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UNIT -I

STEADY STRESSES AND VARIABLE STRESS IN MACHINE MEMBES

Introduction to the design process - factor influencing machine design, selection of materials based on mechanical properties – Direct, Bending and torsional stress equations – Impact and shock loading – calculation of principle stresses for various load combinations, eccentric loading – Design of curved beams – crane hook and 'C' frame - Factor of safety - theories of failure – stress concentration – design for variable loading – Soderberg, Goodman and Gerber relations

MACHINE DESIGN

Machine design is the creation of new and better machines and improving the existing one. A new or better machine is one which is more economical in the overall cost of production and operation.

CLASSIFICATION OF MACHINE DESIGN

- 1. Adaptive design: The designers work is concerned with adaptation of existing design.
- 2. Development design: This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea.
- 3. New design: This type of design needs a lot of research technical ability and designers and creative thinking.

GENERAL CONSIDERATION IN MACHINE DESIGN

- (i) Type of load and stresses caused by the load.
- (ii) Motion o parts
- (iii) Selection of materials
- (iv) Frictional resistance and lubrication
- (v) Convenient and economical features
- (vi) Safety of operation

FACTORS INFLUENCING MACHINE DESIGN

- (i) Strength and stiffness
- (ii) Surface finish and tolerances
- (iii) Manufacturability
- (iv) Ease of handling
- (v) Working atmosphere
- (vi) Cooling and lubrication
- (vii) Safety
- (viii) Noise requirement
- (ix) Cost

BENDING STRESS IN STRAIGHT BEAMS

Consider a straight beam subjected to a bending moment M as shown in figure. The following assumption are usually made delivering the bending formula

- (i) The material of the beam is perfectly homogeneous and isotropic.
- (ii) The material of the beam obeys Hooks law.
- (iii) The Young's modulus is same in tension and compression.
- (iv) The loads are applied in the plane of bending.



Figure 1.1 Bending Stress in Straight Beams

When a beam is subjected to the bending moment the fibers on the upper side of the beam will be compress and lower side elongate due to tension. The surface between the top and bottom fibers are neither shorten nor lengthened. Such a surface is called neutral surface. The intersection of the neutral surface with any normal cross section of the beam is known is neutral axis. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

M- Bending moment acting at the given section

 σ - bending stress

I- moment of inertia of the cross section about the neutral axis

y- Distance from the neutral axis to the extreme fiber

E- Youngs modulus of the material of the beam

R- Radius of curvature of the beam

BENDING STRESS IN CURVED BEAMS

In straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution is linear. But in curved beams the neutral axis of the cross section is shifted towards the centre of curvature of the beam causing a nonlinear distribution of stress. Application of curved beam principle is used in crane hooks, chain links planers etc.



Figure 1.2 Bending Stress in Curved Beams

Consider a curved beam subjected to a bending moment M as shown in figure. The general expression for bending stress (σ_b) in a curved beam at any fibre at a distance y from the neutral axis is

$$\sigma_b = \frac{M}{Ae} \left(\frac{y}{R_n - y} \right)$$

- M- Bending moment acting at the given section about the centroidal axis
- A- Area of cross-section
- e- Radius of curvature of the neutral axis
- R- Radius of curvature of the centroidal axis
- R_n radius of curvature of the neutral axis

y- Distance from the neutral axis to the fiber under consideration. It is positive for the distances towards the center of curvature and negative for the distance away from the center of curvature.

STRESS CONCENTRATION

When every a machine component changes the shape of cross section the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration.

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in figure. From the figure the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole.



 $K_t = \frac{\max imumstress}{noinalstress}$

Kt depends upon material and geometry of the part.

Methods of Reducing Stress Concentration

- Avoiding sharp corners
- Providing fillets
- > Use of multiple holes instead of single hole.
- Undercutting the shoulder part

PRINCIPLE STRESSES AND PRINCIPLE PLANES

The planes which have no shear stress are known as principle planes (τ =0).

The normal stresses acting on the principle planes are known as principle stresses.

Two principle stresses are

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$
$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Maximum shear stress

$$\tau_{\max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

APPLICATION OF PRINCIPLE STRESSES AND PRINCIPLE PLANES

Maximum tensile stress

$$\sigma_{t(\max)} = \frac{\sigma_t}{2} + \frac{1}{2} \left[\sqrt{(\sigma_t)^2 + 4\tau^2} \right]$$

Maximum compressive stress

$$\sigma_{c(\max)} = \frac{\sigma_c}{2} + \frac{1}{2} \left[\sqrt{(\sigma_c)^2 + 4\tau^2} \right]$$

Maximum shear stress

$$\tau_{\max} = \frac{1}{2} \left[\sqrt{(\sigma_t)^2 + 4\tau^2} \right]$$

THEORIES OF FAILURE

Stress produce in a body due to the application of the load is beyond the elastic limit the permanent deformation occurs in the body. If the load is removed the body will not retain its

original shape. There are some permanent deformations in the body. Whenever permanent deformation occurs in the body the body is said to be failed. The failure of a component due to increase of tensile stress or due to other quantities such as shear stress and strain energy also attain definite values and any one of these may be deciding factor of the failure have advanced to explain the cause of failure.

According to the important theories the failure takes place when a certain limiting value is reached by one of the following

(i) Maximum principal stress (or) maximum normal stress (or) Ranking theory

Failure occurs when the maximum normal stress is equal to the tensile yield strength.

