Module
2

DC Circuit

Lesson
9

Analysis of dc resistive network in presence of one non-linear element

Objectives

- To understand the volt (V) – ampere (A) characteristics of linear and nonlinear elements.
- Concept of load-line and analysis of dc resistive network having a single non-linear element using load-line analysis.

L.9.1 Introduction

The volt-ampere characteristic of a linear resistance is a straight line through the origin as in fig. 3.2 (see Lesson-3) but the characteristic for non-linear element for example, diodes or lamps is not linear as in fig. 3.3 (lesson-3). Temperature effects cause much non-linearity in lamps that are made of metals. Most materials resistance increases (or decreases) with rise (or fall) in temperature. On the other hand, most nonmetals resistance decreases or increases with the rise or fall in temperature. The typical tungsten lamp resistance rises with temperature. Note, as the voltage across the lamp increases, more power is dissipated and in turn rising the filament temperature. Further note, that the increments of voltage produce smaller increments of current that causes increase resistance in the filament element. Opposite effects can be observed in case of carbon filament lamp or silicon carbide or thermistor. Additional increments of voltage produce large increments of current that causes decrease resistance in the element. Fig.9.1 shows the characteristics of tungsten and carbon filaments.
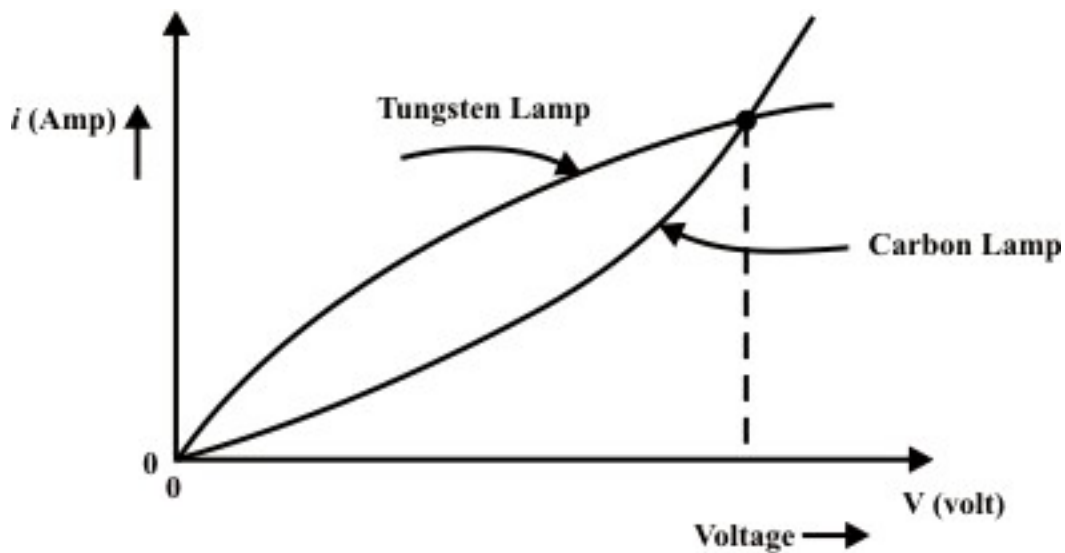

Fig. 9.1: Temperature effects produce two kinds of circuit nonlinearities
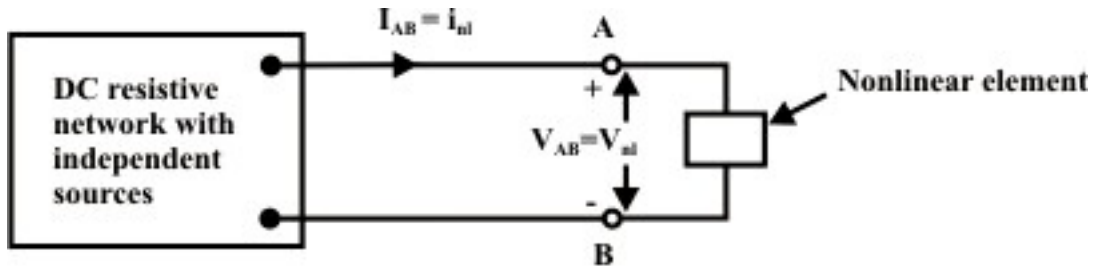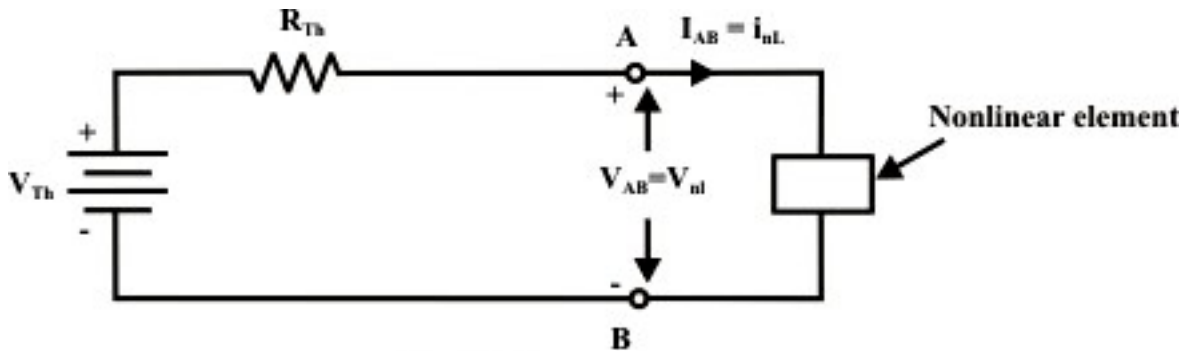
Fig. 9.2(a)

