LECTURE NOTE

COURSE CODE-BCE 306

STRUCTURAL ANALYSIS 2

BCE 306 - STRUCTURAL ANALYSIS –II

Module – I

Introduction to Force and Displacement methods of structural analysis, Analysis of continuous beam and plane frame by slope deflection method and moment distribution method.

Module –II

Analysis of continuous beam and simple portals by Kani's method, Analysis of two pinned and fixed arches with dead and live loads, suspension cable with two pinned stiffening girders.

Module – III

Plastic Analysis: Plastic modulus, shear factor, plastic moment of resistance, load factor, plastic analysis of continuous beam and simple rectangular portals, Application of upper and lower bound theorems

Module – IV

Matrix method of analysis: flexibility and stiffness method, Application to simple trusses and beam

Reference Books

1. Indeterminate Structures by J.S. Kenney
2. Indeterminate Structures By C.K. Wang.
3. Matrix methods of Structural Analysis By Pandit and Gupta

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INTRODUCTION TO FORCE AND DISPLACEMENT METHODS OF STRUCTURAL ANALYSIS

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure. In the analysis of indeterminate structure it is necessary to satisfy the equilibrium equations (implying that the structure is in equilibrium) compatibility equations (requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis

2. Displacement method of analysis

In the force method of analysis,primary unknown are forces.In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium.

In the displacement method of analysis,the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3. Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2. Moment distribution method 3. Kani’s method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy- kinematic indeterminacy

Governing equations-compatibility equations	Governing equations-equilibrium equations
Force displacement relations- flexibility matrix	Force displacement relations- stiffness matrix

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

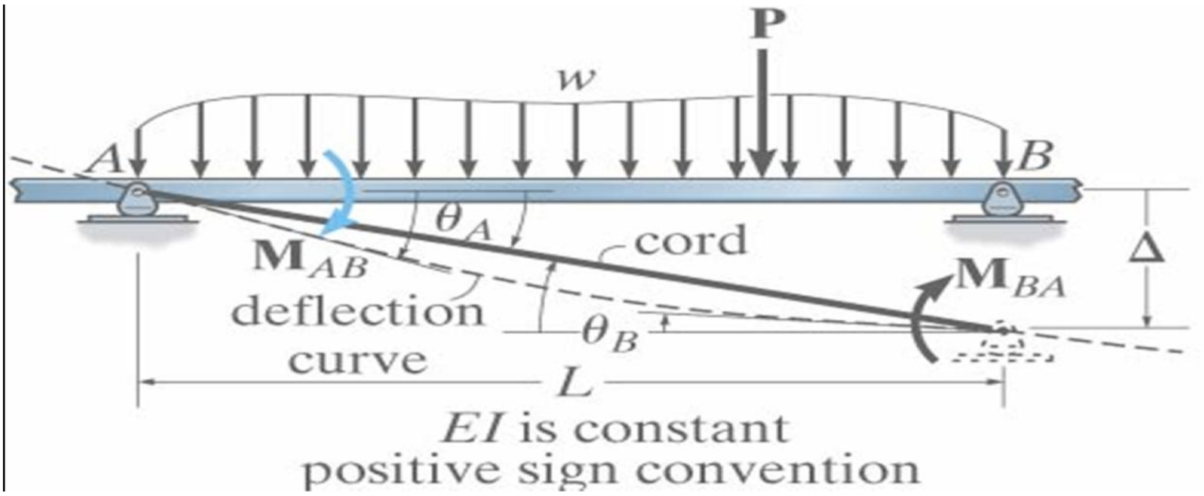
SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements.

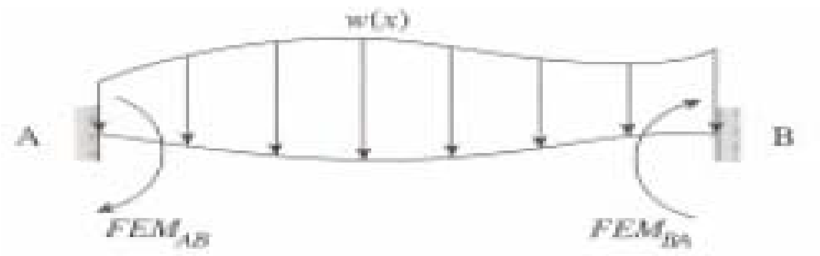
The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames. In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known. The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

Fundamental Slope-Deflection Equations:

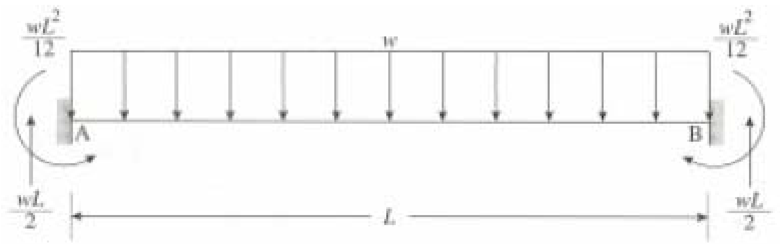
The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments M_{AB} & M_{BA} in terms of its three degrees of freedom, namely its angular displacements Θ_A& Θ_B and linear displacement Δ which could be caused by relative settlements between the supports. Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement Δ will be considered positive since this displacement causes the chord of the span and the span's chord angle to rotate clockwise. The slope deflection equations can be obtained by using principle of superposition by considering separately the moments developed at each supports due to each of the displacements Θ_A& Θ_B & Δ and then the loads.



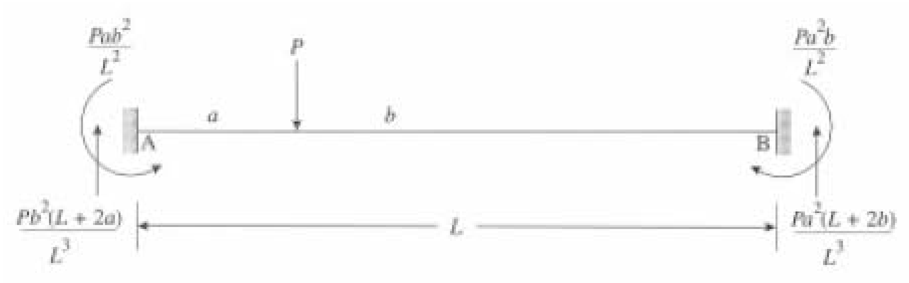
Case A: fixed-end moments



$M_{AB} = FEM_{AB}$, $M_{BA} = FEM_{BA}$



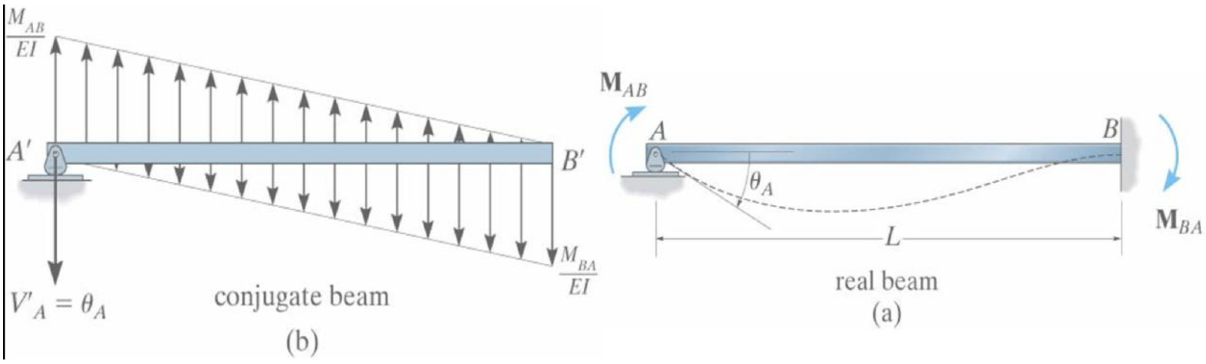
$FEM_{AB} = -\frac{wL^2}{12}$, $FEM_{BA} = \frac{wL^2}{12}$



$$\mathrm{FEM_{AB}} = -\ \frac{\mathrm{Pab^2}}{\mathrm{L^2}},\ \mathrm{FEM_{BA}} = \ \frac{\mathrm{Pa^2b}}{\mathrm{L^2}}$$

Case B: rotation at A, θ_A (angular displacement at A)

Consider node A of the member as shown in figure to rotate θ_A while its far end B is fixed. To determine the moment M_{AB} needed to cause the displacement, we will use conjugate beam method. The end shear at A acts downwards on the beam since θ_A is clockwise.