 $\sigma_1(\text{or}) \sigma_2(\text{or}) \sigma_3(\text{which is maximum}) = \sigma_y/n \text{ (for ductile material)}$

 $\sigma_1(\text{or}) \sigma_2(\text{or}) \sigma_3(\text{which is maximum}) = \sigma_u/n$ (for brittle material)

Where σ_v -yield stress, σ_u -ultimate stress, n-factor of safety

This theory is based on failure in tensile or compression and ignores the possibility of failure due to shearing stress, ductile material mostly fail by shearing. So this theory is used for brittle material.

(ii) Maximum shear theory (or) Guest's theory (or) Coloumb theory

Failure occurs when the maximum shear stress developed in the machine member becomes equal to the maximum shear stress at yielding in a tensile test.

$$(\sigma_1 - \sigma_2)$$
 or $(\sigma_2 - \sigma_3)$ or $(\sigma_3 - \sigma_1) = \sigma_y/n$

This theory is mostly used for ductile materials.

(iii) Maximum strain theory (or) Venant's theory

Failure occurs when the maximum strain in the member equal in the tensile yield strain.

$$\sigma_1$$
- $\nu(\sigma_2+\sigma_3)$ (or) σ_2 - $\nu(\sigma_3+\sigma_1)$ (or) σ_3 - $\nu(\sigma_1+\sigma_2) = \sigma_y/n$

v- Poisson ratio

(iv) Maximum strain energy theory

Failure is induced in the member when the strain energy stored per unit volume of the member becomes equal to the strain energy per unit volume at the yield point.

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = (\sigma_y/n)^2$$

(v) Distortion energy theory (Vonmiseshenky theory)

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \sigma_2 - \sigma_2 \sigma_3 - \sigma_3 \sigma = (\sigma_v/n)^2$$

DESIGN OF VARIABLE LOADING

Consider a rotating beam of circular cross section and carrying a load of W, this load induces stresses in the beam which are cyclic in nature.

Upper fibers of beam(a) under compression and lower fiber (B) tensile after half revolution the point B occupies the position of point A and point A occupies the point of B. thus point B is now compression and point A is tensile.



- The stresses which vary from one value of compressive to same value of tensile or vice versa are known as completely reversed or cyclic stresses.
- The stresses which vary from a minimum value to a maximum value of same nature is called fluctuating stresses.
- The stresses which vary from zero to a certain maximum value are called repeated stresses.
- The stresses which vary from a minimum value to a maximum value of the opposite nature is called alternative stresses (from a certain minimum compressive to a maximum tensile or minimum tensile to a certain maximum compressive).



NOTCH SENSITIVITY (q)

This is defined as the degree to which the actual stress concentration effect compares with theoretical stress concentration effect.

ENDURANCE LIMIT

It is defined as maximum value of completely reversed bending stress which a polished specimen can withstand without failure for infinite number of cycles.

FACTORS AFFECTING ENDURANCE STRENGTH

Load factor (K_L)

- 1. Surface finish factor(K_{SF})
- 2. Size factor(K_{SZ})
- 3. Reliability factor(K_R)
- 4. Miscellaneous factors(K)

UNIT -II

DESIGN OF SHAFT AND COUPLINGS

Design of solid and hollow shafts based on strength, rigidity and critical speed – Design of keys and key ways - Design of rigid and flexible couplings – Introduction to gear and shock absorbing couplings - design of knuckle joints.

SHAFT

A shaft is a rotating machine element which is used to transmit power from one place to other place.

Carbon steels of grade 40C8, 45C8, 50C4, 50C12 are normally used as shaft materials.

Material properties

- ➢ It should have high strength
- ➢ It should have good machinability.
- > It should have low notch sensitivity factor.
- ➢ It should have good heat treatment properties.
- ➢ It should have high wear resistance.

TYPES OF SHAFT

1. Transmission shaft:

These shafts transmit power between the source and machines absorbing power. The counter shafts, line shafts, overhead shafts all shafts are transmission shafts.

2. Machine shafts:

These shafts from an integral part of the machine itself.

Stresses in shaft

Following stresses are induced in the shaft.

1. Shear stress due to transmission of torque

2. Bending stress due to forces acting upon machine elements like gears, pulleys etc.

3. Stresses due to combined torsional and bending loads.

DESIGN OF SHAFTS

The shaft may be designed on the basis of 1. Strength 2. Rigidity and stiffness

In designing shaft on the basis of strength the following cases may be consider

- 1. Shafts subjected to twisting moment only
- 2. Shaft subjected to bending moment only
- 3. Shaft subjected to combined twisting moment and bending moment
- 4. Shaft subjected to fluctuating loads

1. SHAFTS SUBJECTED TO TWISTING MOMENT ONLY

$$\frac{T}{J} = \frac{\tau}{r}$$
$$T = \frac{\pi}{16} \times \tau \times d^3$$

For hollow section

$$T = \frac{\pi}{16} \times \tau \times d_o^3 \times \left(1 - k^4\right)$$

$$k = \frac{d_i}{d_o}$$
 Where d_i=inside diameter, d_o= outside diameter

Twisting moment may be obtained by using the following relation

$$T = \frac{P \times 60}{2 \times \pi \times N}$$

In case of belt drives

$$T=(T_1-T_2) R$$

T₁- Tension in the tight side

T₂- Tension in the slack side

R- Radius of the pulley

2. SHAFT SUBJECTED TO BENDING MOMENT ONLY

The bending moment equation is

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

M- Bending moment

I- moment of inertia of cross sectional area of the shaft about the axis of rotation

