqrt{0.42857} = 230 \times 0.65465$$

$$V_{O\ RMS} = 150.570V$$

$$I_{O\ RMS} = \frac{V_{O\ RMS}}{Z} = \frac{V_{O\ RMS}}{R_L} = \frac{150.570V}{50\Omega} = 3.0114A$$

$$P_O = I^2_{O\ RMS} \times R_L = 3.0114^2 \times 50 = 453.426498W$$

Input Power Factor $P.F = \sqrt{k}$

$$PF = \sqrt{\frac{n}{m+n}} = \sqrt{\frac{30}{70}} = \sqrt{0.4285}$$

$$PF = 0.654653$$

Average Thyristor Current Rating

$$I_{T\ Avg} = \frac{I_m}{\pi} \times \left(\frac{n}{m+n} \right) = \frac{k \times I_m}{\pi}$$

where $$I_m = \frac{V_m}{R_L} = \frac{\sqrt{2} \times 230}{50} = \frac{325.269}{50}$$

$I_m = 6.505382A$ = Peak (maximum) thyristor current.

$$I_{T\ Avg} = \frac{6.505382}{\pi} \times \left(\frac{3}{7} \right)$$

$$I_{T\ Avg} = 0.88745A$$

RMS Current Rating of Thyristor

$$I_{T\ RMS} = \frac{I_m}{2} \sqrt{\frac{n}{m+n}} = \frac{I_m}{2} \sqrt{k} = \frac{6.505382}{2} \times \sqrt{\frac{3}{7}}$$

$$I_{T\ RMS} = 2.129386A$$

3.9 Type of Ac Voltage Controllers

The ac voltage controllers are classified into two types based on the type of input ac supply applied to the circuit.

- Single Phase AC Controllers.
- Three Phase AC Controllers.

Single phase ac controllers operate with single phase ac supply voltage of 230V RMS at 50Hz in our country. Three phase ac controllers operate with 3 phase ac supply of 400V RMS at 50Hz supply frequency.

Each type of controller may be sub divided into

- Uni-directional or half wave ac controller.
- Bi-directional or full wave ac controller.

In brief different types of ac voltage controllers are

- Single phase half wave ac voltage controller (uni-directional controller).
- Single phase full wave ac voltage controller (bi-directional controller).
- Three phase half wave ac voltage controller (uni-directional controller).
- Three phase full wave ac voltage controller (bi-directional controller).

3.9.1 Applications of Ac Voltage Controllers

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Industrial heating & Domestic heating.
- Transformer tap changing (on load transformer tap changing).
- Speed control of induction motors (single phase and poly phase ac induction motor control).
- AC magnet controls.

3.10 Principle of AC Phase Control

The basic principle of ac phase control technique is explained with reference to a single phase half wave ac voltage controller (unidirectional controller) circuit shown in the below figure.

The half wave ac controller uses one thyristor and one diode connected in parallel across each other in opposite direction that is anode of thyristor T_1 is connected to the cathode of diode D_1 and the cathode of T_1 is connected to the anode of D_1. The output voltage across the load resistor 'R' and hence the ac power flow to the load is controlled by varying the trigger angle 'α'.

The trigger angle or the delay angle 'α' refers to the value of ωt or the instant at which the thyristor T_1 is triggered to turn it ON, by applying a suitable gate trigger pulse between the gate and cathode lead.

The thyristor T_1 is forward biased during the positive half cycle of input ac supply. It can be triggered and made to conduct by applying a suitable gate trigger pulse only during the positive half cycle of input supply. When T_1 is triggered it conducts and the load current flows through the thyristor T_1, the load and through the transformer secondary winding.

By assuming T_1 as an ideal thyristor switch it can be considered as a closed switch when it is ON during the period $\omega t = \alpha$ to π radians. The output voltage across the load follows the input supply voltage when the thyristor T_1 is turned-on and when it conducts from $\omega t = \alpha$ to π radians. When the input supply voltage decreases to zero at $\omega t = \pi$, for a resistive load the load current also falls to zero at $\omega t = \pi$ and hence the thyristor T_1 turns off at $\omega t = \pi$. Between the time period $\omega t = \pi$ to 2π, when the supply voltage reverses and becomes negative the diode D_1 becomes forward biased and hence turns ON and conducts. The load current flows in the opposite direction during $\omega t = \pi$ to 2π radians when D_1 is ON and the output voltage follows the negative half cycle of input supply.

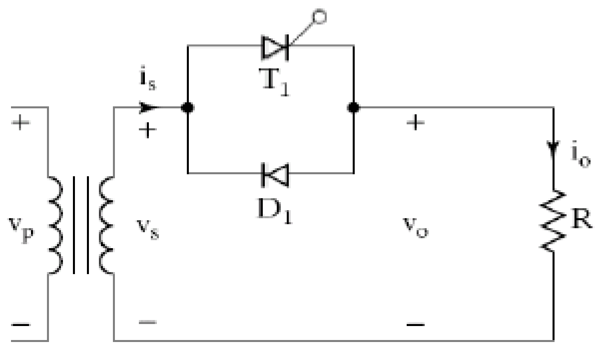


Fig 3.9 Halfwave AC phase controller (Unidirectional Controller)

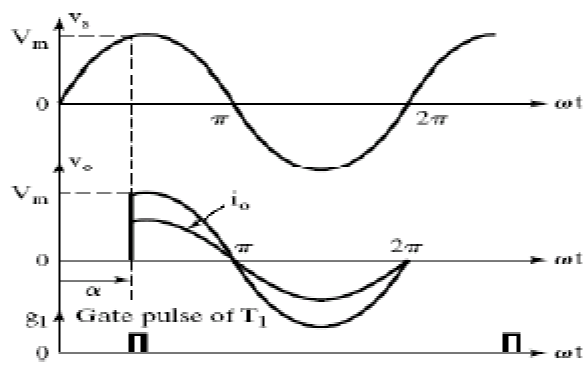


Fig 3.10 Output Waveforms

Equations Input AC Supply Voltage across the Transformer Secondary Winding.

$$v_s = V_m \sin \omega t$$

$$V_S = V_{in\ RMS} = \frac{V_m}{\sqrt{2}}$$ = RMS value of secondary supply voltage.

Output Load Voltage

$$v_o = v_L = 0\ ;\ \text{for}\ \omega t = 0\ \text{to}\ \alpha$$

$$v_o = v_L = V_m \sin \omega t\ ;\ \text{for}\ \omega t = \alpha\ \text{to}\ 2\pi\ .$$

Output Load Current

$$i_o = i_L = \frac{v_o}{R_L} = \frac{V_m \sin \omega t}{R_L}\ ;\ \text{for}\ \omega t = \alpha\ \text{to}\ 2\pi\ .$$

$$i_o = i_L = 0\ ;\ \text{for}\ \omega t = 0\ \text{to}\ \alpha\ .$$

(i) To Derive an Expression for rms Output Voltage $V_{O\ RMS}$.

$$V_{O\ RMS} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m^{\,2}\sin^2\omega t.d\ \ \omega t\ \right]}$$

$$V_{O\ RMS} = \sqrt{\frac{V_m^{\,2}}{2\pi}\left[\int_{\alpha}^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right).d\ \ \omega t\ \right]}$$

$$V_{O\ RMS} = \sqrt{\frac{V_m^{\,2}}{4\pi}\left[\int_{\alpha}^{2\pi} 1-\cos 2\omega t\ .d\ \ \omega t\ \right]}$$

$$V_{O\ RMS} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\int_{\alpha}^{2\pi} d\ \ \omega t\ -\int_{\alpha}^{2\pi}\cos 2\omega t.d\omega t\right]}$$

$$V_{O\ RMS} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\ \omega t\ \Big/_{\alpha}^{2\pi}-\left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}\right]}$$

$$V_{O\ RMS} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\ 2\pi-\alpha\ -\left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{2\pi}}$$

$$V_{O\ RMS} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\ 2\pi-\alpha\ -\left\{\frac{\sin 4\pi}{2}-\frac{\sin 2\alpha}{2}\right\}} \qquad ;\sin 4\pi = 0$$

$$V_{O\ RMS} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\ 2\pi-\alpha\ +\frac{\sin 2\alpha}{2}}$$

$$V_{O\ RMS} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\ 2\pi-\alpha\ +\frac{\sin 2\alpha}{2}}$$

$$V_{O\ RMS} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\ 2\pi-\alpha\ +\frac{\sin 2\alpha}{2}\right]}$$

$$V_{O\ RMS} = V_{i\ RMS}\sqrt{\frac{1}{2\pi}\left[\ 2\pi-\alpha\ +\frac{\sin 2\alpha}{2}\right]}$$

$$V_{O\,RMS} = V_S\sqrt{\frac{1}{2\pi}\left[\ 2\pi - \alpha\ + \frac{\sin 2\alpha}{2}\right]}$$

Where, $V_{i\,RMS} = V_S = \frac{V_m}{\sqrt{2}}$ = RMS value of input supply voltage (across the transformer secondary winding).