Fig. 9.2(b)

Let us consider a simple circuit shown in fig. L.9.2(a) that consists of independent sources, combination of linear resistances, and a nonlinear element. It is assumed that the nonlinear element characteristics either defined in terms of current ($i(t)$) (flowing through it) and voltage ($v_{nl}(t)$) (across the nonlinear element) relationship or *Voltage* ($v_{nl}(t)$) and *current* (i) relationships of nonlinear element can be expressed as mathematical expression or formula. For example, consider the actual (non-ideal) $V - I$ relationship of the typical diode can be expressed as $I = I_0 \left(e^{V/a} - 1 \right)$, where '$a$' is constant (for germanium diode 'a'=0.026 and silicon diode 'a'=0.052). Assume that the network (fig.9.2(a)) at the terminals 'A *and* B' is replaced by an equivalent Thevenin network as shown in fig. 9.2(b). From an examination of the figure one can write the following expression:

$V_{Th} = I_{AB} \times R_{Th} + V_{AB}$ $or,\ V_{Th} - I_{AB} \times R_{Th} = V_{AB} \Rightarrow$ Thevenin terminal voltage = load voltage.

If the nonlinear element characteristic is given (note, no any analytical expression is available) then one can adopt graphical method called load-line analysis to determine the branch variables ($I_{AB} = i_{nl}$ *and* $V_{AB} = v_{nl}$) of nonlinear element as shown in fig. L.9.2(a). This resulting solution is frequently referred to as the operating point (Q) for the nonlinear element characteristic (in the present discussion, we consider a nonlinear element is a resistor R_2). This method is quite simple and useful to analysis the circuit while the load has a nonlinear $V - I$ characteristic. It is very easy to draw the source characteristic using the intercepts at points $v(t) = V_{AB} = V_{Th}$, $i = i_{AB} = 0$ (*open circuit condition*)   and $v(t) = V_{AB} = 0,\ i = i_{AB} =$

$\dfrac{V_{Th}}{R_{Th}} = I_N$ (*short* − *circuited at A & B ter* min *als*) in two axes. It is obvious that the values of voltage (V_{AB}) and current (I_{AB}) at the terminals of the source are exactly same as the voltage across and current in the load as indicated in fig. 9.2(a). The point of intersection of the load and the source characteristic represents the only condition where voltage and current are same for both source and load elements. More-specifically, the intersection of source characteristic and load characteristic represents the solution of voltage across the nonlinear element and current flowing through it or operating point (Q)of the circuit as shown in fig.9.2(c). Application of load-line analysis is explained with the following examples.
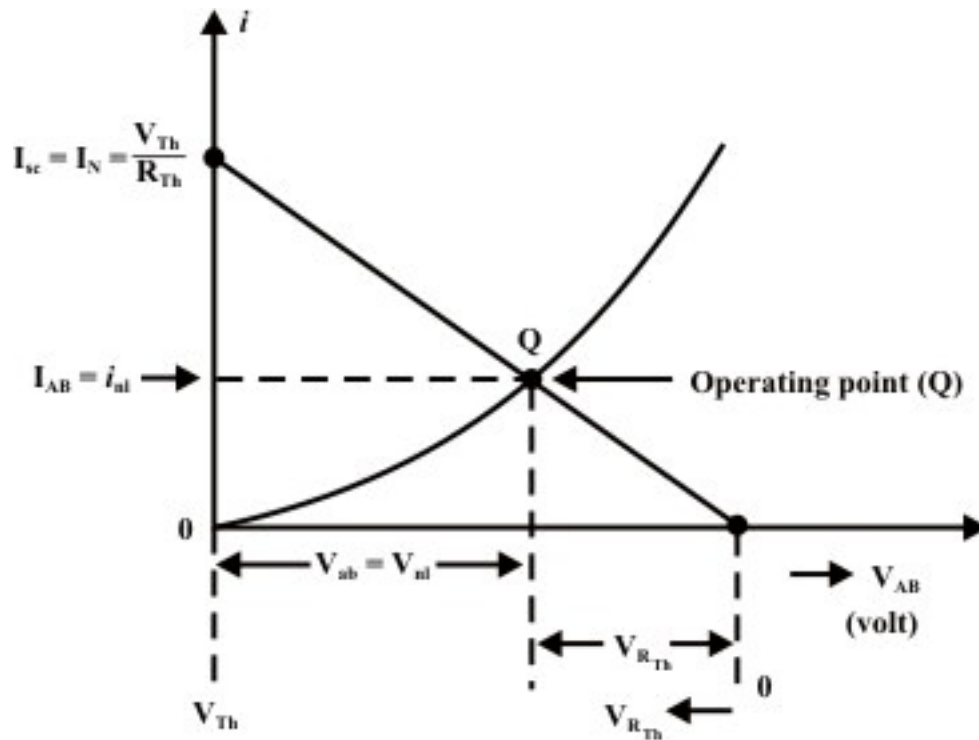

Fig. 9.2 (c)

L.9.3 Application of load-line method

Example-L.9.1: The volt-ampere characteristic of a non-linear resistive element connected in the circuit (as shown Fig.9.3(a)) is given in tabular form.