 σ_b - Bending stress

For round solid shaft

$$M = \frac{\pi}{32} \times \sigma_b \times d^3$$

For hollow shaft

$$M = \frac{\pi}{32} \times \sigma_b \times d_o^3 \times (1 - k^4)$$

3. SHAFT SUBJECTED TO COMBINED TWISTING MOMENT AND BENDING MOMENT

When the shaft is subjected to combined twisting moment ad bending moment then the shaft must be designed on the basic of two moments simultaneously

For solid shaft

$$\frac{\pi}{16} \times \tau_{\max} \times d^3 = \sqrt{\left(M^2 + T^2\right)}$$

$$\frac{\pi}{32} \times \sigma_{b(\max)} \times d^3 = \frac{1}{2} \left(M + \sqrt{M^2 + T^2} \right)$$

For hollow shaft

$$\frac{\pi}{16} \times \tau_{\max} \times d_o^{3} \times (1 - k^{4}) = \sqrt{(M^{2} + T^{2})}$$
$$\frac{\pi}{32} \times \sigma_{b(\max)} \times d_o^{3} \times (1 - k^{4}) = \frac{1}{2} (M + \sqrt{M^{2} + T^{2}})$$

KEY

A key is a piece of mildsteel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.

TYPES OF KEYS

1. Sunk key, 2.Saddle key, 3.Tangent key, 4. Round key 5.Splines

SUNK KEYS

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley.

TYPES OF SUNK KEYS

1. Rectangular sunk key



2. Square sunk key

The only difference from the rectangular sunk key is the width and thickness is equal

w=t=d/2

3. Parallel sunk key

The parallel sunk key may be of rectangular or square cross section. The cross section is uniform in width and thickness throughout length.

4. Gib head key

A gib head key is similar to a square or rectangular key but it has a head at one end, generally at the larger end of the taper sunk key. The gib head is used for driving the key while assembling or disassembling.

5. Feather key

Feather key is used where it is necessary to slide a keyed gear, pulley assembly along the shaft. Keys are tight fitted or screwed on the shaft.

6. Woodruff key

A woodruff key is used to transmit small amount of torque in automotive and machine tool industries. The keyway in the shaft is milled in a curved shape whereas the keyway in the hub is usually straight. The main advantage of this key is that it will align itself in the keyway.

TYPES SADDLE KEYS

1. Flat saddle key

A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft.

2. Hollow saddle key

A hollow saddle key is a tapper key which fits in the keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft.

Forces acting on a sunk key

1. Forces due to tight fit of the key and thus compressive stress is induced.

2. Force due to torque transmitted by the shaft and this force produced shearing and crushing stresses in the key.

EFFECT OF KEYWAYS

The keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross sectional area of the shaft. Torsional strength of shaft is reduced.

The following relation for the weakening effect of the keyway is based on the experiments results by H.F.Moore.

$$e = 1 - 0.2 \left(\frac{w}{d}\right) - 1.1 \left(\frac{h}{d}\right)$$

e- shaft strength factor. It is the ratio of strength of shaft with keyway to the strength of shaft without keyway.

w-width of the keyway

d-diameter of the shaft

h-depth of keyway(thicknessof key/2)

DESIGN OF COUPLING

Shaft couplings are used in machinery for several purposes

1. To provide for connection of shaft of units those are manufactured separately.

2. To provide for misalignment of the shaft or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against over loads.

REQUIREMENT OF A GOOD SHAFT COUPLING

1. It should be easy to connect or disconnect.

- 2. It should transmit the full power from one shaft to the other shaft without losses.
- 3. It should hold the shaft in perfect alignment.
- 4. It should have no projecting parts.

TYPES OF SHAFT COUPLINGS

1. Rigid coupling

It is used to connect two shafts which are perfectly aligned. The types are

- Sleeve (or) muff coupling
- Clamp(or) split muff (or) compression coupling
- ➢ Flange coupling
- 2. Flexible coupling

It is used to connect two shafts having lateral and angular misalignments. The types are

- Bushed pin type coupling
- ➤ Universal coupling
- > Oldham coupling

SLEEVE (or) MUFF COUPLING

It is made of cast iron. It consists of a hollow cylinder whose inner diameter is that same as that of the shaft. It is fitted over the ends of two shafts by means of a gib head key. The power transmitted from one shaft two other shafts by means of a key and a sleeve.

Outer diameter of sleeve D=2d+13mm

Length of sleeve L=3.5d

d- diameter of shaft



Figure 2.1 muff coupling

DESIGN OF MUFF COUPILNG

1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

2. Design for key

The length of coupling key is at least equal to the length of the sleeve. The coupling key is usually made into two parts so that the length of key in each shaft

$$l = \frac{L}{2}$$

after that the induced shearing and crushing stresses may be checked.

$$T = l \times w \times \tau \times \frac{d}{2}$$
$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

CLAMP (or) COMPRESSION COUPLING

In this case the muff or sleeve is made into two halves are bolted together. The halves of the muff are made of cast iron. The shaft end is made to abut each other and a single key is fitted directly in the keyway of both the shaft. Both the halves are held together by means of mildsteel bolts and nuts. The number of bolt may be two or four or six.