(ii) To Calculate the Average Value (Dc Value) Of Output Voltage

$$V_{O\,dc} = \frac{1}{2\pi}\int_{\alpha}^{2\pi} V_m \sin\omega t.d\ \ \omega t$$

$$V_{O\,dc} = \frac{V_m}{2\pi}\int_{\alpha}^{2\pi} \sin\omega t.d\ \ \omega t$$

$$V_{O\,dc} = \frac{V_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{2\pi}\right]$$

$$V_{O\,dc} = \frac{V_m}{2\pi}\ \ -\cos 2\pi + \cos\alpha \qquad ;\ \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi}\ \cos\alpha - 1 \qquad ;\ V_m = \sqrt{2}V_S$$

Hence $V_{dc} = \frac{\sqrt{2}V_S}{2\pi}\ \cos\alpha - 1$

When 'α' is varied from 0 to π. V_{dc} varies from 0 to $\frac{-V_m}{\pi}$

3.10.1 Disadvantages of single phase half wave ac voltage controller.

- The output load voltage has a DC component because the two halves of the output voltage waveform are not symmetrical with respect to '0' level. The input supply current waveform also has a DC component (average value) which can result in the problem of core saturation of the input supply transformer.
- The half wave ac voltage controller using a single thyristor and a single diode provides control on the thyristor only in one half cycle of the input supply. Hence ac power flow to the load can be controlled only in one half cycle.
- Half wave ac voltage controller gives limited range of RMS output voltage control.

These drawbacks of single phase half wave ac voltage controller can be over come by using a single phase full wave ac voltage controller.

3.10.2 Applications of rms Voltage Controller

- Speed control of induction motor (polyphase ac induction motor).

- Heater control circuits (industrial heating).
- Welding power control.
- Induction heating.
- On load transformer tap changing.
- Lighting control in ac circuits.
- Ac magnet controls.

3.11 Single Phase Full Wave Ac Voltage Controller (Ac Regulator) or Rms Voltage Controller with Resistive Load

Single phase full wave ac voltage controller circuit using two SCRs or a single triac is generally used in most of the ac control applications. The ac power flow to the load can be controlled in both the half cycles by varying the trigger angle"**.**

The RMS value of load voltage can be varied by varying the trigger angle ' '**.** The input supply current is alternating in the case of a full wave ac voltage controller and due to the symmetrical nature of the input supply current waveform there is no dc component of input supply current i.e., the average value of the input supply current is zero.

A single phase full wave ac voltage controller with a resistive load is shown in the figure below. It is possible to control the ac power flow to the load in both the half cycles by adjusting the trigger angle' '. Hence the full wave ac voltage controller is also referred to as to a bi-directional controller.

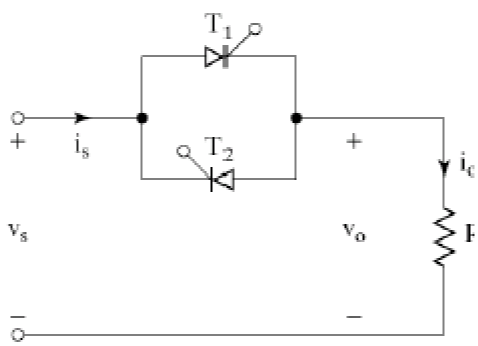


Fig 3.10 Single phase full wave ac voltage controller (Bi-directional Controller) using SCRs

The thyristor T_1 is forward biased during the positive half cycle of the input supply voltage. The thyristor T_1 is triggered at a delay angle of 'α' $0 \le \alpha \le \pi$ radians . Considering the ON thyristor T_1 as an ideal closed switch the input supply voltage appears across the load resistor R_L and the output voltage $v_O = v_S$ during $\omega t = \alpha$ to π radians. The load current flows through the ON thyristor T_1 and through the load resistor R_L in the downward direction during the conduction time of T_1 from $\omega t = \alpha$ to π radians.

At $\omega t = \pi$, when the input voltage falls to zero the thyristor current (which is flowing through the load resistor R_L) falls to zero and hence T_1 naturally turns off . No current flows in the circuit during $\omega t = \pi$ to $\pi + \alpha$.

The thyristor T_2 is forward biased during the negative cycle of input supply and when thyristor T_2 is triggered at a delay angle $\pi + \alpha$, the output voltage follows the negative halfcycle of input from $\omega t = \pi + \alpha$ to 2π . When T_2 is ON, the load current flows in the reverse direction (upward direction) through T_2 during $\omega t = \pi + \alpha$ to 2π radians. The time interval (spacing) between the gate trigger pulses of T_1 and T_2 is kept at π radians or 180^0. At $\omega t = 2\pi$ the input supply voltage falls to zero and hence the load current also falls to zero and thyristor T_2 turn off naturally.

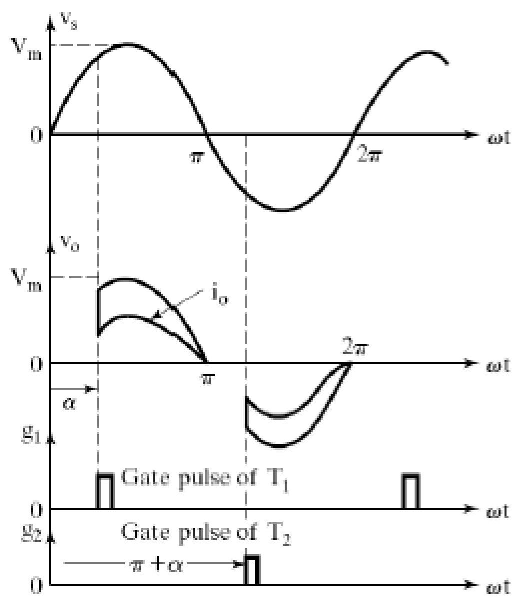


Fig 3.11 Waveforms of single phase full wave ac voltage controller

Equations

Input supply voltage

$$v_S = V_m \sin \omega t = \sqrt{2} V_S \sin \omega t \ ;$$

Output voltage across the load resistor R_L ;

$$v_O = v_L = V_m \sin \omega t \ ;$$

for $\omega t = \alpha$ to π and $\omega t = \pi + \alpha$ to 2π

Output load current

$$i_O = \frac{v_O}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t \ ;$$

for $\omega t = \alpha$ to π and $\omega t = \pi + \alpha$ to 2π

(i) To Derive an Expression for the Rms Value of Output (Load) Voltage

The RMS value of output voltage (load voltage) can be found using the expression

$$V^2_{O\,RMS} = V^2_{L\,RMS} = \frac{1}{2\pi}\int_0^{2\pi} v_L^2\, d\left(\omega t\right);$$

For a full wave ac voltage controller, we can see that the two half cycles of output voltage waveforms are symmetrical and the output pulse time period (or output pulse repetition time) is π radians. Hence we can also calculate the RMS output voltage by using the expression given below.

$$V^2_{L\,RMS} = \frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t.d\omega t$$

$$V^2_{L\,RMS} = \frac{1}{2\pi}\int_0^{2\pi} v_L^2.d\left(\omega t\right);$$

$v_L = v_O = V_m \sin\omega t$; For $\omega t = \alpha$ to π and $\omega t = \left(\pi+\alpha\right)$ to 2π