Table: volt-ampere characteristic of non-linear element

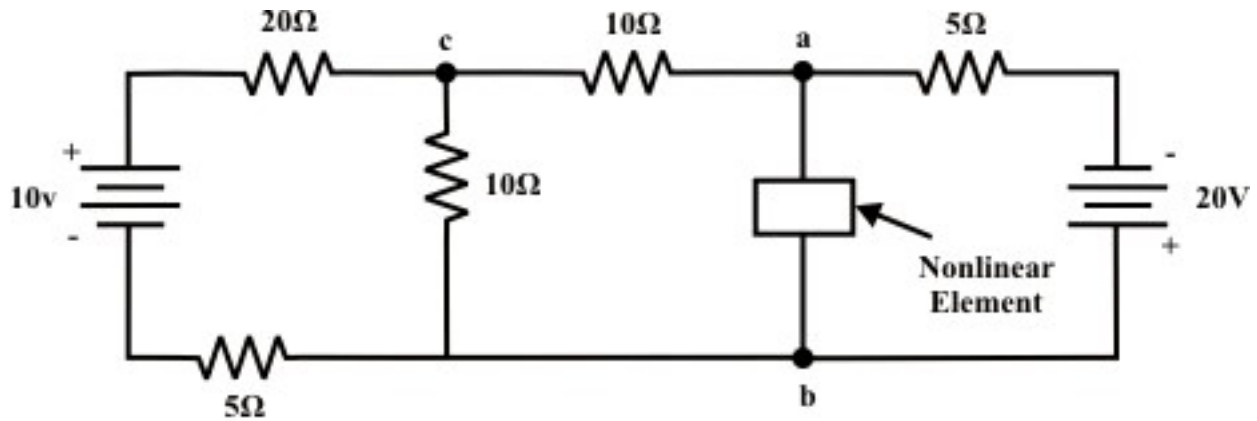| V_{nl}	0	2	4	6	8	10	12	14	15
I_{nl}	0	0.05	0.1	0.2	0.6	1.0	1.8	2.0	4.0

Fig. 9.3 (a) A single non-linear element in the dc resistive network

   (i)   Calculate the voltage drop V_{ab} across the non-linear element.

   (ii)   For the same circuit, if the non-linear element is replaced by a linear resistance R_L, find the choice of R_L that will absorb maximum power delivered by the independent sources.

Solution:
 (i) The Thevenin equivalent circuit across the terminals "$'a'$ and $'b'$ of fig.9.3(b) can be obtain using 'node-voltage' method (or one can apply any method to find V_{Th}).
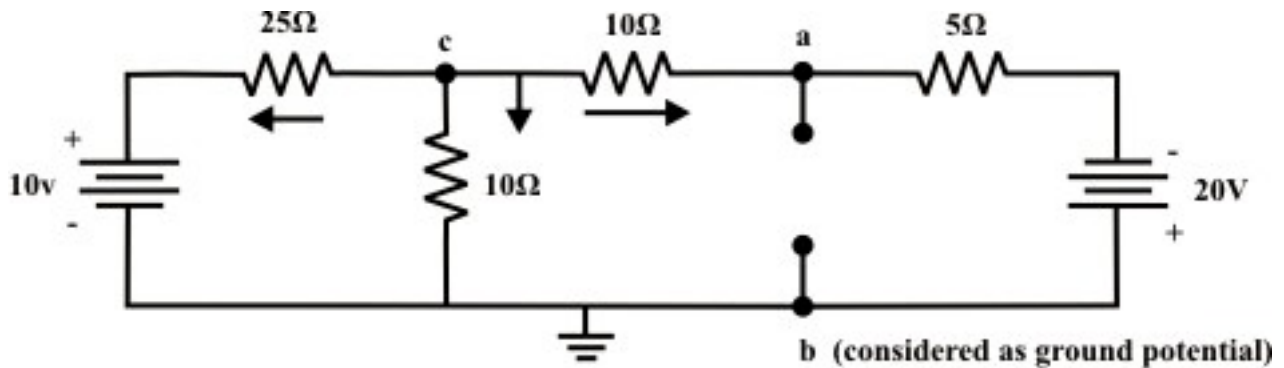

Fig. 9.3 (b)

Node voltage at 'c'

$$\frac{V_c - 10}{25} + \frac{V_c}{10} + \frac{V_c + 20}{15} = 0 \Rightarrow V_c = -4.52\,volt.$$

Current through 'c-a' branch $= \dfrac{V_c - (-20)}{15} = \dfrac{-4.52 + 20}{15} = 1.032\,A$

Voltage across the terminals 'a' and 'b'=Thevenin equivalent voltage (V_{Th}) = $-4.52 - 10 \times 1.032 = -14.48\,volt.$ (note, point 'b' is higher potential than the point 'a').

Thevenin resistance (R_{Th}):

$$R_{Th} = \left[(25 \| 10) + 10\right] \| 5 = (7.14 + 10) \| 5 = 3.87\,\Omega$$

Thevenin equivalent circuit for the network as shown in fig.9.3(b) is given below:
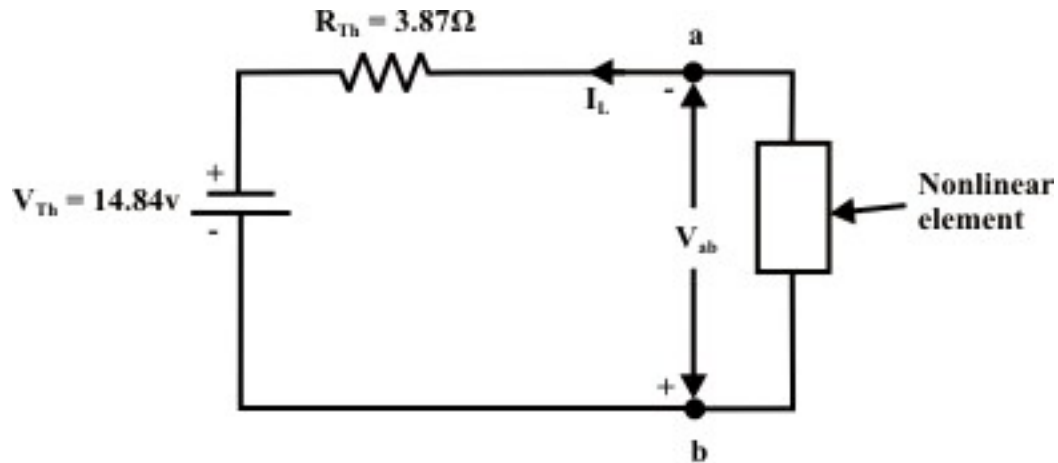