Figure 2.2 compression coupling

Diameter of muff D=2d+13mm

Length of muff or sleeve L=3.5d

DESIGN OF CLAMP (or) COMPRESSION COUPILNG

1. Design for sleeve

The sleeve is designed by considering it as a hollow shafts

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

2. Design for key

The length of coupling key is at least equal to the length of the sleeve. The coupling key is usually made into two parts so that the length of key in each shaft

$$l = \frac{L}{2}$$

after that the induced shearing and crushing stresses may be checked.

$$T = l \times w \times \tau \times \frac{d}{2}$$
$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

3. Design of clamping bolts

Force exerted by each bolt

$$=\frac{\pi}{4}\times d_b^2\times \sigma_t$$

Force exerted by each side of the shaft

$$=\frac{\pi}{4} \times d_b^2 \times \sigma_t \times \frac{n}{2}$$

Torque transmitted by the coupling

$$T = \frac{\pi}{16} \times \mu \times d_b^2 \times \sigma_t \times n \times d$$

Where

T-torque transmitted by the shaft

d-diameter of shaft

d_b- root or effective dia of bolt

n- number of bolt

 $\sigma\text{-}\text{Permissible}$ stress for bolt

 $\mu\text{-}coefficent$ of friction between the muff and shaft

L- length of muff

FLANGE COUPLING

A flange coupling usually applied to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess. This helps to bring the shaft into line and to maintain alignment. The two flanges are coupled together by means of bolt and nuts.



Figure 2.3 flange coupling

1. Design for hub

The hub is designed by considering it as a hollow shaft

$$T = \frac{\pi}{16} \times \tau_c \times \left(\frac{D^4 - d^4}{D}\right)$$

D=2*d

Length of hub L=1.5d

2. Design for key

The key is designed with equal properties and then checked for shearing and crushing stress. The length of key is taken equal to the length of hub

3. Design for flange

$$T = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

t_f- thickness of flange(d/2)

4. Design for bolt

The bolts are subjected to shear stress due to torque transmitted. The number of bolts (n) depends upon the diameter of shaft and pitch circle diameter is taken

Torque transmitted

$$T = \frac{\pi}{4} \times d_1^2 \times \tau_b \times n \times \frac{D_1}{2}$$

d₁- diameter of bolt

for crushing

$$T = \left(n \times d_1 \times t_f \times \sigma_{cb}\right) \frac{D_1}{2}$$

KNUCKLE JOINT

A knuckle joint is used to connect two rods which are under the action of tensile loads. It consists of mainly three elements a fork or double eye rod, a single eye rod and knuckle pin. Its use may be found in the link of a cycle chain, tie rod joint for roof truss.



Figure 2.4 knuckle joint

Dimension of various parts of the knuckle joint

d- diameter of rod

d₁- diameter of pin

outer dia of eye d₂=2d

diameter of knukle pin head and coller $d_3=1.5d$

thickness of single eye or rod end t=1.25d

thickness of fork $t_1=0.75d$

thickness of pin head t₂=0.5d

UNIT-III

DESIGN OF FASTNERS AND WELDED JOINTS

Threaded fastners - Design of bolted joints including eccentric loading – Design of welded joints for pressure vessels and structures - theory of bonded joints.

INTRODUCTION

Welding is the most commonly used process for permanently joining machine parts. Welding is done by fusing the metallic parts with heat at their junction with or without pressure.

Advantages of welding over riveting

- 1. Possibility of joining curvilinear parts
- 2. Cost is less
- 3. Tightness of joint
- 4. Noiseless process
- 5. Greater strength

DESIGN OF WELDED JOINTS

In order to determine the strength of fillet joint, it is assumed that the section of fillet is a right angle triangle ABC with hypotenuse AC making equal angles with other two sides AB and BC. The length of each side is known as leg or size of the weld and the perpendicular distance of hypo tenuous from intersection of legs (BD) is known as the throat thickness.

In the triangle BDA

 $\frac{BD}{AB} = \sin \theta$ $BD = AB \sin \theta$

=s sin45=0.707s

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STRENGTH OF TRANSVERSE FILLET WELDED JOINT

The fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength.



(a) Single transverse fillet weld.



(b) Double transverse fillet weld.

Assumed that the weld section is right angle triangle ABC with hypotenuse AC making equal with other two sides AB and BC

t- throat thickness

s- thickness of plate

l-length of weld

throat thickness

minimum aera of the weld

 σ_t is the allowable tensile stress for the weld then tensile strength of joint for single fillet weld

$$P = 0.707 s \times l \times \sigma_t$$

Tensile strength of joint for double fillet weld

$$P = 2 \times 0.707 \, s \times l \times \sigma_{t}$$

STRENGTH OF PARALLEL FILLET WELD

The parallel filet weld joints are designed for shear strength. Consider a double parallel fillet weld as shown in figure



Shear strength of joint due to double parallel fillet weld

$$P = 2 \times 0.707 s \times l \times \tau$$

$$=1.414 \times s \times l \times \tau$$

Combination of single transverse and double parallel fillet weld then the strength of joint

$$P = 0.707 s \times l_1 \times \sigma_t + 1.414 \times s \times l_2 \times \tau$$

STRESS CONCENTRATION FACTOR FOR WELDED JOINTS

The reinforcement provided to the weld produces stress concentration at the junction of the weld and parent metal. When the parts are subjected to fatigue loading the stress concentration factor as given in the following table is taken into account

Type of joint	Stress concentration factor
Reinforced butt welding	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corner	2.0

AXIALLY LOADED UNSYMMETRICAL WELD SECTIONS

Unsymmetrical sections such as angles, channels T-sections etc, welded on the flange edges are loaded axially as shown in figure. In such case the length of weld should be proportioned in such a way that the sum of resisting moments of the welds about the gravity axis is zero.