Hence,

$$V^2_{L\,RMS} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi}\left(V_m \sin\omega t\right)^2 d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}\left(V_m \sin\omega t\right)^2 d\left(\omega t\right)\right]$$

$$= \frac{1}{2\pi}\left[V_m^2\int_{\alpha}^{\pi}\sin^2\omega t.d\left(\omega t\right) + V_m^2\int_{\pi+\alpha}^{2\pi}\sin^2\omega t.d\left(\omega t\right)\right]$$

$$= \frac{V_m^2}{2\pi}\left[\int_{\alpha}^{\pi}\frac{1-\cos 2\omega t}{2}d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}\frac{1-\cos 2\omega t}{2}d\left(\omega t\right)\right]$$

$$= \frac{V_m^2}{2\pi\times 2}\left[\int_{\alpha}^{\pi}d\left(\omega t\right) - \int_{\alpha}^{\pi}\cos 2\omega t.d\left(\omega t\right) + \int_{\pi+\alpha}^{2\pi}d\left(\omega t\right) - \int_{\pi+\alpha}^{2\pi}\cos 2\omega t.d\left(\omega t\right)\right]$$

$$= \frac{V_m^2}{4\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi} + \left(\omega t\right)\Big/_{\pi+\alpha}^{2\pi} - \left[\frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi} - \left[\frac{\sin 2\omega t}{2}\right]_{\pi+\alpha}^{2\pi}\right]$$

$$= \frac{V_m^2}{4\pi}\left[\left(\pi-\alpha\right) + \left(\pi-\alpha\right) - \frac{1}{2}\left(\sin 2\pi - \sin 2\alpha\right) - \frac{1}{2}\left(\sin 4\pi - \sin 2\left(\pi+\alpha\right)\right)\right]$$

$$= \frac{V_m^2}{4\pi}\left[2\left(\pi-\alpha\right) - \frac{1}{2}\left(0 - \sin 2\alpha\right) - \frac{1}{2}\left(0 - \sin 2\left(\pi+\alpha\right)\right)\right]$$

$$= \frac{V_m^2}{4\pi}\left[2\ \pi-\alpha\ +\frac{\sin 2\alpha}{2}+\frac{\sin 2\ \ \pi+\alpha}{2}\right]$$

$$= \frac{V_m^2}{4\pi}\left[2\ \pi-\alpha\ +\frac{\sin 2\alpha}{2}+\frac{\sin 2\ \ \pi+\alpha}{2}\right]$$

$$= \frac{V_m^2}{4\pi}\left[2\ \pi-\alpha\ +\frac{\sin 2\alpha}{2}+\frac{\sin\ \ 2\pi+2\alpha}{2}\right]$$

$$= \frac{V_m^2}{4\pi}\left[2\ \pi-\alpha\ +\frac{\sin 2\alpha}{2}+\frac{1}{2}\ \ \sin 2\pi.\cos 2\alpha+\cos 2\pi.\sin 2\alpha\ \right]$$

$$\sin 2\pi = 0 \quad \& \quad \cos 2\pi = 1$$

Therefore,

$$V_{L\ RMS}^2 = \frac{V_m^2}{4\pi}\left[2\ \pi-\alpha\ +\frac{\sin 2\alpha}{2}+\frac{\sin 2\alpha}{2}\right]$$

$$= \frac{V_m^2}{4\pi}\left[2\ \pi-\alpha\ +\sin 2\alpha\right]$$

Taking the square root, we get

$$V_{L\ RMS} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\ 2\pi-2\alpha\ +\sin 2\alpha\right]}$$

$$V_{L\ RMS} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\left[\ 2\pi-2\alpha\ +\sin 2\alpha\right]}$$

$$V_{L\ RMS} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\ 2\pi-2\alpha\ +\sin 2\alpha\right]}$$

$$V_{L\ RMS} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[2\left\{\ \pi-\alpha\ +\frac{\sin 2\alpha}{2}\right\}\right]}$$

$$V_{L\ RMS} = V_S\sqrt{\frac{1}{\pi}\left[\ \pi-\alpha\ +\frac{\sin 2\alpha}{2}\right]}$$

Maximum RMS voltage will be applied to the load when $\alpha = 0$, in that case the full sine wave appears across the load. RMS load voltage will be the same as the RMS supply voltage $= \frac{V_m}{\sqrt{2}}$. When α is increased the RMS load voltage decreases.

$$V_{L\ RMS}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[\pi - 0 + \frac{\sin 2\times 0}{2}\right]}$$

$$V_{L\ RMS}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[\pi + \frac{0}{2}\right]}$$

$$V_{L\ RMS}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i\ RMS} = V_S$$

The output control characteristic for a single phase full wave ac voltage controller with resistive load can be obtained by plotting the equation for $V_{O\ RMS}$

3.12 **Single Phase Full Wave Ac Voltage Controller (Bidirectional Controller) With RL Load**

In this section we will discuss the operation and performance of a single phase full wave ac voltage controller with RL load. In practice most of the loads are of RL type. For example if we consider a single phase full wave ac voltage controller controlling the speed of a single phase ac induction motor, the load which is the induction motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

A single phase full wave ac voltage controller circuit (bidirectional controller) with an RL load using two thyristors 1 *T* and 2 *T* (1 *T* and 2 *T* are two SCRs) connected in parallel is shown in the figure below. In place of two thyristors a single Triac can be used to implement a full wave ac controller, if a suitable Traic is available for the desired RMS load current and the RMS output voltage ratings.

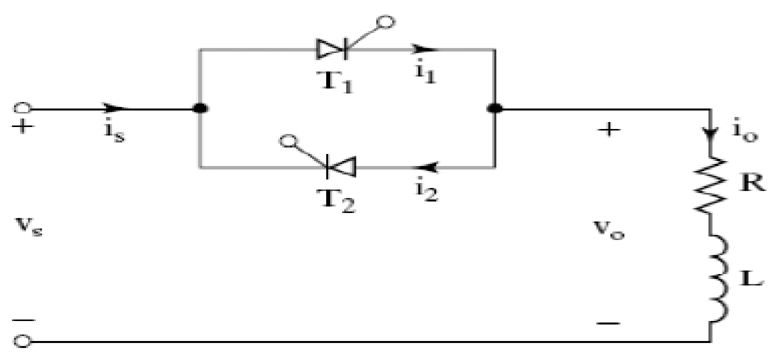


Fig 3.12 Single phase full wave ac voltage controller with RL load

The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_O flows through the thyristor T_1 and through the load in the downward direction. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_O flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = \beta - \alpha$, which depends on the delay angle α and the load impedance angle ϕ. The waveforms of the input supply voltage, the gate trigger pulses of T_1 and T_2, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.