Fig. 9.3(c): Therenin equivalent circuit of fig. 9.3 (b)

To construct the source characteristic (load-line), we examine the extreme conditions under which a given source may operate. If the nonlinear load is removed (i.e. terminal 'a' and 'b' is open-circuited), the terminal voltage will be equal to the Thevenin voltage (V_{Th}). On the other hand, if the nonlinear resistance is short-circuited, the current flow through the 'a' and 'b' terminal is $I_{sc} = I_N = \dfrac{V_{Th}}{R_{Th}}$.

The operating point of the circuit is found from the intersection of source characteristic (load-line) and load characteristic curves as shown in fig.9.3(d). From this graph (see fig.9.3(d)), one can find the current flowing through the nonlinear element $I_L = 1.15\,A$ and the voltage across the nonlinear element is $V_{ab} = 10.38\ Volt$.
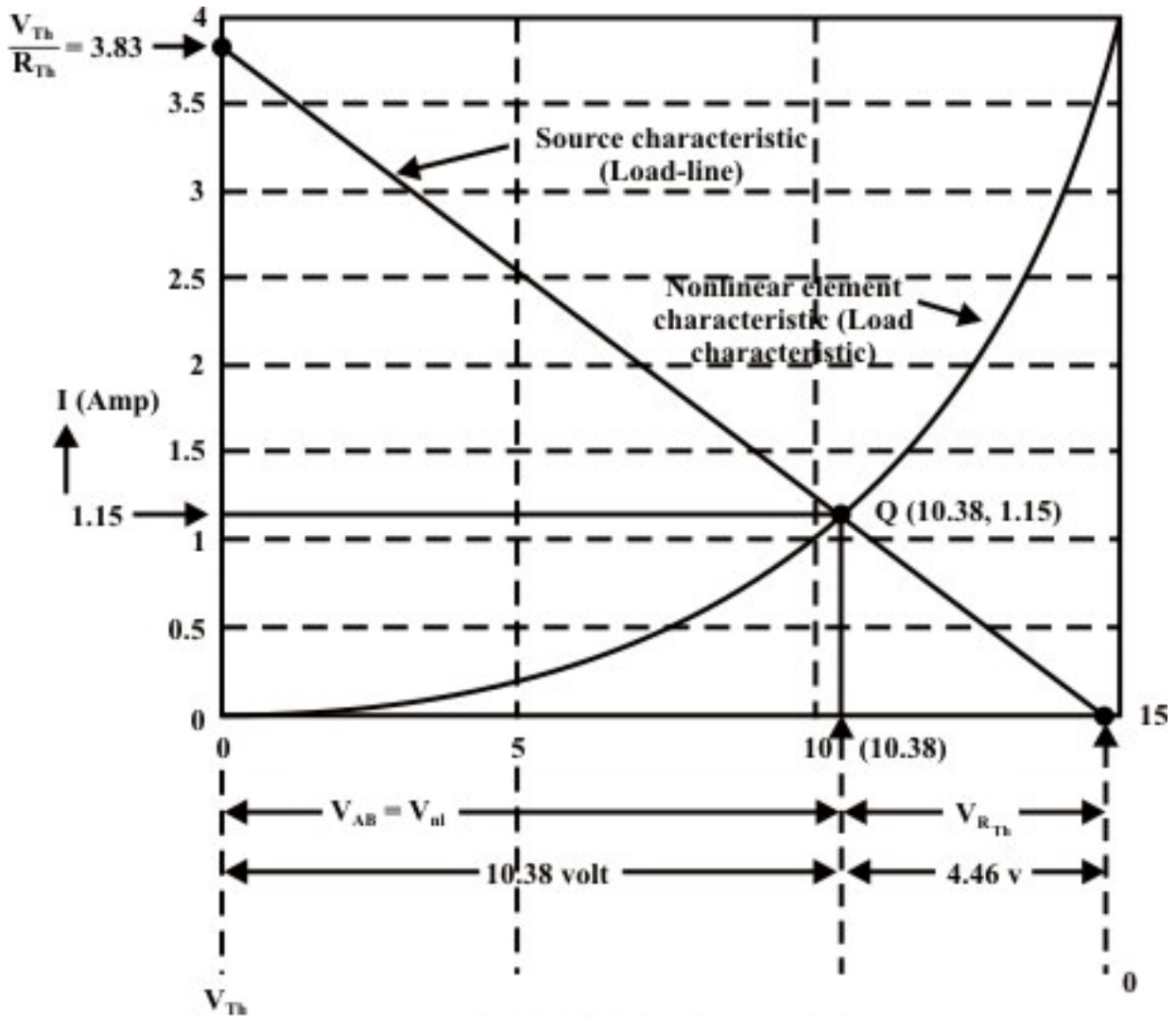
Fig. 9.3 (d): Load-line analysis

(ii) If the nonlinear resistance is replaced by a linear resistance, the maximum power transfer condition is achieved when the linear load resistance $R_L = R_{Th} = 3.87\,\Omega$. Under such condition, the network will deliver the maximum power to the load and the corresponding maximum power is given by $P_{max} = \dfrac{V_{Th}^2}{R_{Th}} = \dfrac{14.84^2}{4 \times 3.87} = 14.23\ W$

Remarks: The primary limitation of this method (load-line analysis) is accuracy, due to its graphical nature and thus it provides an approximate solution of the circuit.