 $l_{a}\text{-}$ length of weld at the top

 l_{b} - length of weld at the bottom

l- total length of weld

a- distance of top weld from gravity axis

b- distance of bottom weld from gravity axis

$$l_a = \frac{l \times b}{a + b}$$
, $l_b = \frac{l \times a}{a + b}$

$$l = l_a + l_b$$

ECCENTRICALLY LOADED WELD JOINTS

When a welded joint is eccentrically loaded the principle stress will be applied because the welded part undergoing direct load and a bending moment

Maximum normal stress

$$\sigma_{t(\max)} = \frac{\sigma_b}{2} + \frac{1}{2} \left[\sqrt{(\sigma_b)^2 + 4\tau^2} \right]$$

Maximum shear stress

$$\tau_{\max} = \frac{1}{2} \left[\sqrt{(\sigma_b)^2 + 4\tau^2} \right]$$

Case (i)

Consider a t-T-joint fixed at one end and subjected to an eccentric load P at a distance e, the joint subjected to two types of stresses



Figure 3.4 eccentrically loaded weld joint

- (i) Direct shear due to the shear force (P)
- (ii) Bending stress due to bending moment P*e

Aera of throat (A)=throat thickness*length of weld

=t*l*2

Shear stress in the weld

$$\tau = \frac{P}{A}$$

Section modulus of the weld metal through the throat

$$z = \frac{s \times l^2}{4.242}$$

Bending stress

$$=\frac{M}{z} = \frac{P \times e}{\frac{s \times l^2}{4.242}}$$

Case (ii)

When a welded joint is loaded eccentrically as shown in figure subjected to two types of stresses

- 1. Direct or primary shear stress
- 2. Shear stress due to turning moment



Figure 3.5 eccentrically loaded weld joint

P- Eccentric load

e- Eccentricity (i.e.) perpendicular distance between the line of action of load and center of gravity(G) of throat section

l- length of single weld

t- throat thickness

Direct or primary shear stress

$$\tau_1 = \frac{P}{1.414 \times s \times l}$$

Shear stress due to turning moment (i.e.) secondary shear stress

$$\tau_1 = \frac{P \times e \times r_2}{J}$$

The polar moment of inertia of the throat area (A) about the center of gravity (G) is obtained by the parallel axis theorem

x- perpendicular distance between two parallel axis

$$J = 2A\left(\frac{l_2}{12} + x^2\right)$$

$$A = 0.707 \times s \times l$$

Resultant shear stress at A

$$\tau_A = \sqrt{\tau_1^2 + \tau_2^2 + (\tau_1 \times \tau_2 \times \cos \theta)}$$

 $\theta\text{-angle}$ between τ_1 and τ_2

$$\cos\theta = \frac{r_1}{r_2}$$

SCREWED FASTENER

A screw thread is obtained by cutting a continuous helical groove on a cylindrical surface (external thread). The threaded portion engages with corresponding threaded hole (internal thread) forming screwed fastener. Screwed fasteners such as bolts, studs and nuts in combination, machine screws are used for fastening components that require frequent assembly and disassembly. Screwed fasteners are to hold parts together and to transmit power.

SCREW THREAD NOMENCLATURE

1. Major diameter

This is the largest diameter of a screw thread, touching the crests on external thread or the roots on internal thread. This is also called nominal diameter.

2. Minor diameter

This is the smallest diameter of a screw thread, touching the roots or core of external thread or the crests of internal thread. This is also called core or root diameter.

3. Pitch diameter

This is the diameter of an imaginary cylinder, passing through the threads at the points where the width of thread is equal to the space between threads.

4. Pitch

It is the distance measured parallel to the axis, between corresponding points on adjacent screw threads.

5. Lead

It is the distance, a screw advances axially in one turn. For a single threaded screw, the pitch and lead are equal. For a double threaded screw the lead is twice the pitch and so on.

FORMS OF THREADS

Bureau of Indian Standards (BIS) adopts ISO (International Organization for Standards) metric threads which are followed by number of countries.

1. V-Thread

This thread profile has a larger contact area providing more frictional resistance to motion. It is used where effective positioning is required.

2. British Standard Whitworth (B.S.W) thread

This thread is adopted in Britain in inch units. The profile has rounded ends making it less liable to damage than sharp V-threads.

3. Square thread

This is an ideal thread form for power transmission. In this as the threaded flank is at right angle to the axis. The normal force between the threads acts parallel to the axis with zero radial components. This enables the nut to transmit very high pressure as in case of a screw jack and other similar applications.

4. Buttress thread

This thread form is combination of V-thread and square thread. It exhibits the advantages of square threads like the ability to transmit power and low frictional resistance and the strength of a V-thread. It is used where power transmission takes place in one direction only.

5. ACME thread

It is a modification form of square thread. It is much stronger than square thread because of the wider base and it is easy to cut. The inclined sides of thread facilitate quick and easy engagement and disengagement as for example the split nut with the lead screw of lathe.