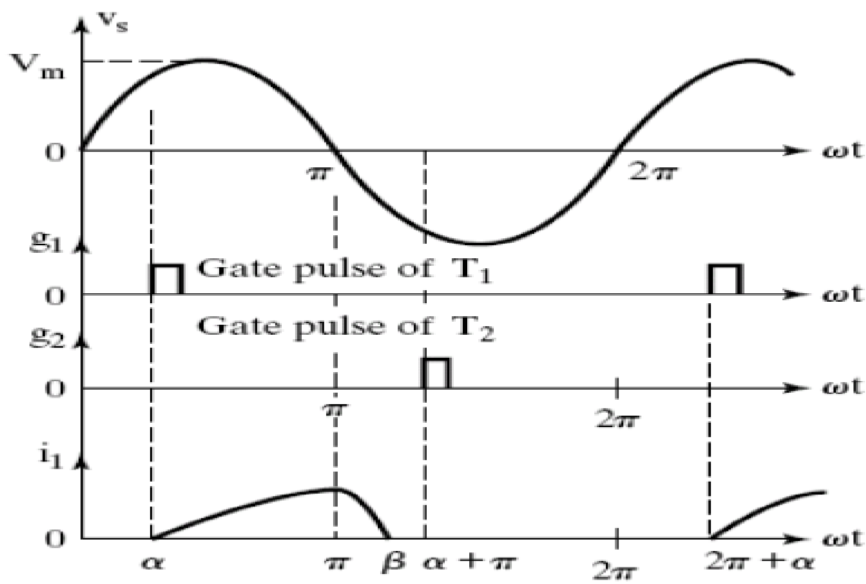


Fig 3.13 Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

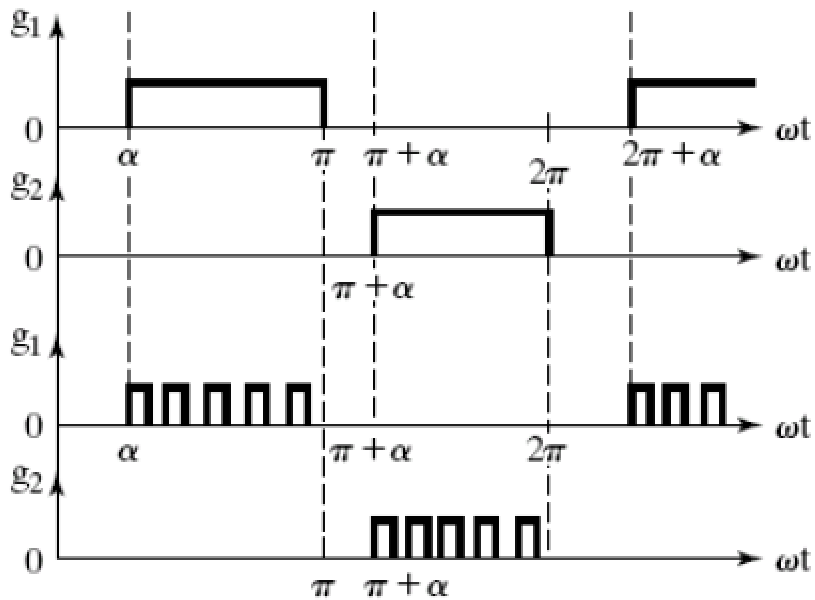
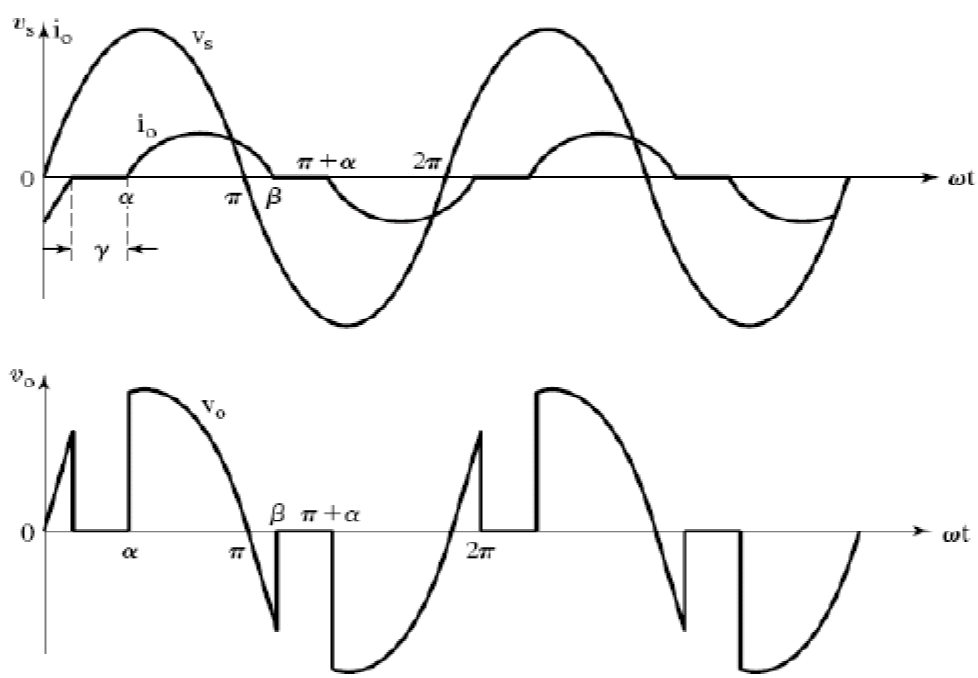


Fig 3.14 Gating Signals

Waveforms of single phase full wave ac voltage controller with RL load for $\alpha > \phi$. Discontinuous load current operation occurs for $\alpha > \phi$ and $\beta < \pi + \alpha$;

i.e., $\beta - \alpha < \pi$, conduction angle $< \pi$.



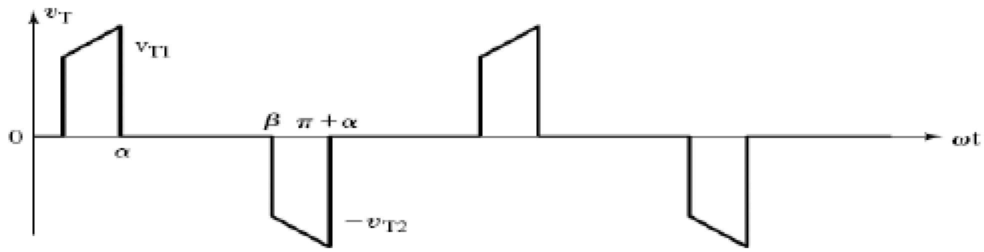


Fig 3.15 Waveforms of Input supply voltage, Load Current, Load Voltage and Thyristor Voltage across T $_1$

Extinction angle $\beta = \pi + \phi = \pi + \alpha$; for the case when $\alpha = \phi$

Conduction angle $\delta = \beta - \alpha = \pi$ radians $= 180^0$; for the case when $\alpha = \phi$

Each thyristor conducts for 180^0 (π radians) . T_1 conducts from $\omega t = \phi$ to $\pi + \phi$ and provides a positive load current. T_2 conducts from $\pi + \phi$ to $2\pi + \phi$ and provides a negative load current. Hence we obtain a continuous load current and the output voltage waveform appears as a continuous sine wave identical to the input supply voltage waveform for trigger angle $\alpha \leq \phi$ and the control on the output is lost.

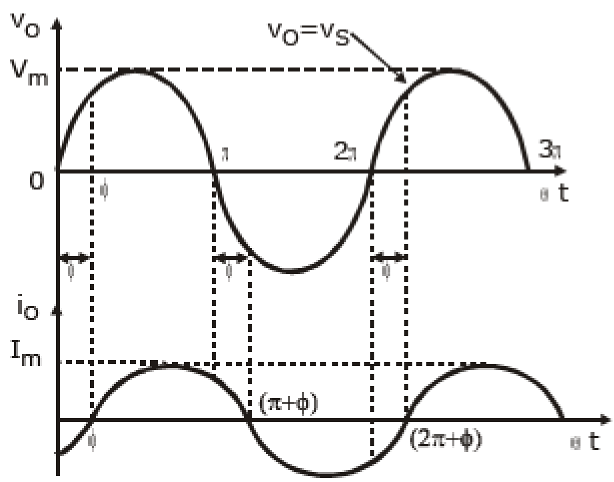


Fig 3.16 Output Voltage And Output Current Waveforms For A Single Phase Full Wave Ac Voltage Controller With Rl Load $\alpha \leq \phi$

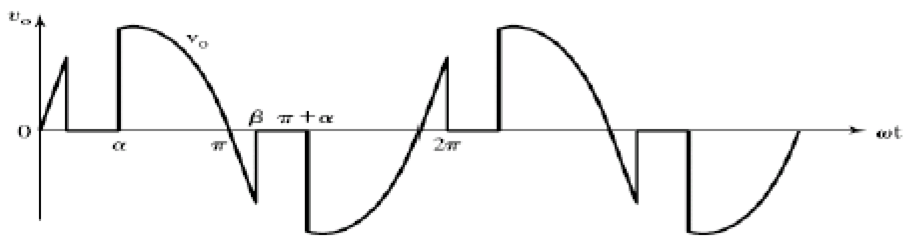
Thus we observe that for trigger angle, the load current tends to flow continuously and we have continuous load current operation, without any break in the load current waveform and we obtain output voltage waveform which is a continuous sinusoidal waveform identical to the input supply voltage waveform. We lose the control on the output voltage for $\alpha \leq \phi$ as the output voltage becomes equal to the input supply voltage and thus we obtain,

$$V_{O\,RMS} = \frac{V_m}{\sqrt{2}} = V_S \;\; ; \text{ for } \alpha \le \phi$$

Hence,

RMS output voltage = RMS input supply voltage for $\alpha \le \phi$

(ii) To Derive an Expression For rms Output Voltage $V_{O\,RMS}$ of a Single Phase Full-Wave Ac Voltage Controller with RL Load.



When $\alpha > \emptyset$, the load current and load voltage waveforms become discontinuous as shown in the figure above.

$$V_{O\,RMS} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d\left(\omega t\right)\right]^{\frac{1}{2}}$$

Output $v_o = V_m \sin\omega t$, for $\omega t = \alpha$ to β, when T_1 is ON.

$$V_{O\,RMS} = \left[\frac{V_m^2}{\pi}\int_{\alpha}^{\beta}\frac{1-\cos 2\omega t}{2} d\left(\omega t\right)\right]^{1/2}$$

$$V_{O\,RMS} = \left[\frac{V_m^2}{2\pi}\left\{\int_{\alpha}^{\beta} d\left(\omega t\right) - \int_{\alpha}^{\beta}\cos 2\omega t . d\left(\omega t\right)\right\}\right]^{1/2}$$

$$V_{O\,RMS} = \left[\frac{V_m^2}{2\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{\beta} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\beta}\right\}\right]^{1/2}$$

$$V_{O\,RMS} = \left[\frac{V_m^2}{2\pi}\left\{\left(\beta-\alpha\right) - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$V_{O\,RMS} = V_m\left[\frac{1}{2\pi}\left\{\left(\beta-\alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right\}\right]^{1/2}$$

$$V_{O\,RMS} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left\{\left(\beta-\alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right\}\right]^{1/2}$$

The RMS output voltage across the load can be varied by changing the trigger angle α.

For a purely resistive load $L = 0$, therefore load power factor angle $\phi = 0$.

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = 0 \ ;$$

Extinction angle $\beta = \pi$ radians $= 180^0$

Performance Parameters of A Single Phase Full Wave Ac Voltage Controller with Resistive Load

- **RMS Output Voltage** $V_{O\ RMS} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]}$; $\frac{V_m}{\sqrt{2}} = V_S$ = RMS input supply voltage.