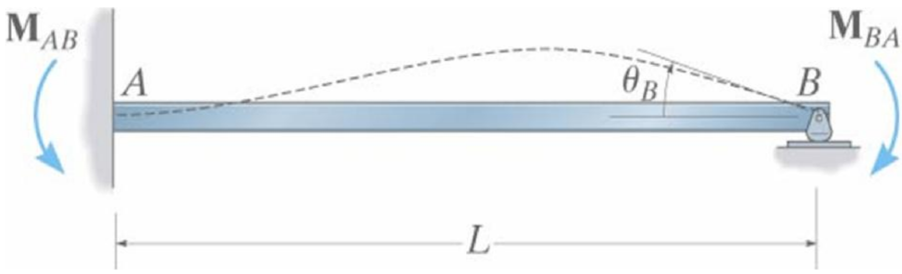
$$\Sigma \mathrm{M'_A} = 0,\quad \left[\frac{1}{2}\frac{\mathrm{M_{AB}}}{\mathrm{EI}}\mathrm{L}\right]\frac{\mathrm{L}}{3} - \left[\frac{1}{2}\frac{\mathrm{M_{BA}}}{\mathrm{EI}}\mathrm{L}\right]\frac{2\mathrm{L}}{3} = 0$$

$$\Sigma \mathrm{M'_B} = 0,\quad \left[\frac{1}{2}\frac{\mathrm{M_{BA}}}{\mathrm{EI}}\mathrm{L}\right]\frac{\mathrm{L}}{3} - \left[\frac{1}{2}\frac{\mathrm{M_{BA}}}{\mathrm{EI}}\mathrm{L}\right]\frac{2\mathrm{L}}{3} + \ \theta_\mathrm{A}\mathrm{L} = 0$$

$$\mathrm{M_{AB}} = \ \frac{4\mathrm{EI}}{\mathrm{L}}\theta_\mathrm{A},\ \mathrm{M_{BA}} = \frac{2\mathrm{EI}}{\mathrm{L}}\theta_\mathrm{A}$$

Case C: rotation at B, θ_B (angular displacement at B)

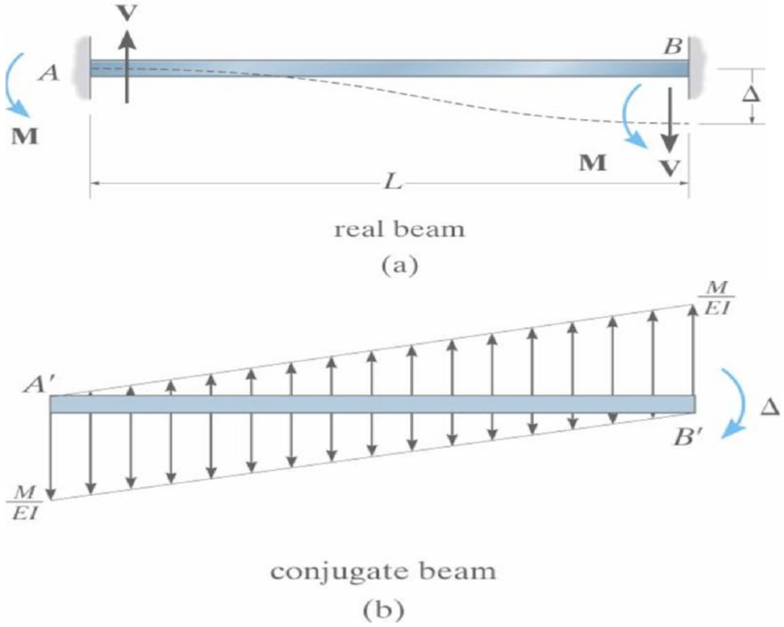
In a similar manner if the end B of the beam rotates to its final position, θ_B while end A is held fixed. We can relate the applied moment M_{BA} to the angular displacement θ_B and the reaction moment M_{AB}.



$M_{AB} = \frac{2EI}{L}\theta_B, \ M_{BA} = \frac{4EI}{L}\theta_B$

Case D: displacement of end B related to end A

If the far node B of the member is displaced relative to A so that so that the chord of the member rotates clockwise (positive displacement) .The moment M can be related to displacement Δ by using conjugate beam method. The conjugate beam is free at both the ends as the real beam is fixed supported. Due to displacement of the real beam at B, the moment at the end B` of the conjugate beam must have a magnitude of Δ.Summing moments about B` we have,



$\Sigma M'_B = 0, \quad \left[\frac{1}{2}\frac{M}{EI}(L)\right]\frac{2L}{3} - \left[\frac{1}{2}\frac{M}{EI}(L)\right]\frac{L}{3} - \Delta = 0$

$M_{AB} = M_{BA} = M = -\frac{6EI}{L^2}\Delta$

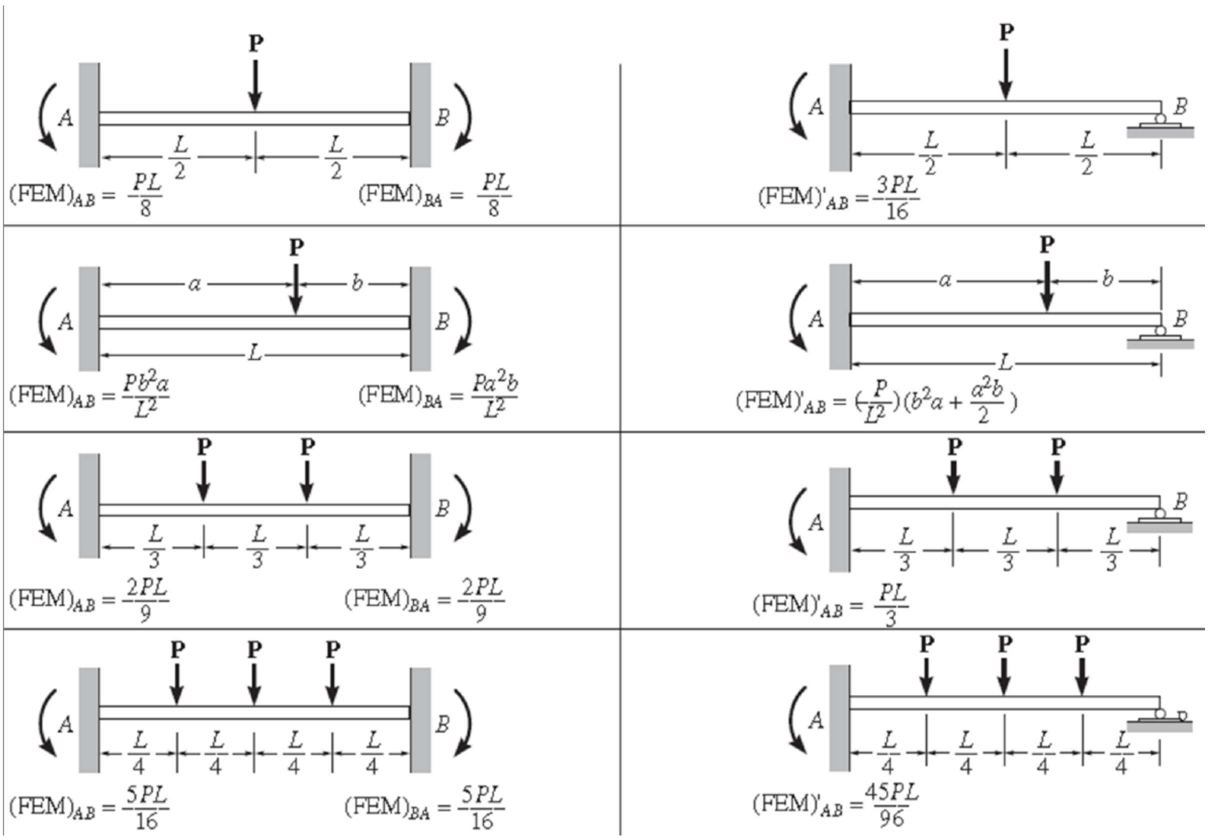
By our sign convention the induced moment is negative, since for equilibrium it acts counter clockwise on the member.