THREAD SERIES

BIS recommends two thread series, coarse and fine series. Based on the relative values of pitches. It must be noted that the concept of quality is not associated with these terms. For any

particular diameter there is only largest pitch called coarse pitch and the rest are designated as fine pitches.

THREAD DESIGNATION

The diameter pitch combination of an ISO metric screw thread is designated by the letter M followed by the value of nominal diameter and pitch, the two values being separated by the sign 'x'. For example a diameter pitch combination of nominal diameter 10mm and pitch 1.25mm is designated as M10x1.25.

STRESSES IN SCREW THREADS

The following types of stresses are induced in screwed fasteners under static loading

- (i) Stresses due to initial tightening of the nut
- (ii) Stresses due to external forces
- (iii) Stresses due to the combination of above

UNIT IV

DESIGN OF ENERGY STORING ELEMENTS

Design of various types of springs, optimization of helical springs -- rubber springs --

Design of flywheels considering stresses in rims and arms, for engines and punching machines.

SPRING

Spring is an elastic body whose function is to distort when loaded and to recover its original shape when the load is removed.

APPLICATION OF SPRINGS

- 1. To apply forces as in brakes, clutches and spring loaded valves.
- 2. To store energy as in watches, toys.
- 3. To measure forces as in spring balance and engine indicators.
- 4. To cushion, absorb or control energy due to either shock or vibration as in car.

TYPES OF SPRINGS

- 1. Helical springs
- 2. Conical and volute spring
- 3. Torsion spring
- 4. Laminated or leaf spring
- 5. Disc or bellevile spring
- 6. Special purpose spring

HELICAL SPRINGS

The helicall springs are made up of a wire coiled in the form of helix and is primarily intended for tensile or compressive loads. The cross section of the wire from which the spring made may be circular, square or rectangular. The two forms of helical springs are compression spring and helical tension springs.



(a) Compression helical spring.



(b) Tension helical spring.

TERMS USED IN COMPRESSION SPRING

SOLID LENGTH

When the compression spring is compressed until the coils come in contact with each other the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire.

L_S=n'*d

n'- total number of coils

d- diameter of the wire

FREE LENGTH

It is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils.

 $L_F=n^*d+\delta_{max}+0.15 \delta_{max}$

SPRING INDEX

It is defined as the ratio of the mean diameter of the coil to the diameter of the coil to the diameter of the wire.

C=D/d

D- mean diameter of coil

d- diameter of wire

SPRING RATE

It is defined as the load required per unit deflection of the spring.

q=P/y

P- applied load

y- deflection of the spring

PITCH

The pitch of the coil is defined as the axial distance between adjacent coil in uncompressed state.

Pitch length=free length/(n'-1)

ENDS FOR COMPRESSION HELICAL SPRING

In all springs the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, this effect must be taken into account. The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis. It may be noted that part of the coil which is in contact with the seat is in contact with the seat does not contribute to spring action and hence are termed as inactive coils. The turns which impact spring action are known as active turns. As the load increases, the number of

inactive coils also increased due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at usual working loads.



Type of end	Total number of turns (n')	Solid length	Free length
 Plain ends Ground ends Squared ends 	n n n+2	$(n+1) d$ $n \times d$ $(n+3) d$	$p \times n + d$ $p \times n$ $p \times n + 3d$
4. Squared and ground ends	n + 2	(n+2) d	$p \times n + 2d$

n- number of active turns

p-pitch of coils

d-diameter of spring

STRESSES IN HELICAL SPRING

Consider a helical compression spring made of circular wire and subjected to an axial load W

Maximum shear stress induced in the wire = $K_s \times \frac{8 \times W \times D}{\pi d^3}$

Ks- Shear stress factor

$$\mathrm{Ks} = 1 + \frac{1}{2c}$$

When the springs are subjected to static loading the effect of wire curvature may be neglected because yielding of material will relive the stresses. In order to consider the effect of both direct shear as well as curvature of the wire wahl's stress factor is introduced.

Maximum shear stress introduce in wire $\tau_{\text{max}} = \frac{K \times 8 \times W \times D}{\pi d^3}$

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

DEFLECTION OF HELICAL SPRING OF CIRCULAR WIRE

$$y = \frac{8 \times W \times D^3 \times n}{Gd^4}$$

STIFFNESS OF SPRING (or) SPRING RATE

$$q = \frac{W}{\delta} = \frac{Gd^4}{8D^3n}$$

LEAF SPRING

The laminated or leaf spring consists of a number of flat plates of varying lengths held together by means of clamps and bolts. These are mostly used in automobiles.



Figure 4.2 Leaf spring

A leaf spring commonly used in automobiles is of semielliptical form. It is built up of a number of plates. The leaves are usually given an initial curvature or cambered so that they will tend to straighten under the load. The leaves are held together by means of a band shrunk around them at the center or by a bolt passing through the center. Since the band exerts stiffening and strengthening effect, therefore the effective length of the spring for bending will be overall length of spring minus width of band. The spring is clamped to the axle housing by means of U bolts. The longest leaf known as main leaf or master leaf has its ends formed in the shape of an eye through which the bolts are passed to secure the spring to its support. Usually the eyes through which the spring is attached to the hanger or shackle, are provided with bushings of some antifriction materials such as bronze or rubber. The other leaves of the springs are known as graduated leaves. Rebound clips are located at intermediate positions in the length of the spring rebounds.