- $I_{O\ RMS} = \frac{V_{O\ RMS}}{R_L}$ = RMS value of load current.

- $I_S = I_{O\ RMS}$ = RMS value of input supply current.

- **Output load power**

$$P_O = I_{O\ RMS}^2 \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{V_S \times I_S} = \frac{I_{O\ RMS}^2 \times R_L}{V_S \times I_{O\ RMS}} = \frac{I_{O\ RMS} \times R_L}{V_S}$$

$$PF = \frac{V_{O\ RMS}}{V_S} = \sqrt{\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]}$$

- **Average Thyristor Current,**

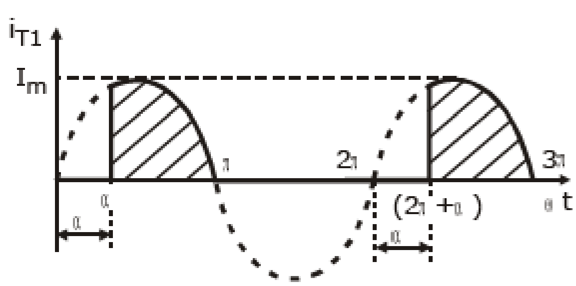


Fig 3.17 Thyristor Current Waveform

$$I_{T\ Avg} = \frac{1}{2\pi}\int_{\alpha}^{\pi} i_T\, d\ \omega t = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t . d\ \omega t$$

$$I_{T\,Avg} = \frac{I_m}{2\pi}\int_{\alpha}^{\pi}\sin\omega t.d\ \omega t\ = \frac{I_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{\pi}\right]$$

$$I_{T\,Avg} = \frac{I_m}{2\pi}\ -\cos\pi + \cos\alpha\ = \frac{I_m}{2\pi}\ 1+\cos\alpha$$

- **Maximum Average Thyristor Current, for** $\alpha = 0$,

$$I_{T\,Avg} = \frac{I_m}{\pi}$$

- **RMS Thyristor Current**

$$I_{T\,RMS} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t.d\ \omega t\ \right]}$$

$$I_{T\,RMS} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\ \pi-\alpha\ + \frac{\sin 2\alpha}{2}\right]}$$

- **Maximum RMS Thyristor Current, for** $\alpha = 0$,

$$I_{T\,RMS} = \frac{I_m}{2}$$

In the case of a single phase full wave ac voltage controller circuit using a Triac with resistive load, the average thyristor current $I_{T\,Avg} = 0$. Because the Triac conducts in both the half cycles and the thyristor current is alternating and we obtain a symmetrical thyristor current waveform which gives an average value of zero on integration.

3.13 PROBLEMS

1. A single phase full wave controller has an input voltage of 120 V (RMS) and a load resistance of 6 ohm. The firing angle of thyristor is 90^o. Find RMS output voltage, Power output ,Input power factor, Average and RMS thyristor current.

Solution

$$\alpha = \frac{\pi}{2} = 90^0, \quad V_S = 120\text{ V}, \quad R = 6\Omega$$

RMS Value of Output Voltage

$$V_O = V_S\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_O = 120\left[\frac{1}{\pi}\left(\pi-\frac{\pi}{2}+\frac{\sin 180}{2}\right)\right]^{\frac{1}{2}}$$

$$V_O = 84.85\text{ Volts}$$

RMS Output Current

$$I_O = \frac{V_O}{R} = \frac{84.85}{6} = 14.14 \text{ A}$$

Load Power

$$P_O = I_O^2 \times R$$

$$P_O = 14.14^2 \times 6 = 1200 \text{ watts}$$

Input Current is same as Load Current

Therefore $I_S = I_O = 14.14$ Amps

Input Supply Volt-Amp $= V_S I_S = 120 \times 14.14 = 1696.8\ VA$

Therefore

$$\text{Input Power Factor} = \frac{\text{Load Power}}{\text{Input Volt-Amp}} = \frac{1200}{1696.8} = 0.707\ \ lag$$

Each Thyristor Conducts only for half a cycle

Average thyristor current $I_{T\ Avg}$

$$I_{T\ Avg} = \frac{1}{2\pi R}\int_{\alpha}^{\pi} V_m \sin\omega t . d\ \ \omega t$$

$$= \frac{V_m}{2\pi R}\ 1 + \cos\alpha \quad ; \quad V_m = \sqrt{2} V_S$$

$$= \frac{\sqrt{2} \times 120}{2\pi \times 6}\ 1 + \cos 90\ = 4.5 \text{ A}$$

RMS thyristor current $I_{T\ RMS}$

$$I_{T\ RMS} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}\frac{V_m^2 \sin^2\omega t}{R^2} d\ \ \omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi R^2}\int_{\alpha}^{\pi}\frac{1 - \cos 2\omega t}{2} d\ \ \omega t}$$

$$= \frac{V_m}{2R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2} V_s}{2R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2} \times 120}{2 \times 6}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin 180}{2}\right)\right]^{\frac{1}{2}} = 10 \text{ Amps}$$

2. A single phase half wave ac regulator using one SCR in anti-parallel with a diode feeds 1 kW, 230 V heater. Find load power for a firing angle of 45^0.

Solution

$$\alpha = 45^0 = \frac{\pi}{4}, \quad V_S = 230 \text{ V} \; ; \; P_O = 1KW = 1000W$$

At standard rms supply voltage of 230V, the heater dissipates 1KW of output power

Therefore

$$P_O = V_O \times I_O = \frac{V_O \times V_O}{R} = \frac{V_O^2}{R}$$

Resistance of heater

$$R = \frac{V_O^2}{P_O} = \frac{230^{\;2}}{1000} = 52.9\Omega$$

RMS value of output voltage

$$V_O = V_S\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}} \qquad ; \text{for firing angle } \alpha = 45^0$$

$$V_O = 230\left[\frac{1}{2\pi}\left(2\pi - \frac{\pi}{4} + \frac{\sin 90}{2}\right)\right]^{\frac{1}{2}} = 224.7157 \text{ Volts}$$

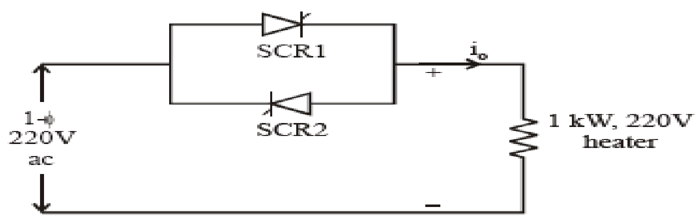
RMS value of output current

$$I_O = \frac{V_O}{R} = \frac{224.9}{52.9} = 4.2479 \text{ Amps}$$

Load Power

$$P_O = I_O^2 \times R = \; 4.25^{\;2} \times 52.9 = 954.56 \text{ Watts}$$

3. Find the RMS and average current flowing through the heater shown in figure. The delay angle of both the SCRs is 45^0.



Solution

$$\alpha = 45^0 = \frac{\pi}{4}, \quad V_s = 220 \text{ V}$$

Resistance of heater

$$R = \frac{V^2}{R} = \frac{220^{\ 2}}{1000} = 48.4\Omega$$

Resistance value of output voltage

$$V_O = V_S\sqrt{\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]}$$

$$V_O = 220\sqrt{\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{4} + \frac{\sin 90}{2}\right)\right]}$$

$$V_O = 220\sqrt{\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{4} + \frac{1}{2}\right)\right]} = 209.769 \text{ Volts}$$

RMS current flowing through heater $= \frac{V_O}{R} = \frac{209.769}{48.4} = 4.334$ Amps

Average current flowing through the heater $I_{Avg} = 0$

3.14 CYCLOCONVERTER

3.14.1 Basic Principle of Operation

The basic principle of operation of a cyclo-converter is explained with reference to an equivalent circuit shown in Fig. . Each two-quadrant converter (phase-controlled) is represented as an alternating voltage source, which corresponds to the fundamental voltage component obtained at its output terminals. The diodes connected in series with each voltage source, show the unidirectional conduction of each converter, whose output voltage can be either positive or negative, being a two-quadrant one, but the direction of current is in the direction as shown in the circuit, as only thyristors − unidirectional switching devices, are used in the two converters.Normally, the ripple content in the output voltage is neglected.