Example-L.9.2: Fig.9.4(a) shows that a nonlinear element (diode) is included in the circuit.
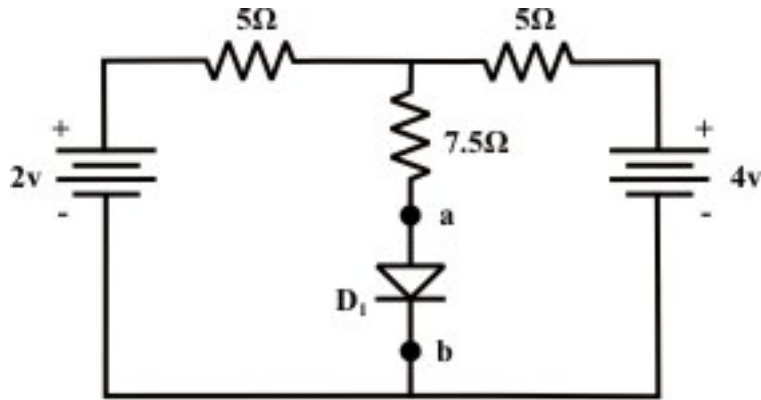
Fig. 9.4 (a)

Calculate the current flowing through the diode, voltage across the diode and the power dissipated in the diode using (i) load-line analysis (graphical) technique (ii) analytical method. The volt-ampere characteristic of the diode is given by the expression

$$I = I_0 \left(e^{V_d/a} - 1 \right) = 10^{-6} \left(e^{V_d/0.026} - 1 \right)$$ (9.1)

Solution:

    (i)   Load-line analysis method

The volt-ampere characteristics of the diode are given in tabular form using the equation (9.1).

Table: V_d = voltage across the diode in volt, I = current in diode in mA

V_d	0 v	0.1 v	0.2 v	0.25 v	0.3 v	0.32 v	0.35 v
I	0 mA	0.046mA	2.19mA	15mA	102.6mA	268mA	702mA

Step-1: Thevenized circuit

Remove the nonlinear element (diode) and replace the rest of the circuit (as shown in fig.9.4(b)) by a thevenin equivalent circuit (see fig.9.4(c)).
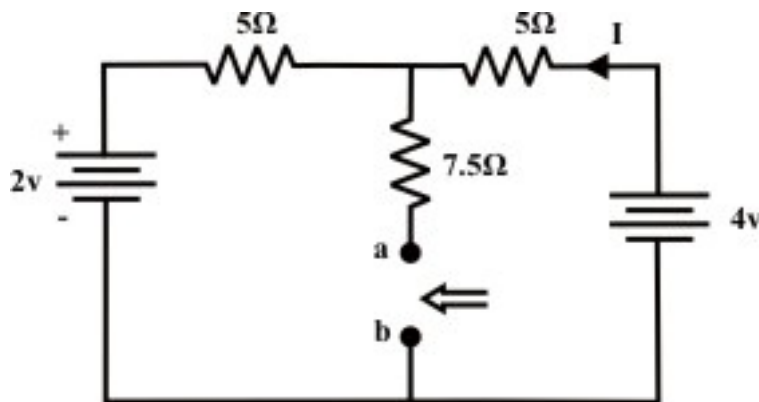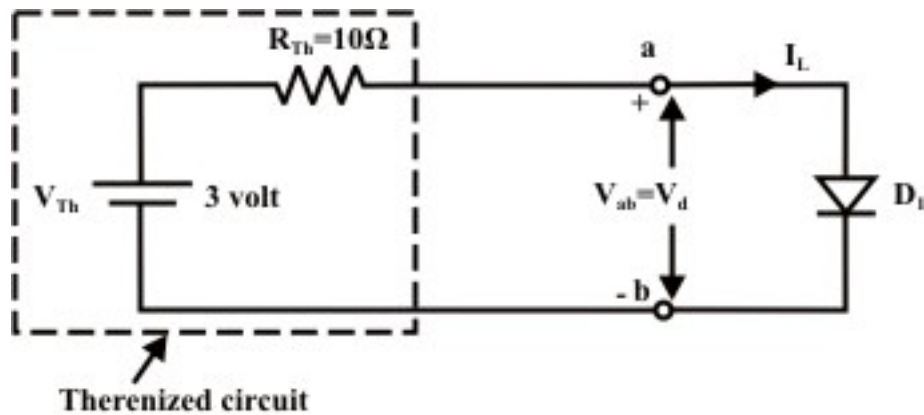

Fig. 9.4 (b)

Fig. 9.4(c) Thevenin equivalent circuit of fig. L.9.4(b)

Applying KVL around the closed path of the circuit as shown in fig.9.4(b), we get

$$4-(5+5)\times I-2=0 \Rightarrow I =\frac{2}{10}=0.2\,A$$

$V_{Th}=4-5\times0.2=3\,volt.; \quad R_{Th}=(5\|5)+7.5=10\Omega$

The equivalent Thevenin circuit is shown in fig.9.4(c).