2L1- length of span (or) overall length of spring

l-width of band (or) distance between centers of U bolt, ineffective length of springh

effective length of spring 2L=2L1-L

Length of smallest leaf= $\frac{Effectivel ength}{n-1}$ + ineffectiv elength

Length of next leaf= $\frac{Effectivel ength}{n-1} \times 2 + ineffectiv elength$

Length of master leaf= $2L_1 + \pi (d+t) * 2$

Bending stress in the spring $\sigma_b = \frac{6WL}{nbt^2}$

Deflection in the leaves
$$\delta = \frac{6WL^3}{nEbt^3}$$

When the leaves are not initially stressed therefore maximum stress or bending stress for

full length leaves
$$\sigma_F = \frac{18WL}{bt^2(2n_G + 3n_F)}$$

n- number of leaves

n_G- number of graduated leaves

n_F- number of extra full length leaves

For graduated leaves $\sigma_G = \frac{12WL}{bt^2(2n_G + 3n_F)}$

The deflection in the full length and graduated leaves $\delta = \frac{12WL^3}{Ebt^3(2n_G + 3n_F)}$

Initialgap (or) Nip $C = \frac{2WL^3}{nEbt^3}$

Load on the bolt to close the nip $P_b = \frac{2Wn_G n_F}{n(2n_G + 3n_F)}$

DESIGN OF FLYWHEEL

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and release it during the period when the requirement of energy is more than supply.



Figure 4.3 Turning moment diagram for single cylinder double acting steam engine

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider a turning moment diagram for a single cylinder double acting steam engine. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle. a little consideration will show that the turning moment is zero when the crank angle is zero. It rises to maximum value when crank angle reaches 90° and it again zero when crank angle reaches 180°. This is shown by curve abc in figure and it represents the turning moment for outstroke. The curve cde is turning moment diagram for instroke and is somewhat similar to the curve abc. The work done is the product of turning moment and angle turned, therefore the area of the turning moment diagram represents the work done per revolution.

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called coefficient of fluctuation of speed.

Coefficient of fluctuation of speed $C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{(N_1 + N_2)}$

$$C_{s} = \frac{\omega_{1} - \omega_{2}}{\omega} = \frac{2(\omega_{1} - \omega_{2})}{(\omega_{1} + \omega_{2})}$$
 in terms of angular speeds
$$C_{s} = \frac{v_{1} - v_{2}}{v} = \frac{2(v_{1} - v_{2})}{(v_{1} + v_{2})}$$
 in terms of angular velocity

Maximum fluctuation of energy Δ_E = maximum energy-minimum energy

coefficent of fluctuation of energy $C_E = \frac{max imumfluctuation of energy}{workdone percycle}$

work done per cycle=T_{mean}*\theta

 θ =angle turned in radians per revolution

 $=2\pi$ in case of stream engine and two stroke I.C engine

 $=4\pi$ in case of four stroke I.C engines

energy stored in flywheel rim

Maximum fluctuation of energy
$$\Delta_{\rm E} = I\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right) = I\omega^2 C_s = mk^2 \omega^2 C_s = 2EC_s$$

k may be equal to mean radius of rim(R) because the thickness of rim is very small as compared to the diameter of rim substituting k=R

 $\Delta_{\rm E} = mR^2\omega^2 C_s = mv^2 C_s$

Mass of the flywheel rim $m = 2\pi RA\rho$

A=b*t

b-width of rim , t- thickness of rim

UNIT V

DESIGN OF BEARINGS AND MISCELLANEOUS ELEMENTS

Sliding contact and rolling contact bearings -- Design of hydrodynamic journal bearings, McKee's Eqn., Sommerfield Number, Raimondi & Boyd graphs, -- Selection of Rolling Contact bearings -- Design of Seals and Gaskets -- Design of Connecting Rod.

A bearing is a machine element which supports another moving machine element. It permits a relative motion between the contact surfaces of the members while carrying the load. Due to the relative motion between the surfaces a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are indirect contact there will be rapid wear. In order to reduce frictional resistance and wear resistance in some cases to cases carry away the heat generated a layer of fluid may be provided.

Depend upon the nature of contact

(i) Sliding contact bearing

The sliding takes place along the surfaces of contact between the moving element and fixed element

(ii) Rolling contact bearing

The steel balls or rollers are interposed between the moving and fixed element. The balls offer rolling friction at the two points for each ball or roller.

JOURNAL BEARING

A sliding contact bearing that supports a load in a radial direction is known as journal bearing. It consists of two main parts, a shaft and a hollow cylinder. The portion of the shaft inside the hollow cylinder also known as bearing is called as journal. In most applications the journal rotates while the bearing is stationary. However there are some applications where the journal is stationary and the bearing rotates and even somewhere both the journal and bearing

rotates. This journal bearing may be classified as full journal bearing and partial journal bearing depending upon whether the journal is fully or partially covered by bearing.

TERMS USED IN HYDRODYNAMIC JOURNAL BEARING

1. Diameter clearance: It the different between the diameter of journal and the bearing.

c=D-d

2. Radial clearance: it is the different between the radial of the bearing and the journal.

 $c_1 = R - r = D - d/2$

3. Diameteral clearance ratio: It is the ratio of the diameteral clearance to the diameter of the journal.

=c/d=(D-d)/d

4. Eccentricity: It is the radial distance between the center of the bearing and the displaced center of the bearing under load.