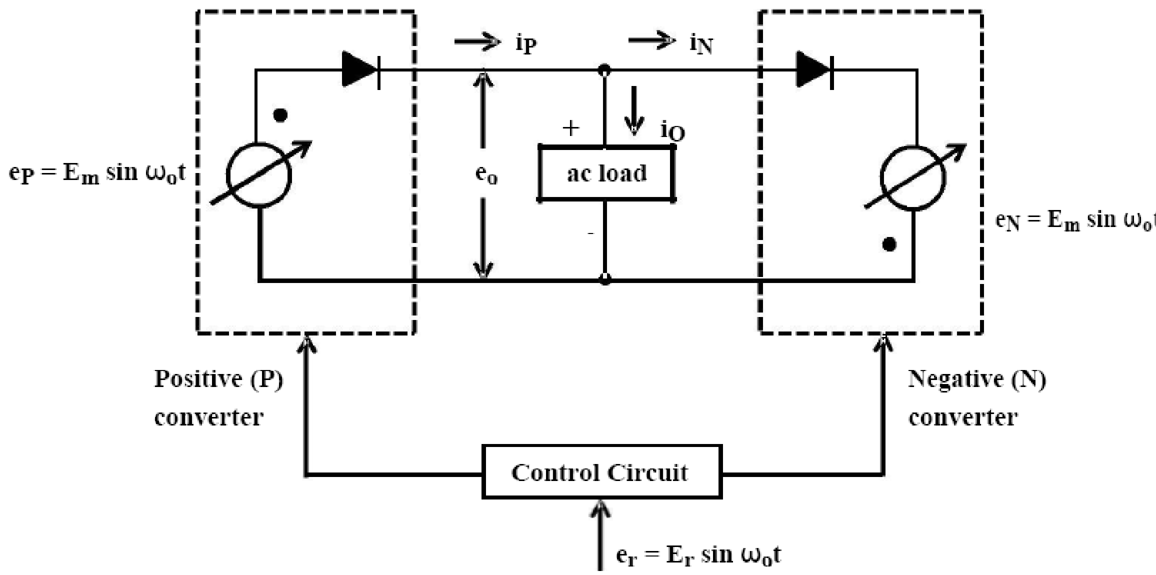


Fig 3.18 Equivalent circuit of cyclovonverter

The control principle used in an ideal cyclo-converter is to continuously modulate the firing angles of the individual converters, so that each produces the same sinusoidal (ac) voltage at its output terminals. Thus, the voltages of the two generators have the same amplitude, frequency and phase, and the voltage of the cyclo-converter is equal to the voltage of either of these generators. It is possible for the mean power toflow either ‘to’ or ‘from’ the output terminals, and the cyclo-converter is inherently capable of operation with loads of any phase angle − inductive or capacitive. Because of the uni-directional current carrying property of the individual converters, it is inherent that the positive half-cycle of load current must always be carried by the positive converter, and the negative half-cycle by the negative converter, regardless of the phase of the current with respect to the voltage. This means that each two-quadrant converter operates both in its rectifying (converting) and in its inverting region during the period of its associated half-cycle of current.

3.15 Single-phase to Single-phase Cyclo-converter

The circuit of a single-phase to single-phase cyclo-converter is shown in Fig. . Two full-wave fully controlled bridge converter circuits, using four thyristors for each bridge, are connected in opposite direction (back to back), with both bridges being fed from ac supply (50 Hz). Bridge 1 (P – positive) supplies load current in the positive half of the output cycle, while bridge 2 (N – negative) supplies load current in the negative half. The two bridges should not conduct together as this will produce short -circuit at the input. In this case, two thyristors come in series with each voltage source. When the load current is positive, the firing pulses to the thyristors of bridge 2 are inhibited, while the thyristors of bridge 1 are triggered by giving pulses at their gates at that time. Similarly, when the load current is negative, the thyristors of bridge 2 are triggered by giving pulses at their gates, while the firing pulses to the thyristors of bridge 1 are inhibited at that time. This is the circulating-current free mode of operation. Thus, the firing angle control scheme must be such that only one converter conduct at a time, and the change over of firing pulses from one converter to the other, should be periodic according to the output frequency. However, the firing angles the thyristors in both converters should be the same to produce a symmetrical output.

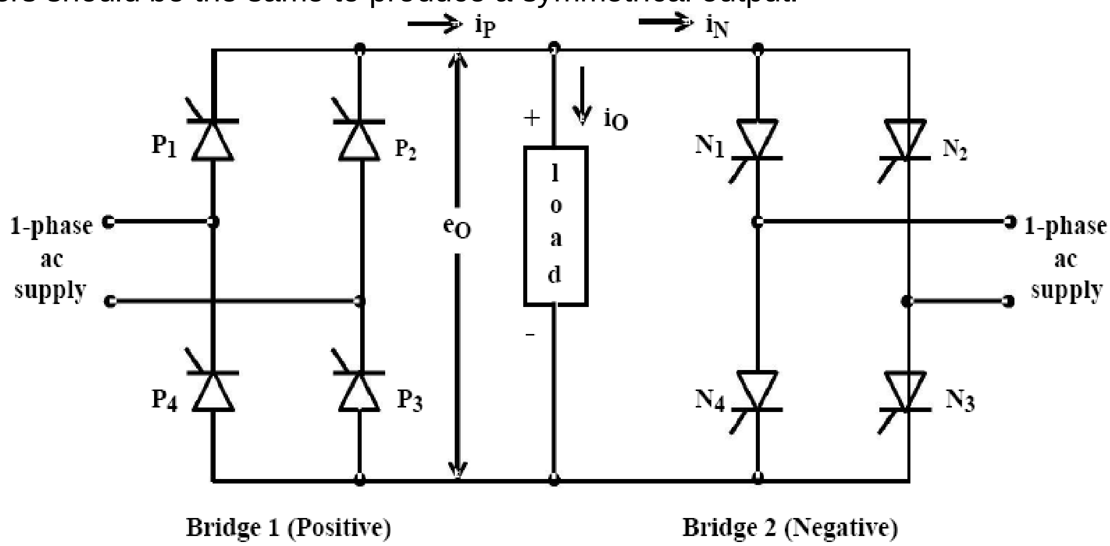


Fig 3.19 Single-phase to single-phase cycloconverter (using thyristor bridges)

When a cyclo-converter operates in the non-circulating current mode, the control scheme is
complicated, if the load current is discontinuous. The control is somewhat simplified, if some
amount of circulating current is allowed to flow between them. In this case, a circulating current
limiting reactor is connected between the positive and negative converters, as is the case with
dual converter, i.e. two fully controlled bridge converters connected back to back, in circulatingcurrent
mode. The readers are requested to refer to any standard text book. This circulating

current by itself keeps both converters in virtually continuous conduction over the whole control
range. This type of operation is termed as the circulating-current mode of operation. The operation of the cyclo-converter circuit with both purely resistive (R), and inductive (R-L) loads
is explained.

Resistive (R) Load: For this load, the load current (instantaneous) goes to zero, as the input
voltage at the end of each half cycle (both positive and negative) reaches zero (0). Thus, the conducting thyristor pair in one of the bridges turns off at that time, i.e. the thyristors undergo natural commutation. So, operation with discontinuous current takes place, as current flows in the load, only when the next thyristor pair in that bridge is triggered, or pulses are fed at respective gates. Taking first bridge 1 (positive), and assuming the top point of the ac supply as positive with the bottom point as negative in the positive half cycle of ac input, the oddnumbered thyristor pair, P1 & P3 is triggered after phase delay ($\alpha 1$), such that current starts flowing through the load in this half cycle. In the next (negative) half cycle, the other thyristor pair (even- numbered), P2 & P4 in that bridge conducts, by triggering them after suitable phase delay from the start of zero-crossing. The current flows through the load in the same direction, with the output voltage also remaining positive. This process continues for one more half cycle
(making a total of three) of input voltage ($f1$ =50 Hz). From three waveforms, one combined positive half cycle of output voltage is produced across the load resistance, with its frequency being one-third of input frequency ($f_2 = f_1 / 3 = 16^2$ Hz)following points may be noted.The firing angle (α) of the converter is first decreased, in this case for second cycle only, and then again increased in the next (third) cycle, as shown in Fig. This is, because only three cycles for each half cycle is used. If the output frequency needed is lower, the number of cycles is to be increased, with the firing angle decreasing for some cycles, and then again increasing in the subsequent cycles, as described earlier.