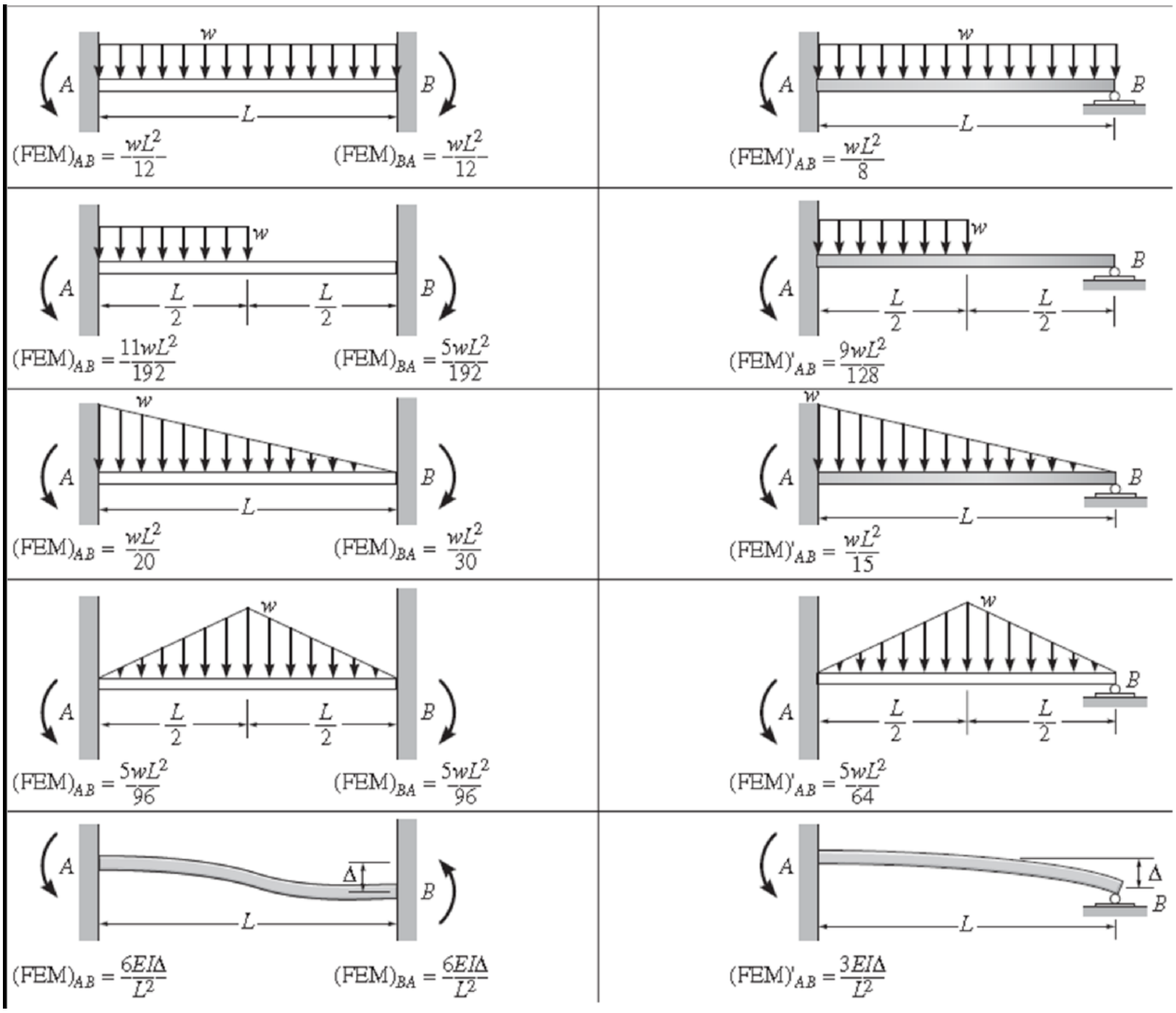
If the end moments due to the loadings and each displacements are added together, then the resultant moments at the ends can be written as,

$$M_{AB} = \frac{2EI}{L}\left[2\Theta_A + \Theta_B - \frac{3\Delta}{L}\right] + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}\left[2\Theta_B + \Theta_A - \frac{3\Delta}{L}\right] + FEM_{BA}$$

Fixed end moment table



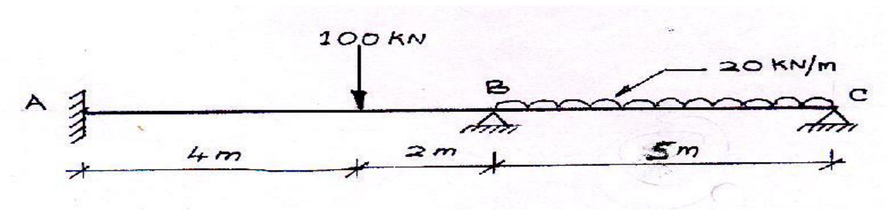


General Procedure OF Slope-Deflection Method

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so we can work out the reactions & draw the bending moment & shear force diagram.

Numerical Examples

1. Q. Analyze two span continuous beam ABC by slope deflection method. Then draw Bending moment & Shear force diagram. Take EI constant.



Fixed end moments are

$$M_{AB=}\ -\frac{Wab^2}{L^2} = -\frac{100\ \times\ 4\ \times 2^2}{6^2} = \ -44.44\ \text{KNm}$$

$$M_{BA=}\ \frac{Wa^2b}{L^2} = \frac{100\ \times\ 4^2 \times\ 2}{6^2} = \ 88.89\text{KNm}$$

$$M_{BC=}\ -\frac{wL^2}{12} = -\ \frac{20\ \times\ 5^2}{12} = \ -41.67\text{KNm}$$

$$M_{CB\ =}\ \frac{wL^2}{12} = \frac{20\ \times\ 5^2}{12} = 41.67\ \text{KNm}$$

Since A is fixed $\Theta_A = 0$ & Θ_B& $\Theta_c \neq 0$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -44.44 + \frac{2EI}{6}\theta_B = -44.44 + \frac{EI}{3}\theta_B \quad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 88.89 + \frac{4EI}{6}\theta_B = 88.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \quad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \quad \cdots\cdots(4)$$

In all the above 4 equations there are only 2 unknowns θ_B& θ_C and accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$

$M_{CB} = 0$ as end C is simply supported.

$$M_{BA} + M_{BC} = 88.89 + \tfrac{2EI}{3}\theta_B \quad - 41.67 + \tfrac{4EI}{5}\theta_B + \tfrac{2EI}{5}\theta_C = 47.22 + \tfrac{22}{15}EI\theta_B + \tfrac{2}{5}EI\theta_C = 0 \quad \cdots\cdots(5)$$

$$M_{CB} = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B = 0 \quad \cdots\cdots(6)$$

Solving the equations (5) & (6), we get

$$\theta_B = -\frac{20.83}{EI}$$

$$\theta_C = -\frac{41.67}{EI}$$

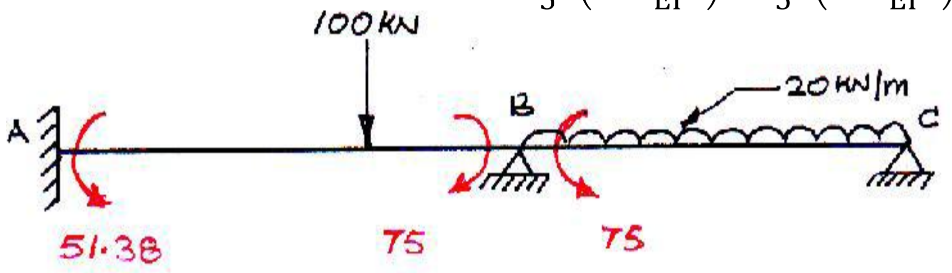
Substituting the values in the slope deflections we have,

$$M_{AB} = -44.44 + \frac{EI}{3}\left(-\frac{20.83}{EI}\right) = -51.38 \text{ KNm}$$

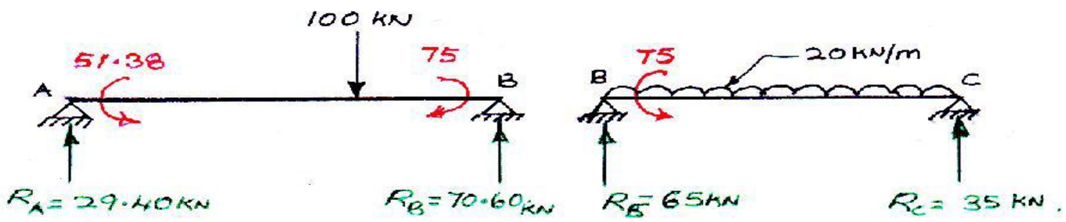
$$M_{BA} = 88.89 + \frac{2EI}{3}\left(-\frac{20.83}{EI}\right) = 75 \text{ KNm}$$

$$M_{BC} = -41.67 + \frac{4EI}{5}\left(-\frac{20.83}{EI}\right) + \frac{2EI}{5}\left(-\frac{41.67}{EI}\right) = -75 \text{ KNm}$$

$$M_{CB} = 41.67 + \frac{4EI}{5}\left(-\frac{41.67}{EI}\right) + \frac{2EI}{5}\left(-\frac{20.83}{EI}\right) = 0$$



Reactions: Consider the free body diagram of the beam



Find reactions using equations of equilibrium.