5. Minimum oil film thickness: It is the minimum distance between the bearing and the journal under complete lubrication condition. It is denoted by ho and occurs at the line of centers.

COEFFICENT OF FRICTION FOR JOURNAL BEARINGS

To determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data

$$\mu = \frac{33}{10^8} \left(\frac{ZN}{p}\right) \left(\frac{d}{c}\right) + k$$

µ- Coefficient of friction

Z- Absolute viscosity of oil

N- Speed of journal in r.p.m

p- bearing pressure on the projected bearing area

d- diameter of journal

l-length of bearing

c- diameter clearance

k- factor to correct for end leakage.

CRITICAL PRESSURE OF THE JOURNAL BEARING

The pressure at which the oil film breaks down so the metal to metal contact begins, is known as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following relation.

$$p = \frac{ZN}{4.75 \times 10^6} \left(\frac{d}{c}\right)^2 \left(\frac{l}{d+l}\right)$$

SOMMERFELD NUMBER

The sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings.

sommerfeld number=
$$\frac{ZN}{p} \left(\frac{d}{c}\right)^2$$

HEAT GENERATED IN A JOURNAL BEARING

The heat generated in a bearing due to the fluid friction and friction of the parts having relative motion.

Qg=WVµ

After the thermal equilibrium is reached heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat will depend upon the temperature difference size and mass of the radiating surface and on the amount of air flowing around the bearing. However for the convenience in bearing design the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing $Q_d = CA(t_b - t_a)$

- C- heat dissipation coefficient
- A- projected area of the bearing
- t_b-temperature of the bearing surface
- t_a- temperature of the surrounding air

DESIGN PROCEDURE FOR JOURNAL BEARING

- 1. Determine the length of bearing.
- 2. Calculate the bearing pressure.
- 3. Select the type of oil used.
- 4.Determine the amount of heat generated.
- 5. Determine the amount of heat rejected.

ROLLING CONTACT BEARING

The advent of automobiles and many high speed machineries make very much use another type of bearings known as rolling contact bearings. The friction produced in these bearing is very low. These bearings also called as antifriction bearings. They differ from sliding contact bearings in their structure and usage.

COMPONENTS OF ROLLING CONTACT BEARINGS

The rolling bearing consists of four main components (1) the inner ring, (2) outer ring, (3) the balls or rollers, (4) the retainers or separators. The inner ring is forced to fitted with machine shaft and outer ring is fitted with machine housing. The shaft rotates because of relative rotations of balls or rollers. The retainers is used to prevent the balls or rollers from ejecting out during operation.

CLASSIFICATION OF ROLLING CONTACT BEARINGS

The rolling contact bearings are classified into two major groups with respect to their structure

- (1) Ball bearings
- (2) Roller bearings

Basically the structure of ball bearings are similar expect that whether the rolling element between the inner ring and outer ring are balls or rollers. Also these ball bearings are many types such as deep grove ball bearings, angular contact bal bearings and so on. Both type of bearing can carry radial loads and axial loads acted individually or in combined form. Generally the ball bearings are used for light loads and the roller bearings are usually used for heavier loads. Also in the case of ball bearings the nature of contact is the point contact hence the friction produced is very less compared to roller bearings where the nature of contact is the line contact which produce more friction.



Figure 5.1 Types of radial ball bearing

SELECTION OF BEARINGS FOR STEADY LOADING

The size of bearing required is judged by the magnitude and nature of applied load, life and reliability. The bearing load is composed of weights involved forces derived from power transmitted and additional force based on method of operation.

$$C = \left(\frac{L}{L_{10}}\right)^{\frac{1}{k}} \times P$$

C- basic dynamic load rating

L- life of bearing in million revolutions

 L_{10} - life of bearing for 90% survival at 1 million revolutions

P- Equivalent load

k- exponent (3 for ball bearing, 10/3 for roller bearings)

 $P = (XF_r + YF_a)S$

Fr- radial load

F_a- axial load

X- radial load factor

Y- axial load factor

S- service factor

SELECTION OF BEARING FOR VARIABLE LOADING

The rolling contact bearing are frequently operate under variable load and speed conditions. This is due to many causes like power fluctuation in electrical machineries or requirement of different cutting forces for different kinds of machining tools, or running with loading and unloading condition as in automobiles. Such as variable loaded bearings are designed by considering all these different loaded conditions of work cycle and not solely upon most sever operating conditions. The work cycle may be divided into a number of portions in each of which operation condition may be taken as constant.

For variable speed

$$P_{m} = \left[\frac{P_{1}^{3}n_{1} + P_{2}^{3}n_{2} + P_{3}^{3}n_{3} + \dots + P_{n}^{3}n_{n}}{\sum n}\right]$$

For variable time

$$P_{m} = \left[\frac{P_{1}^{3}t_{1} + P_{2}^{3}t_{2} + P_{3}^{3}t_{3} + \dots + P_{n}^{3}t_{n}}{\sum t}\right]$$

 P_1 - constant load during n_1 revolution (or) during the period of time t_1

 P_2 - constant load during n_2 revolution (or) during the period of time t_2

 $\Sigma n = n_1 + n_2 + n_3 + \ldots + n_n$

 $\Sigma t{=}t_1{+}t_2{+}t_3{+}{\dots}{+}t_n$