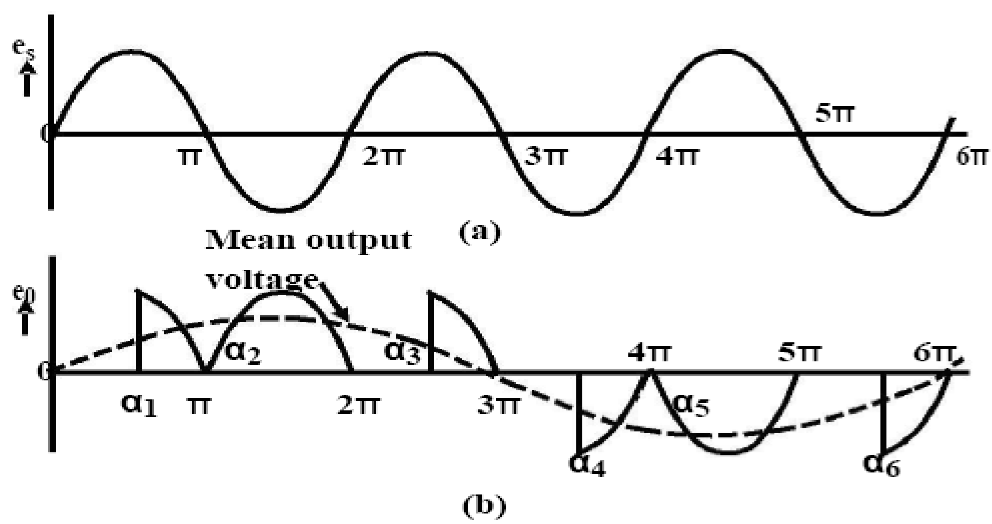


Fig 3.20 Input (a) and output (b) voltage waveforms of a cyclo-converterwith an output frequency of 16^2 Hz for resistive (R) load

To obtain negative output voltage, in the next three half cycles of input voltage, bridge 2 is used. Following same logic, if the bottom point of the ac supply is taken as positive with the top point as negative in the negative half of ac input, the odd-numbered thyristor pair, N1 & N3 conducts, by triggering them after suitable phase delay from the zero -crossing. Similarly, the even-numbered thyristor pair, N2 & N4 conducts in the next half cycle. Both the output voltage and current are now negative. As in the previous case, the above process also continues for three consecutive half cycles of input voltage. From three waveforms, one combined negative half cycle of output voltage is produced, having same frequency as given earlier. The pattern of firing angle − first decreasing and the increasing, is also followed in the negative half cycle. One positive half cycle, along with one negative half cycle, constitute one complete cycle of output (load) voltage waveform, its frequency being 16^2 Hz as stated earlier. The ripple frequency of the output voltage/ current for single–phase full-wave converter is 100 Hz, i.e., double of the input frequency. It may be noted that the load (output) current is discontinuous , as also load (output) voltage . The supply (input) voltage is shown in Fig 3.20 Only one of two thyristor bridges (positive or negative) conducts at a time, giving non-circulating current mode of operation in this circuit

3.15.1 Inductive (R-L) Load

For this load, the load current may be continuous or discontinuous depending on the firing angle and load power factor. The load voltage and current waveforms are shown for continuous and discontinuous load current in Fig 3.21

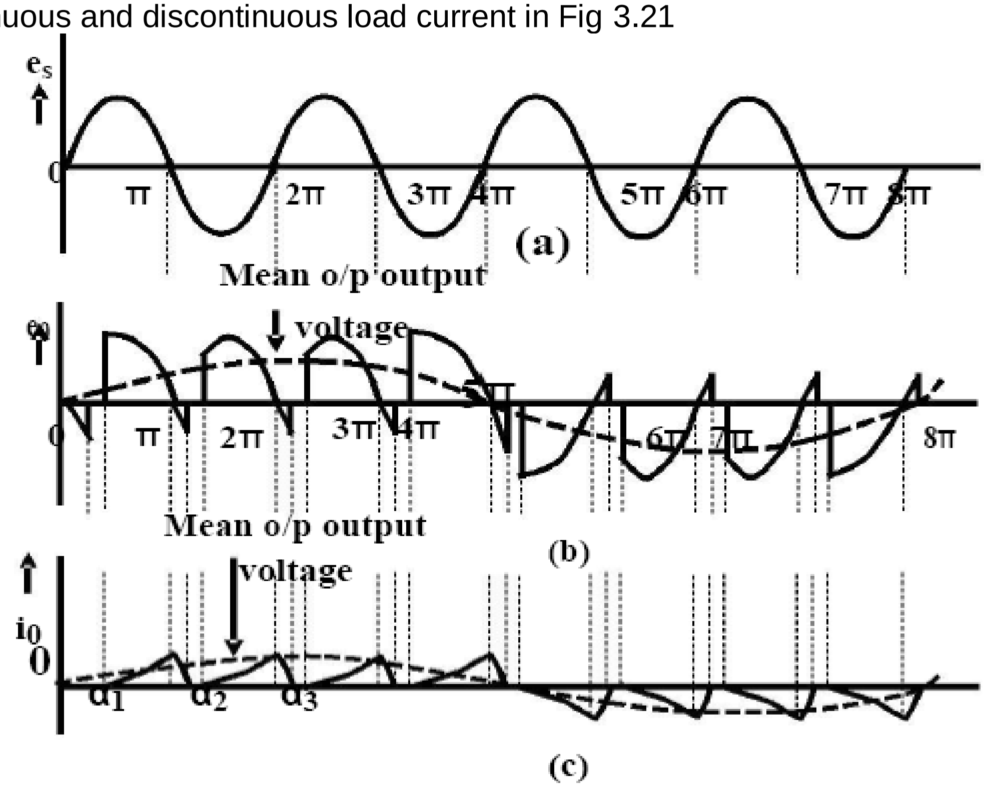


Fig 3.21 Input (a) and output (b) voltage, and current (c) waveforms for a cyclo-converter with discontinuous

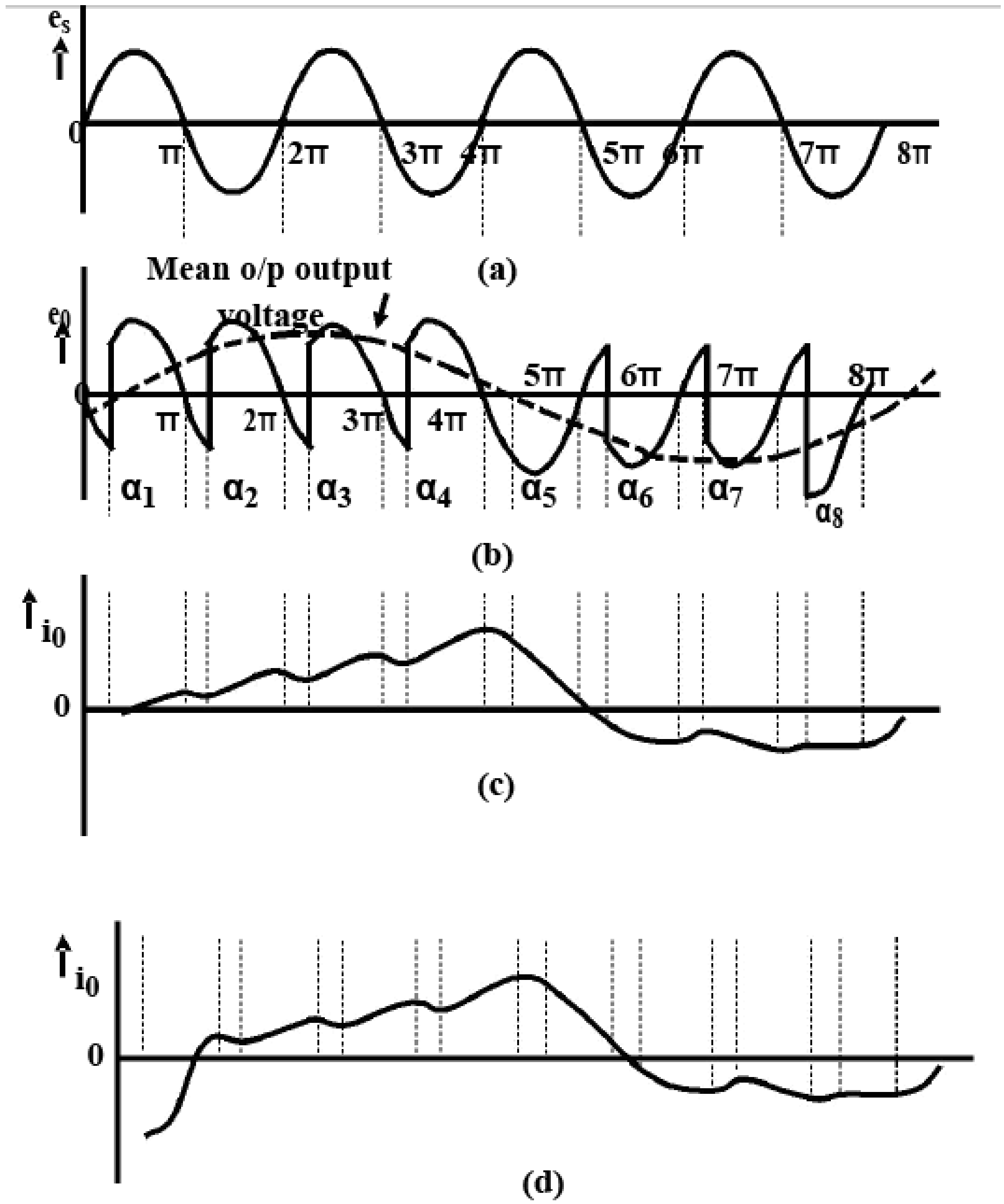


Fig 3.22 Input (a) and output (b) voltage, and current (c, d) waveforms for a cyclo-converter with continuous load current.