Step-2: Load-line analysis
Draw the source and load (diode) characteristics on a same graph paper as shown in fig.L.9.4(d). Note that the Thevenin parameters obtained in step-1 are used to draw the source characteristic.
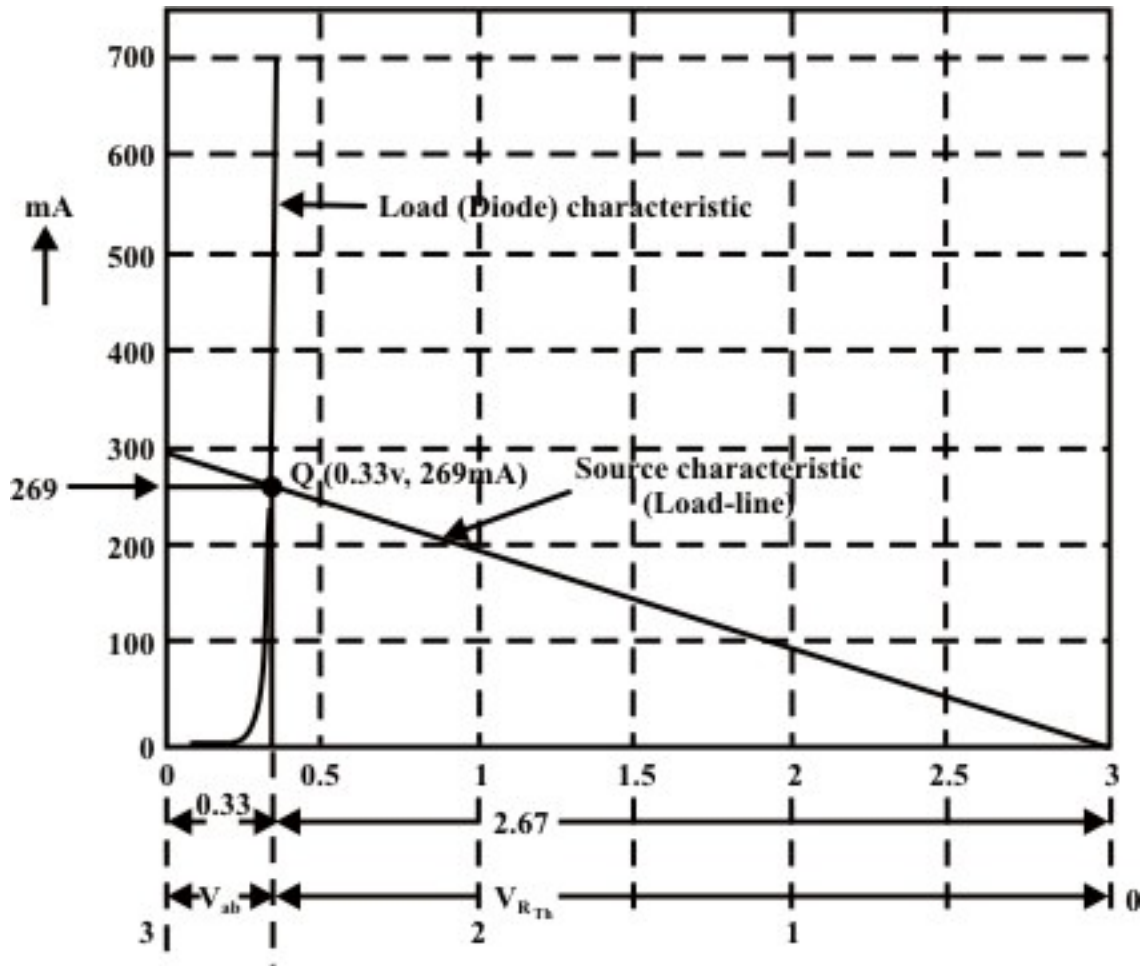
Fig. 9.4 (d): Load-line analysis

The operating point Q in fig.9.4(d) provides the information of the following quantities:

Voltage across the diode $= V_d = V_{ab} = 0.33\,volt.$

Current flowing through the diode $= I_L = 269\,mA$

Power dissipated in the diode $= P_{diode} = V_{ab} \times I_L = 88.77\,mW$

(ii) Analytical method

KVL equation around the loop of fig.L.9.4(c) is written as

$$V_{Th} - I_L \times R_{Th} - V_d = 0 \tag{9.2}$$

$$V_{Th} = I_L \times R_{Th} + V_d \quad \text{or} \quad 3 = 10^{-5}\left(e^{V_d \times 38.46}\right) + V_d \quad (note\ V_{Th} = 3\,V, R_{Th} = 10\Omega)$$

The nonlinear algebraic equation can be solved by using any numerical technique. To solve this equation, one can consider the Newton-Raphson method to the above equation. The above equation is rewritten in the following form

$$f(V_d) = 10^{-5}\left(e^{V_d \times 38.46}\right) + V_d - 3 \tag{9.3}$$

$$\dot{f}(V_d) = 1 + 0.385 \times 10^{-3} \times e^{38.46 \times V_d} \tag{9.4}$$

To solve V_d, Newton's formula is used

$$V_{d(new)} = V_{d(old)} - \left(\frac{f(V_d)}{\dot{f}(V_d)} \right)_{V_d = V_{d(old)}}$$ (9.5)

Initial guess of V_d is assumed as $V_{d(old)} = 0.36V$. The final value of V_d is obtained after four iterations and they are shown below.

Iteration	$V_{d(old)}$	$V_{d(new)}$
1.	0.36 v	0.3996 v
2.	0.3996 v	0.3886 v
3.	0.3886 v	0.3845 v
4.	0.3845 v	0.3842 v

Using equation (9.2), we get $I_L = \dfrac{V_{Th} - V_d}{R_{Th}} = \dfrac{3 - 0.3842}{10} = 261.58\,mA$ and power dissipated in diode $P_{diode} = V_{ab} \times I_L = 0.3842 \times 261.58 = 100.5\,mW$.