Span AB: $\Sigma M_A = 0$, $R_B \times 6 = 100\times4+75-51.38$

$\therefore R_B = 70.60$ KN

$\Sigma V = 0$, $R_A + R_B = 100$KN

$\therefore R_A = 100-70.60=29.40$ KN

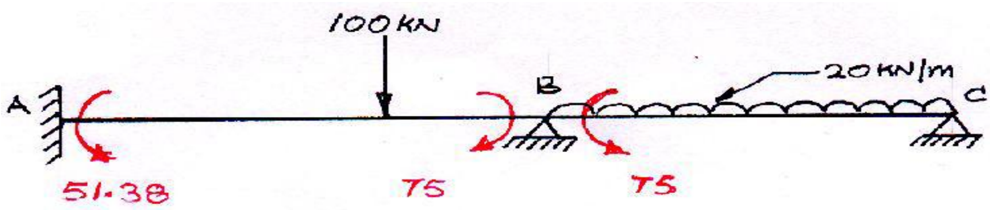
Span BC**:** $\Sigma M_C = 0$, $R_B\times5 = 20\times5\times\frac{5}{2} +75$

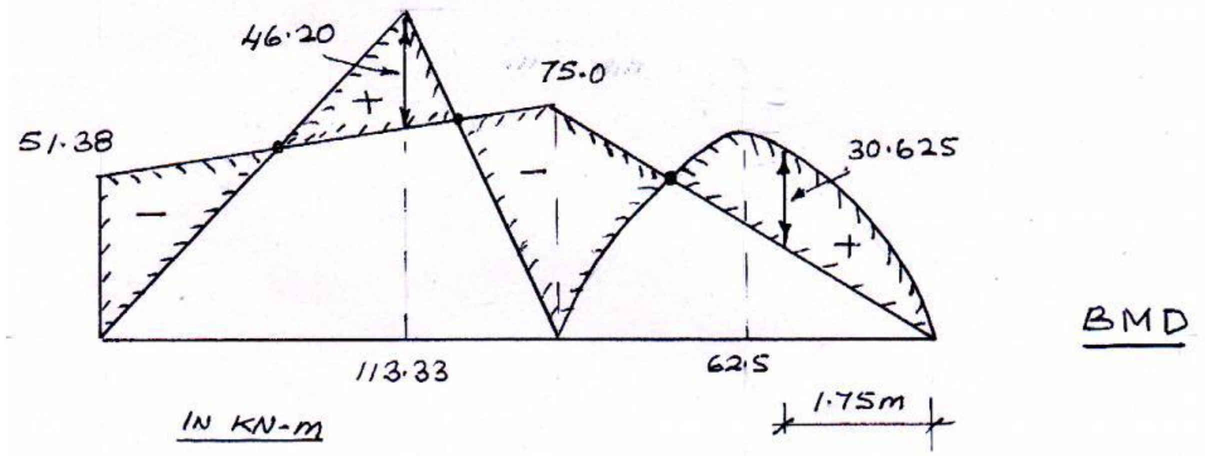
$\therefore R_B = 65$ KN

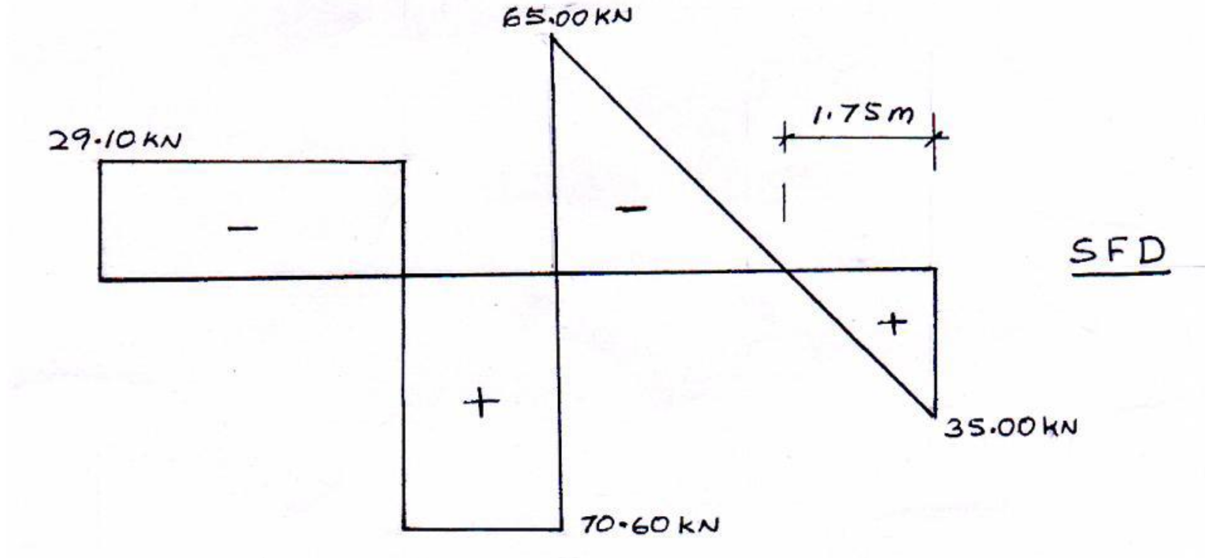
$\Sigma V=0$ $R_B+R_C = 20\times5 = 100$KN

$R_C = 100-65 = 35$ KN

Using these data BM and SF diagram can be drawn







Max BM:

Span AB: Max BM in span AB occurs under point load and can be found geometrically,
M_{max}=113.33-51.38 – $(\frac{75-51.38}{6}) \times 4 = 46.20$KNm

Span BC: Max BM in span BC occurs where shear force is zero or changes its sign. Hence consider SF equation w.r.t C

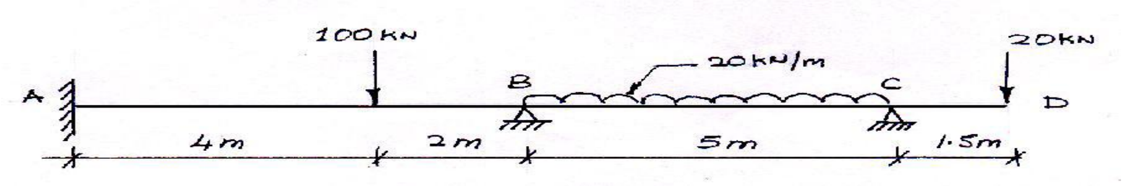
S_x = 35-20x = 0

$x = \frac{35}{20}$=1.75m

Max BM occurs at 1.75m from C

∴ $M_{max} = 35 \times 1.75 - 20\times \frac{1.75^2}{2} = 30.625\ KNm$

2. Q. Analyze continuous beam ABCD by slope deflection method and then draw bending moment diagram. Take EI constant.



$$\Theta_A = 0 \ \& \ \Theta_B \& \ \Theta_c \neq 0$$

$$M_{AB=} -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44 \text{ KNm}$$
$$M_{BA=} \frac{Wa^2b}{L^2} = \frac{100 \times 4^2 \times 2}{6^2} = 88.89 \text{KNm}$$
$$M_{BC=} -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67 \text{KNm}$$
$$M_{CB=} \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67 \text{ KNm}$$
$$M_{CD} = -20 \times 1.5 = -30 \text{ KNm}$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -44.44 + \frac{2EI}{6}\theta_B = -44.44 + \frac{EI}{3}\theta_B \quad \cdots\cdots (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 88.89 + \frac{4EI}{6}\theta_B = 88.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots (2)$$
$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \cdots\cdots (3)$$
$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \cdots\cdots (4)$$
$$M_{CD} = -20 \times 1.5 = -30 \text{ KNm}$$