3.15.2 Discontinuous load current

The load current in this case is discontinuous, as the inductance, L in series with the resistance, R, is low. This is somewhat similar to the previous case, but difference also exists as described. Here, also non-circulating mode of operation takes place, with only one of the bridges − #1 (positive), or #2 (negative), conducting at a time, but two bridges do not conduct at the same time, as this will result in a short circuit. In this case, the output frequency is assumed as (f_2 =12.5 Hz), the input frequency being same as

(f_1 =50 Hz), i.e., $f_1 = 4 \cdot f_2$, or $f_2 = f_1 / 4$ .So, four positive half cycles, or two full cycles of the input to the full-wave bridge converter (#1), are required to produce one positive half cycle of the output waveform, as the output frequency is one-fourth of the input frequency as given earlier. As in the previous case with resistive load, taking bridge 1, and assuming the top point of the ac supply as positive, in the positive half cycle of ac input, the odd-numbered thyristor pair, P1 & P3, is triggered after phase delay ($\theta = \omega t = \alpha 1$), such that current starts flowing the inductive load in this half cycle.But here, the current flows even after the input voltage has reversed (after $\theta = \pi$), till it reaches zero at ($\theta = \beta_1$) with $(\pi + \alpha_2) > \beta_1 > \pi$, due to inductance being present in series with resistance, its value being low. It may be noted that the thyristor pair is, thus, naturally commutated. In the next (negative) half cycle, the other thyristor pair (even-numbered), P_2 & P_4, is triggered at $(\pi + \alpha_2)$. The current flows through the load in the same direction, with the output voltage also remaining positive. The current goes to zero at $(\pi + \beta_2)$, with $(\pi + \alpha_3) > \beta_2 > \pi$. This procedure continues for the next two half cycles, making a total of four positive half cycles. From these four waveforms, one combined positive half cycle of output voltage is produced across the inductive load. The firing angle (α) of the converter is first decreased, in this case for second half cycle only, kept nearly same in the third one, and finally increased in the last (fourth) one, as shown in Fig. .

To obtain negative output voltage, in the next four half cycles of output voltage, bridge 2 is used. Following same logic, if the bottom point of the ac supply is taken as positive in the negative half of ac input, the odd-numbered thyristor pair, N_1 & N_3 conducts, by triggering them after phase delay ($\theta = 4 \cdot \pi + \alpha_1$). The current flows now in the opposite (negative) direction through the inductive load, with the output voltage being also negative. The current goes to zero at ($4 \cdot \pi + \beta_1$), due to load being inductive as given earlier. Similarly, the even-numbered thyristor pair, N_2 & N_4 conducts in the next half cycle, after they are triggered at ($5 \cdot \pi + \alpha_2$). The current goes to zero at ($5 \cdot \pi + \beta_2$). Both the output voltage and current are now negative. As in the previous case, the above process also continues for two more half cycles of input voltage, making a total of four. From these four waveforms, one combined negative half cycle of output voltage is produced with same output frequency. The pattern of firing angle − first decreasing and then increasing, is also followed in the negative half cycle. It may be noted that the load (output) current is discontinuous , as also load (output) voltage The supply (input) voltage is shown in Fig. . One positive half cycle, along with one negative half cycle, constitute one complete cycle of output (load) voltage waveform, its frequency being 12.5 Hz as stated earlier. The ripple frequency remains also same at 100 Hz, with the ripple in load current being filtered by the inductance present in the load.

3.15.3 Continuous load current

As given above, the load current is discontinuous, as the inductance of the load is low. If the inductance is increased, the current will be continuous. Most of the points given earlier are applicable to this case, as described. To repeat, non-circulating mode of operation is used, i.e., only one of the bridges − #1 (positive), or #2 (negative), conducts at a time, but two bridges do not conduct at the same time, as this will result in a short circuit. Also, the ripple frequency in the voltage and current waveforms remains same at

100 Hz. The output frequency is one-fourth of input frequency (50 Hz), i.e., 12.5 Hz. So, for each half-cycle of output voltage waveform, four half cycles of input supply are required. Taking bridge 1, and assuming the top point of the ac supply as positive, in the positive half cycle of ac input, the odd-numbered thyristor pair, P1 & P3, is triggered after phase delay ($\theta = \omega t = \alpha 1$), such that current starts flowing the inductive load in this half cycle. But here, the current flows for about one complete half cycle, i.e., up to the angle, ($\pi + \alpha 1$) or ($\pi + \alpha 2$), whichever is higher, even after the input voltage has reversed, due to the high value of load inductance. In the next (negative) half cycle, the other thyristor pair (even numbered), P2 & P4, is triggered at ($\pi + \alpha\ 2$) . At that time, reverse voltage is applied across each of the conducting thyristors, P1/P3, and the thyristors turn off. The current flows through the load in the same direction, with the output voltage also remaining positive. Also, the current flows for about one complete half cycle, i.e., up to the angle, ($\pi + \alpha 2$) or ($\pi + \alpha 3$), whichever is higher. This procedure continues for the next two half cycles, making a total of four positive half cycles. From these four waveforms, one combined positive half cycle of output voltage is produced across the inductive load. The firing angle (α) of the converter is first decreased, in this case for second half cycle only, kept nearly same in the third one, and finally increased in the last (fourth) one, as shown in Fig. .To obtain negative output voltage, in the next four half cycles of output voltage, bridge 2 is used. Following same logic, if the bottom point of the ac supply is taken as positive in the negative half of ac input, the odd-numbered thyristor pair, N1 & N3 conducts, by triggering them after phase delay ($\theta = 4 \cdot \pi + \alpha 1$). The current flows now in the opposite (negative) direction through the inductive load, with the output voltage being also negative. The current flows for about one complete half cycle, i.e., up to the angle, ($5 \cdot .\pi + \alpha 1$) or ($5 \cdot \pi + \alpha 2$), whichever is higher, as the load is inductive. Similarly, the even-numbered thyristor pair, N2 & N4 conducts in the next half cycle, after they are triggered at ($5 \cdot \pi + \alpha 2$). As described earlier, both the conducting thyristors turn off, as reverse voltage is applied across each of them. Both the output voltage and current are now negative. Also, the current flows for about one complete half cycle, i.e. up to the angle, ($5 \cdot \pi + \alpha 2$) or ($5 \cdot \pi + \alpha 3$), whichever is higher. As in the previous case, the above process also ontinues for two more half cycles of input voltage, making a total of four. From these four waveforms, one combined negative half cycle of output voltage is produced with same output frequency of 12.5 Hz. The pattern of firing angle − first decreasing and then increasing, is also followed in the negative half cycle. It may be observed that the load (output) current is continuous (Fig. 29.6c), as also load (output) voltage (Fig. 29.6b). The load (output) current is redrawn in Fig. 29.6d, under steady state condition, while the supply (input) voltage is shown in Fig. 3.22. One positive half cycle, along with one negative half cycle, constitute one complete cycle of output (load) voltage waveform.

3.15.4 Advantages of Cyclo-converter

1. In a cyclo-converter, ac power at one frequency is converted directly to a lower frequency in a single conversion stage.
2. Cyclo-converter functions by means of phase commutation, without auxiliary forced commutation circuits. The power circuit is more compact, eliminating circuit losses

associated with forced commutation.
3. Cyclo-converter is inherently capable of power transfer in either direction between source and load. It can supply power to loads at any power factor, and is also capable of regeneration over the complete speed range, down to standstill. This feature makes it preferable for large reversing drives requiring rapid acceleration and deceleration, thus suited for metal rolling application.
4. Commutation failure causes a short circuit of ac supply. But, if an individual fuse blows off, a complete shutdown is not necessary, and cyclo-converter continues to function with somewhat distorted waveforms. A balanced load is presented to the ac supply with unbalanced output conditions.
5. Cyclo-converter delivers a high quality sinusoidal waveform at low output frequencies,since it is fabricated from a large number of segments of the supply waveform. This is often preferable for very low speed applications.
6. Cyclo-converter is extremely attractive for large power, low speed drives.