L.9.4 Test your understanding (Marks: 30)

T.1   The volt-ampere characteristic of a unknown device (T) is shown in fig.9.5(a) and it is connected to a resistive circuit of fig.9.5(b).
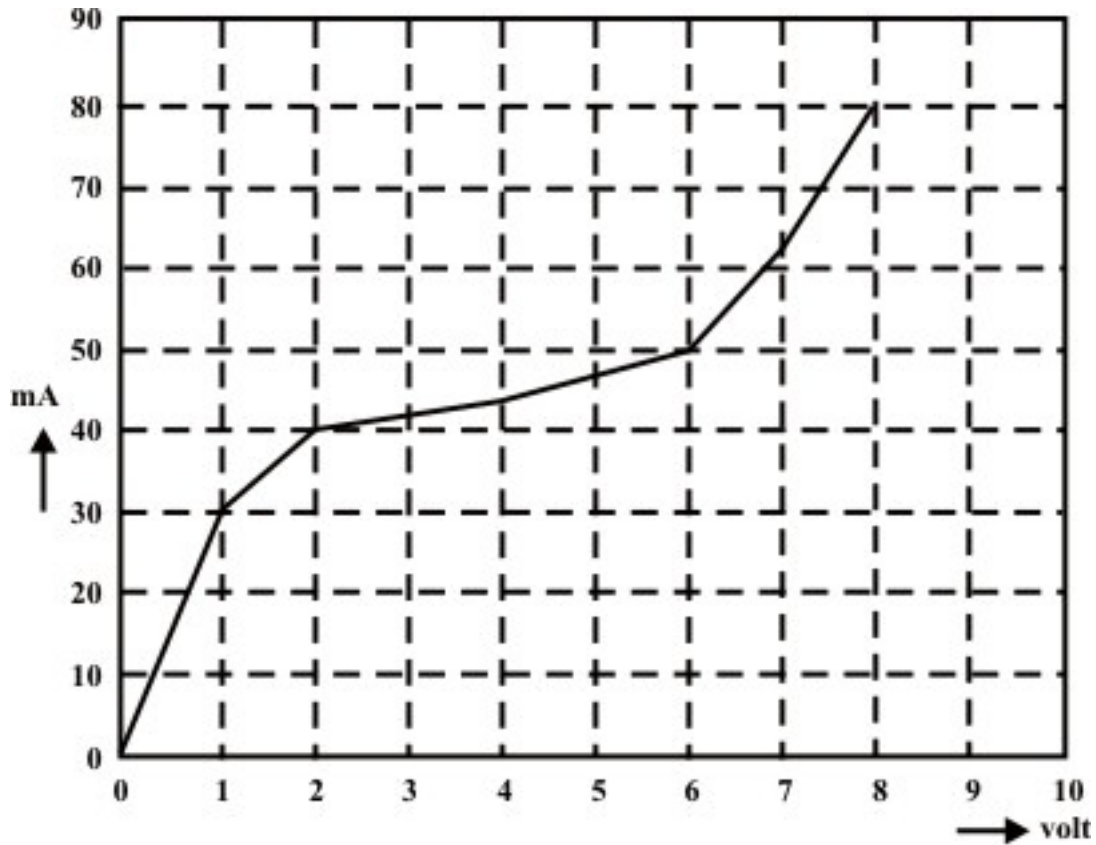
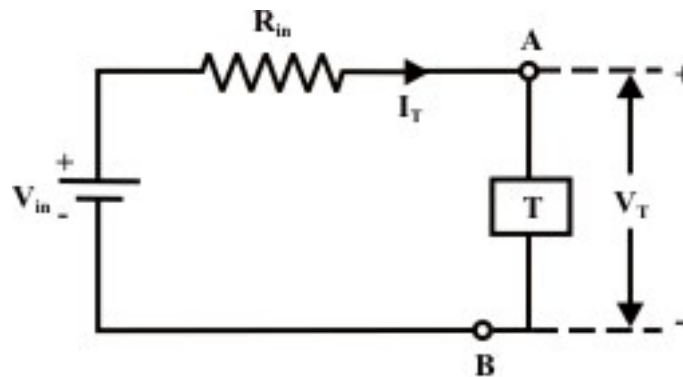Fig. 9.5 (a): Volt-ampere characteristic of unknown device

Fig. 9.5 (b)

(i) If $V_{in} = 10V$, and $R_{in} = 150\Omega$, plot the source characteristics on the voltage-current axes. [3]

(ii) Indicate the operating point on the graph and estimate the values of I_T and V_T (ans. $I_T = 43\,mA$, and $V_T = 3.7V$) [3]