In all the above equations there are only 2 unknowns $\theta_B \& \theta_C$ and accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$
$$M_{BA} + M_{BC} = 88.89 + \tfrac{2EI}{3}\theta_B - 41.67 + \tfrac{4EI}{5}\theta_B + \tfrac{2EI}{5}\theta_C = 47.22 + \tfrac{22}{15}EI\theta_B + \tfrac{2}{5}EI\theta_C = 0 \quad \cdots\cdots \quad (5)$$
$$M_{CB} + M_{CD} = 41.67 + \tfrac{4EI}{5}\theta_C + \tfrac{2EI}{5}\theta_B - 30 = 11.67 + \tfrac{2EI}{5}\theta_B + \tfrac{4EI}{5}\theta_C = 0 \quad \cdots\cdots \quad (6)$$

Solving equations (5) & (6),

$$\theta_B = -\frac{32.67}{EI}$$
$$\theta_C = \frac{1.75}{EI}$$

Substituting the values in the slope deflections we have,

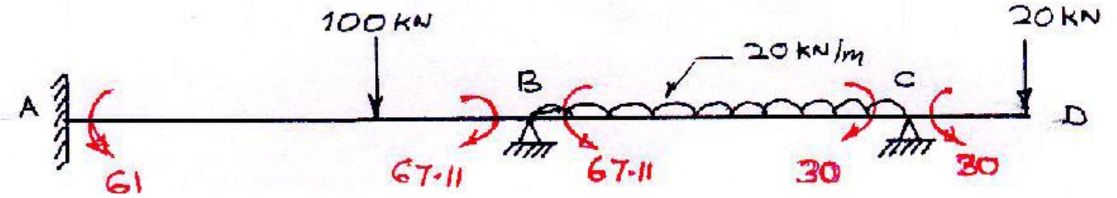
$$M_{AB} = -44.44 + \frac{EI}{3} \times \left(-\frac{32.67}{EI}\right) = -61 \text{ KNm}$$

$$M_{BA} = 88.89 + \frac{2EI}{3} \times \left(-\frac{32.67}{EI}\right) = 67.11 \text{KNm}$$

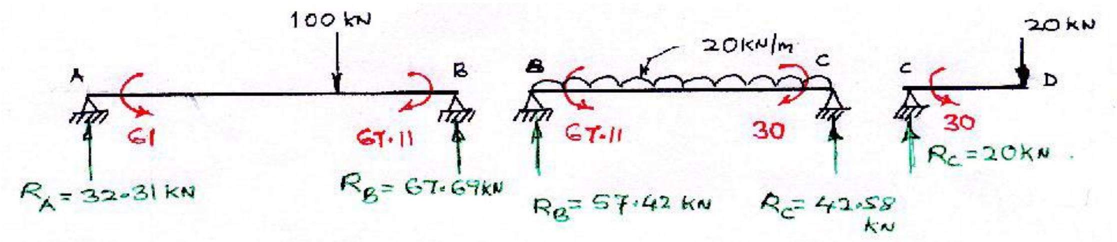
$$M_{BC} = -41.67 + \frac{4EI}{5}\left(-\frac{32.67}{EI}\right) + \frac{2EI}{5}\left(\frac{1.75}{EI}\right) = -67.11 \text{KNm}$$

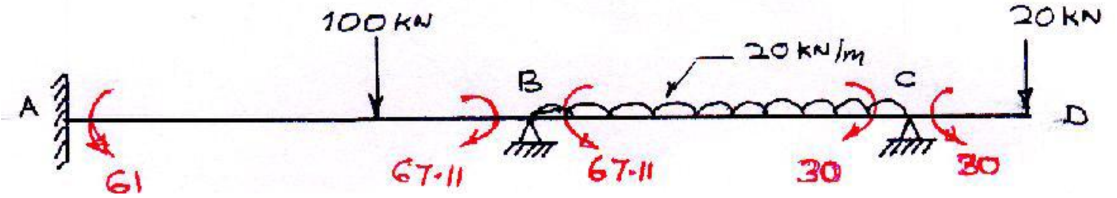
$$M_{CB} = 41.67 + \frac{4EI}{5}\left(\frac{1.75}{EI}\right) + \frac{2EI}{5}\left(-\frac{32.67}{EI}\right) = 30 \text{KNm}$$

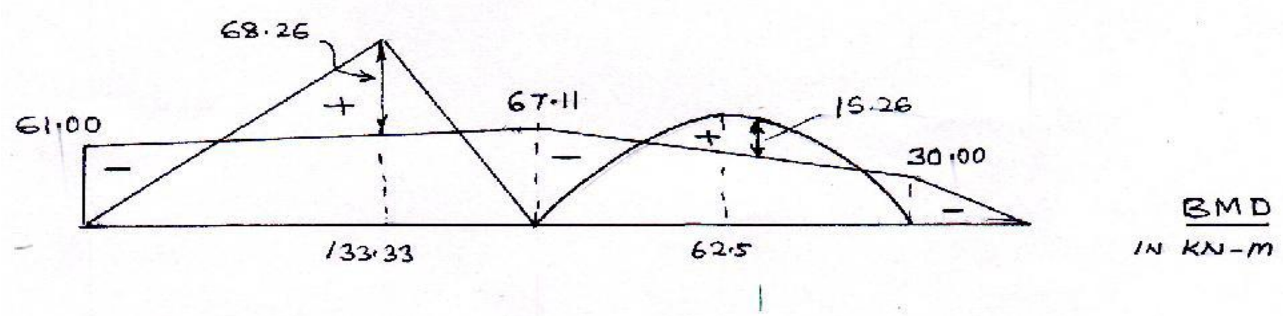
$$M_{CD} = -30 \text{KNm}$$

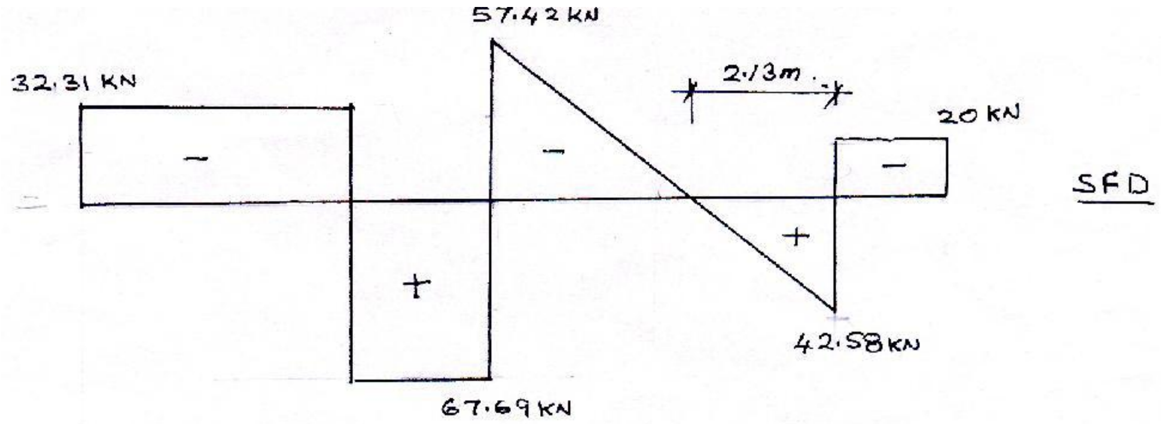


Reactions: Consider free body diagram of beam AB, BC and CD as shown









Span AB:

$R_B \times 6 = 100 \times 4 + 67.11 - 61$

$R_B = 67.69KN$

$R_A = 100 - R_B = 32.31$ KN

Span BC:

$R_C \times 5 = 20 \times \frac{5}{2} \times 5 + 30 - 67.11$

$R_C = 42.58$ KN

$R_B = 20 \times 5 - R_C = 57.42KN$

Maximum Bending Moments:

Span AB: Occurs under point load

$M_{max} = 133.33 - 61 - \frac{67.11-61}{6} \times 4 = 68.26KNm$

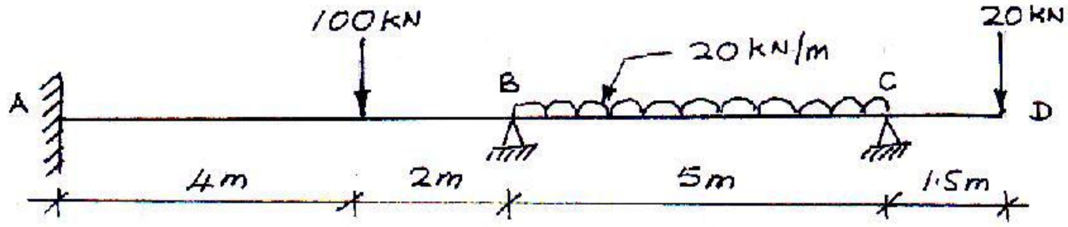
Span BC: Where SF=0, consider SF equation with C as reference

$S_x = 42.58 - 20x = 0$

$x = \frac{42.58}{20} = 2.13m$

$M_{max} = 42.58 \times 2.13 - 20 \times \frac{2.13^2}{2} - 30 = 15.26KNm$

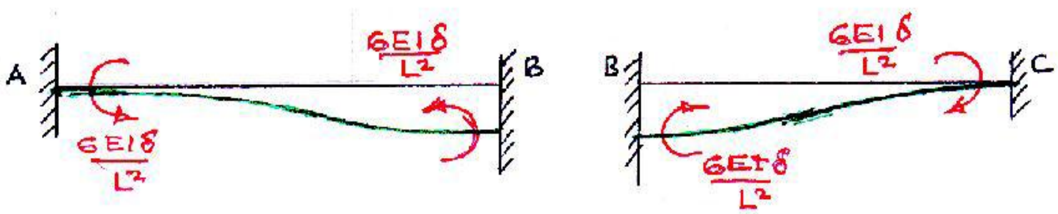
3. Q. Analyse the continuous beam ABCD shown in figure by slope deflection method. The support B sinks by 15mm.Take E =200 $\times 10^5$ KN/m^2 and I =120 $\times 10^{-6}$m^4



A. $\Theta_A = 0$ & Θ_B & $\Theta_c \neq 0$ $\Delta = 15mm$

$$M_{AB=} -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44\ KNm$$
$$M_{BA=} \frac{Wa^2b}{L^2} = \frac{100 \times 4^2 \times 2}{6^2} = 88.89KNm$$
$$M_{BC=} -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67KNm$$
$$M_{CB} = \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67\ KNm$$
$$M_{CD} = -20 \times 1.5 = -30\ KNm$$

FEM due to yield of support B



For span AB:

$$M_{AB} = M_{BA} = -\frac{6EI}{L^2}\Delta = -\frac{6\times200\times10^5\times120\times10^{-6}}{6^2}\frac{15}{1000} = -6KNm$$

For span BC:

$$M_{BC} = M_{CB} = \frac{6EI}{L^2}\Delta = \frac{6\times200\times10^5\times120\times10^{-6}}{5^2}\frac{15}{1000} = 8.64KNm$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{6}\left[2\theta_A + \theta_B - 3\frac{\Delta}{6}\right] = -44.44 + \frac{EI}{3}\theta_B - 6 = -50.44 + \frac{EI}{3}\theta_B \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{6}\left[2\theta_B + \theta_A - 3\frac{\Delta}{6}\right] = 88.89 + \frac{2EI}{3}\theta_B - 6 = 82.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{5}\left[2\theta_B + \theta_C + 3\frac{\Delta}{5}\right] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C + 8.64$$
$$= -33.03 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{5}\left[2\theta_C + \theta_B + 3\frac{\Delta}{5}\right] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B + 8.64$$
$$= 50.31 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \qquad \cdots\cdots(4)$$

$$M_{CD} = -20 \times 1.5 = -30\ KNm$$

In all the above equations there are only 2 unknowns θ_B & θ_C and accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

$$M_{BA} + M_{BC} = 82.89 + \frac{2EI}{3}\theta_B - 33.03 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C = 49.86 + \frac{22}{15}EI\theta_B + \frac{2}{5}EI\theta_C = 0 \quad \cdots\cdots \quad (5)$$

$$M_{CB} + M_{CD} = 50.31 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B - 30 = 20.31 + \frac{2EI}{5}\theta_B + \frac{4EI}{5}\theta_C = 0 \quad \cdots\cdots \quad (6)$$

Solving equations (5) & (6),

$$\theta_B = -\frac{31.35}{EI}$$

$$\theta_C = -\frac{9.71}{EI}$$

Substituting the values in the slope deflections we have,

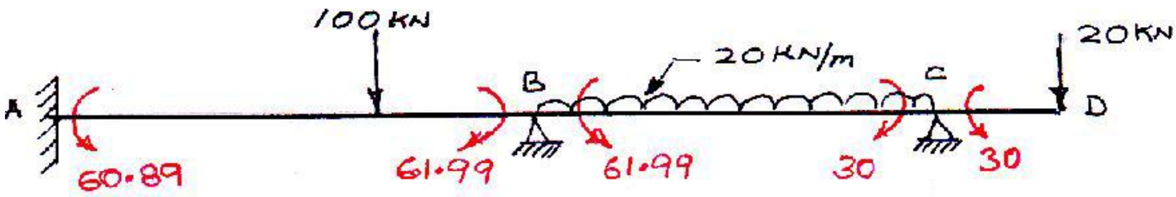
$$M_{AB} = -50.44 + \frac{EI}{3} \times \left(-\frac{31.35}{EI}\right) = -60.89 \text{ KNm}$$

$$M_{BA} = 82.89 + \frac{2EI}{3} \times \left(-\frac{31.35}{EI}\right) = 61.99KNm$$

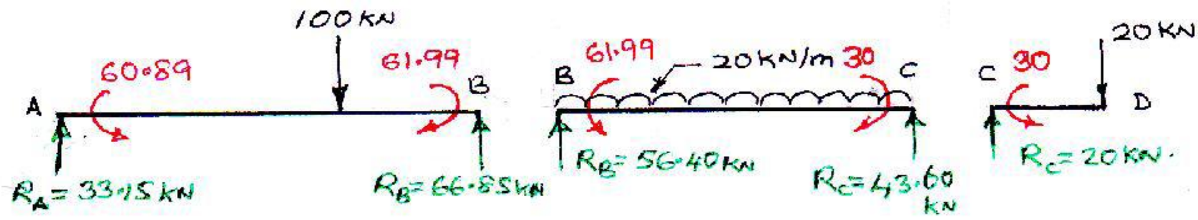
$$M_{BC} = -33.03 + \frac{4EI}{5}\left(-\frac{31.35}{EI}\right) + \frac{2EI}{5}\left(\frac{-9.71}{EI}\right) = -61.99KNm$$

$$M_{CB} = 50.31 + \frac{4EI}{5}\left(\frac{-9.71}{EI}\right) + \frac{2EI}{5}\left(-\frac{31.35}{EI}\right) = 30KNm$$

$$M_{CD} = -30KNm$$



Consider the free body diagram of continuous beam for finding reactions



REACTIONS

Span AB:

$R_B \times 6 = 100 \times 4 + 61.99 - 60.89$

$R_B = 66.85KN$

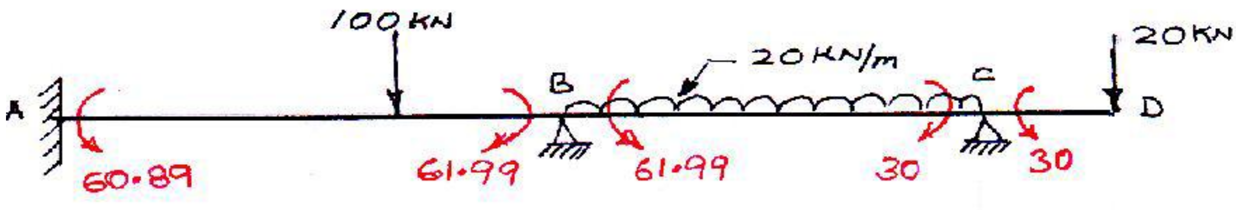
$R_A = 100 - R_B = 33.15$ KN

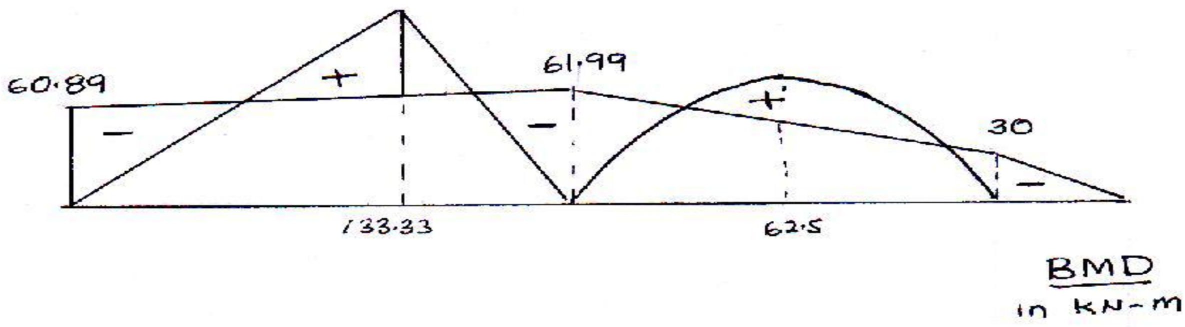
Span BC:

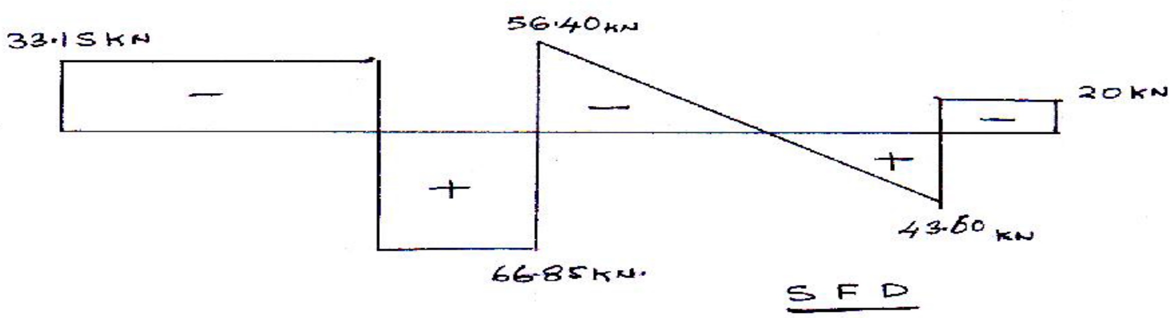
$R_B \times 5 = 20 \times \frac{5}{2} \times 5 + 61.99 - 30$

R_B =56.40 KN

R_C= 20 × 5- R_B = 43.60KN

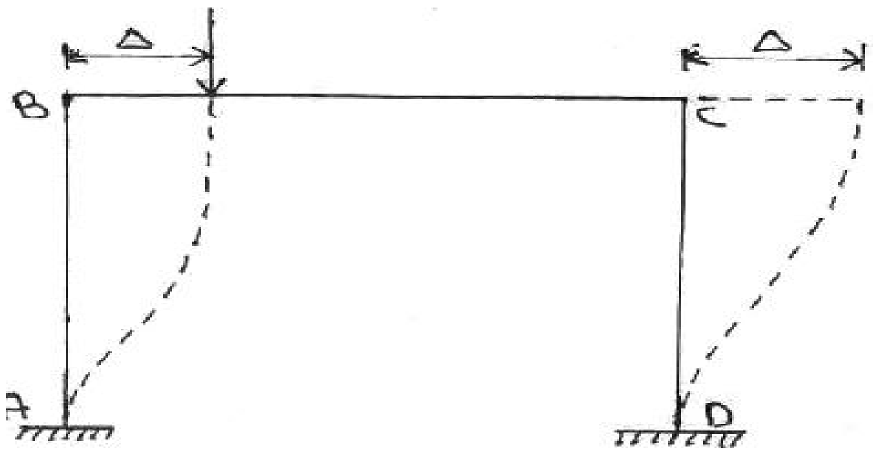




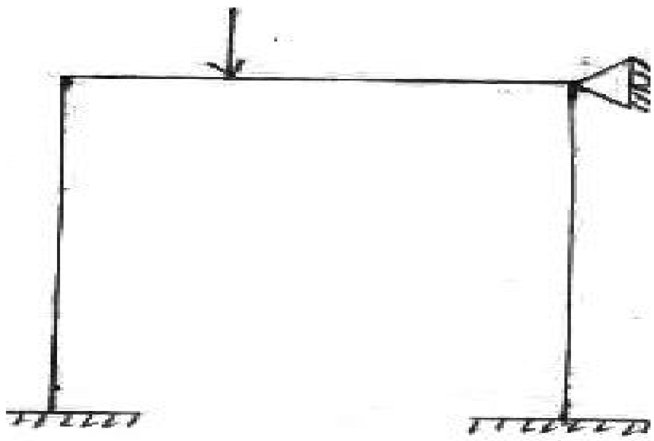


Analysis of frames (without & with sway)

The side movement of the end of a column in a frame is called sway. Sway can be prevented by unyielding supports provided at the beam level as well as geometric or load symmetry about vertical axis.

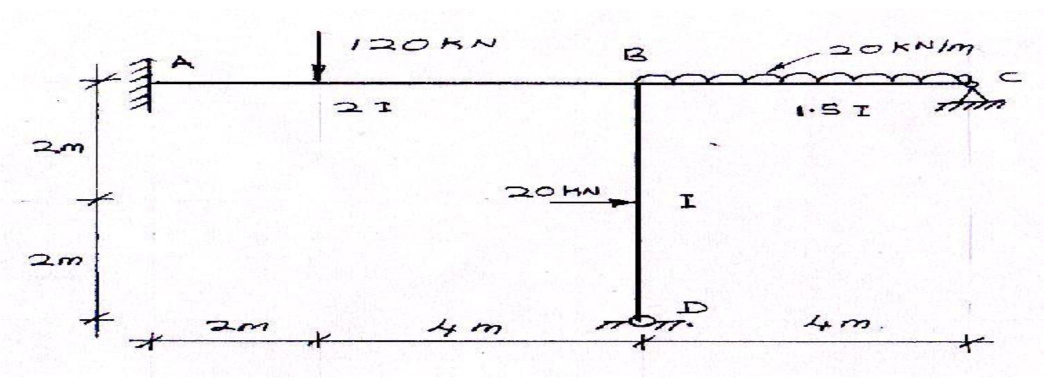


Frame with sway



Sway prevented by unyielding support

4. Q. Analyse the simple frame shown in figure. End A is fixed and ends B & C are hinged. Draw the bending moment diagram.



$$\Theta_A = 0 \;\&\; \Theta_B, \Theta_c \;\&\; \Theta_D \neq 0$$

$$M_{AB=} \; -\frac{Wab^2}{L^2} = -\frac{120\ \times\ 2\ \times 4^2}{6^2} = -106.67\ KNm$$

$$M_{BA=} \; \frac{Wa^2b}{L^2} = \frac{120\ \times\ 2^2 \times\ 4}{6^2} = 53.33KNm$$

$$M_{BC=} \; -\frac{wL^2}{12} = -\frac{20\ \times\ 4^2}{12} = -26.67KNm$$

$$M_{CB\,=} \; \frac{wL^2}{12} = \frac{20\ \times\ 4^2}{12} = 26.67\ KNm$$

$$M_{DB} = -\frac{wL}{8} = -\frac{20 \times 4}{8} = -10\ KNm$$

$$M_{BD} = \frac{wL}{8} = \frac{20 \times 4}{8} = 10\ KNm$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -106.67 + \frac{2E(2I)}{6}\theta_B$$

$$= -106.67 + \frac{2EI}{3}\theta_B \qquad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 53.33 + \frac{2E(2I)}{6}2\theta_B = 53.33 + \frac{4EI}{3}\theta_B \qquad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B \qquad \cdots\cdots(4)$$

$$M_{BD} = M_{FBD} + \frac{2EI}{L}[2\theta_B + \theta_D] = 10 + \frac{2EI}{4}2\theta_B + \frac{2EI}{4}\theta_D$$

$$= 10 + EI\theta_B + \frac{EI}{2}\theta_D \qquad \cdots\cdots(5)$$

$$M_{DB} = M_{FDB} + \frac{2EI}{L}[2\theta_D + \theta_B] = -10 + \frac{2EI}{4}2\theta_D + \frac{2EI}{4}\theta_B$$

$$= -10 + EI\theta_D + \frac{EI}{2}\theta_B \qquad \cdots\cdots(6)$$

In all the above equations there are only 3 unknowns $\theta_B \& \theta_C \& \theta_D$ and accordingly the boundary conditions are

$$M_{BA} + M_{BC} + M_{BD} = 0$$
$$M_{CB} = 0$$
$$M_{DB} = 0$$

$M_{BA} + M_{BC} + M_{BD} = 53.33 + \frac{4EI}{3}\theta_B - 26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C + 10 + EI\theta_B + \frac{EI}{2}\theta_D =$
$36.66 + \frac{23}{6}EI\theta_B + \frac{3}{4}EI\theta_C + \frac{EI}{2}\theta_D = 0 \qquad \cdots\cdots \quad (7)$