T.2 A Thermistor with the volt-ampere characteristic (see fig.9.6(a)) given below is connected into one arm of a bridge as shown in the circuit of fig.9.6(b).
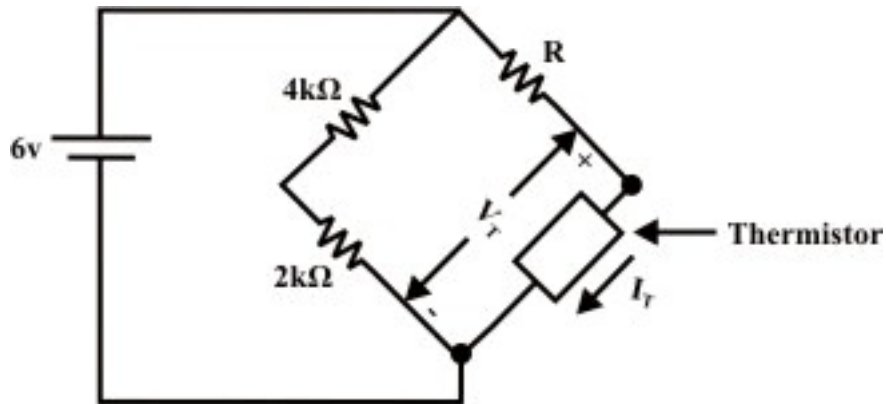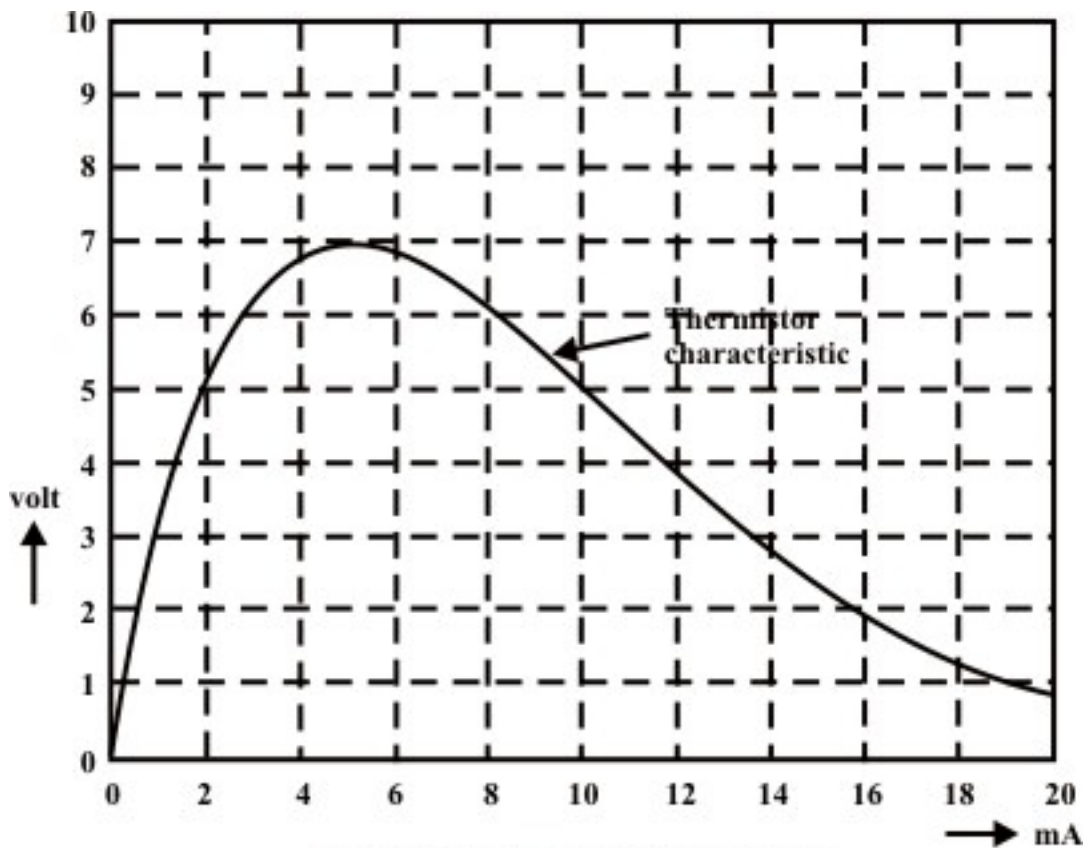
Fig. 9.6 (a)

Fig. 9.6 (b): Thermistor Characteristic

(i) Determine the value of R so that the thermistor operation point is at $V_T = 5$ *Volt and* $I_T = 2\,mA$. Justify the answer using the load-line analysis technique. (ans. $R = 0.5\,k\Omega$) [5]

(ii) Find the new operating point of the thermister when $R = 300\,\Omega$. Subsequently, compute the power dissipated by the thermistor.

(Ans. 5.3 V; 2.4 mA and 12.7 mW) [5+2]

T.9.3 The diode whose volt-ampere characteristic is given by analytical expression

$$I = I_0 \left(e^{V_d/a} - 1 \right) = 10^{-6} \left(e^{V_d/0.026} - 1 \right)$$ is used in the circuit of fig.9.7.
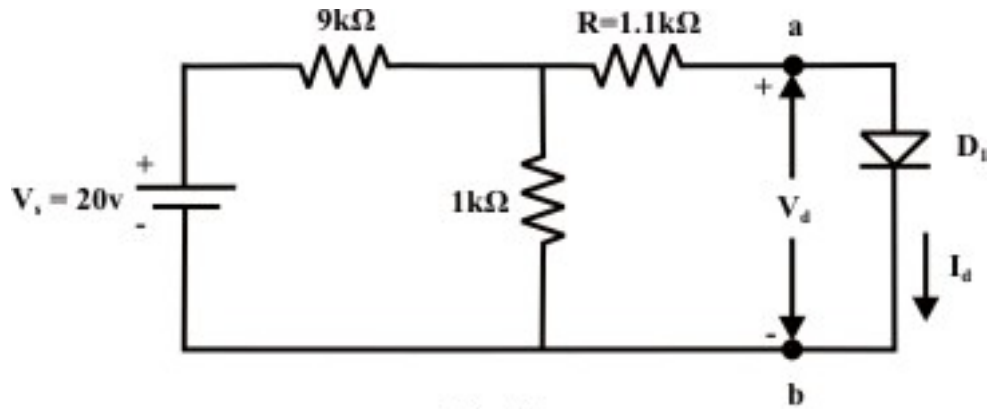

Fig. 9.7

Using analytical method, determine the following

(i) the diode current I_d , voltage V_d and power dissipated by diode.   [5]

(ii) the diode voltage for each case while the supply voltage (V_s) is changed in succession to 30 V, 40 V and 50 V respectively.   [7]