$M_{CB} = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B = 0 \qquad \cdots\cdots \quad (8)$

$M_{DB} == -10 + EI\theta_D + \frac{EI}{2}\theta_B$ =0 $\qquad \cdots\cdots \quad (9)$

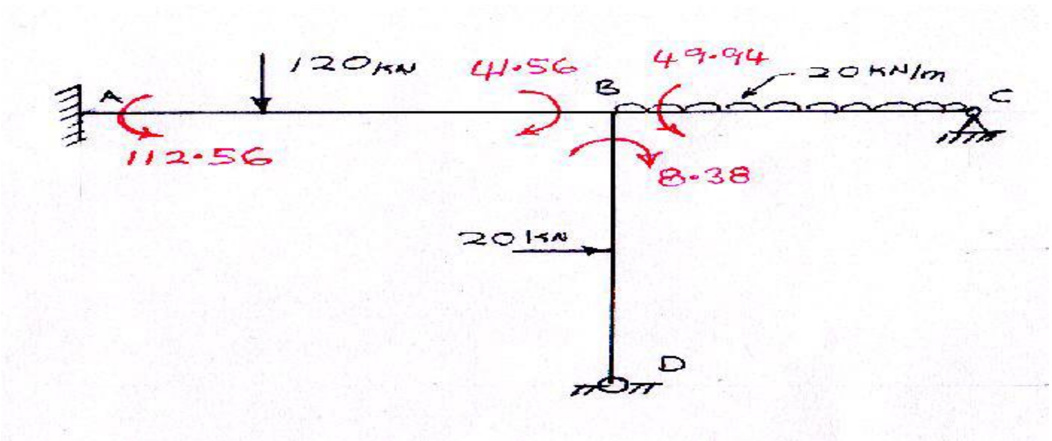
Solving equations (7) & (8) & (9),

$$\theta_B = -\frac{8.83}{EI}$$
$$\theta_C = -\frac{13.36}{EI}$$
$$\theta_D = \frac{14.414}{EI}$$

Substituting the values in the slope deflections we have,

$$M_{AB} = -106.67 + \frac{2}{3}(-8.83) = -112.56KNm$$
$$M_{BA} = 53.33 + \frac{4}{3}(-8.83) = 41.56KNm$$
$$M_{BC} = -26.67 + \frac{3}{2}(-8.83) + \frac{3}{4}(-13.36) = -49.94KNm$$
$$M_{CB} = 26.67 + \frac{3}{2}(-13.36) + \frac{3}{4}(-8.83) = 0$$

$$M_{BD} = 10 - 8.83 + \frac{1}{2}(14.414) = 8.38KNm$$
$$M_{DB} = -10 + \frac{1}{2}(-8.83) + (14.414) = 0$$



REACTIONS:
SPAN AB:

$$R_B = \frac{41.56 - 112.56 + 120 \times 2}{6} = 28.17KN$$
$$R_A = 120 - R_B = 91.83KN$$

SPAN BC:

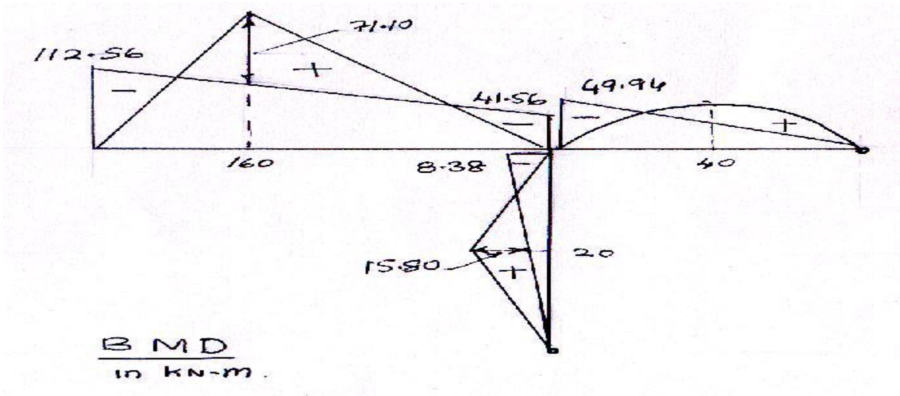
$$\mathrm{R_B} = \frac{49.94 + 20 \times 4 \times 2}{4} = 52.485\mathrm{KN}$$

$$\mathrm{R_C} = 20 \times 4 - \mathrm{R_B} = 27.515\mathrm{KN}$$

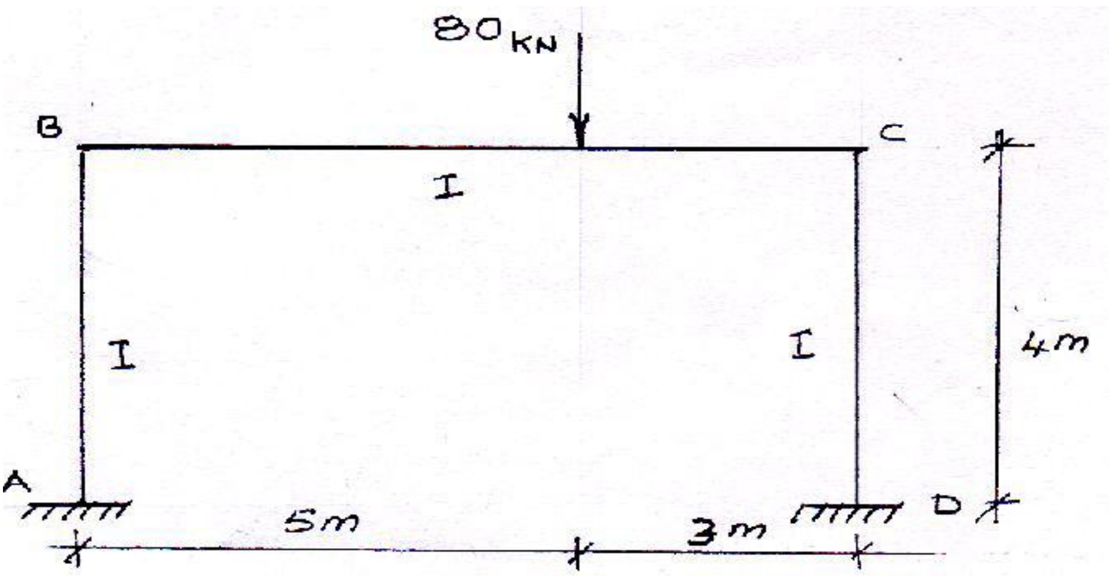
Column BD:

$$H_D = \frac{20 \times 2 - 8.38}{4} = 7.92KN$$

$$H_B = 12.78KN$$



5.Q. Analyse the portal frame and then draw the bending moment diagram



A. This is a symmetrical frame and unsymmetrically loaded, thus it is an unsymmetrical problem and there is a sway ,assume sway to right

$$\theta_A = 0, \theta_D = 0, \theta_B \neq 0, \theta_C \neq 0$$

FEMS:

$$M_{BC=} -\frac{wab^2}{L^2} = -\frac{80 \times 5 \times 3^2}{8^2} = -56.25KNm$$

$$M_{CB=} \frac{wa^2b}{L^2} = \frac{80 \times 3 \times 5^2}{8^2} = 93.75\ KNm$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(0 + \theta_B - 3\frac{\Delta}{L}\right)$$
$$= \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} \qquad \cdots\cdots (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\frac{\Delta}{L}\right]$$
$$= 0 + \frac{2EI}{4}\left(2\theta_B + 0 - 3\frac{\Delta}{4}\right) = EI\theta_B - \frac{3EI}{8}\Delta \qquad \cdots\cdots (2)$$
$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -56.25 + \frac{2EI}{8}(2\theta_B + \theta_C)$$
$$= -56.25 + \frac{EI}{2}\theta_B + \frac{EI}{4}\theta_C \qquad \cdots\cdots (3)$$
$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 93.75 + \frac{2EI}{8}[2\theta_C + \theta_B]$$
$$= 93.75 + \frac{EI}{2}\theta_C + \frac{EI}{4}\theta_B \qquad \cdots\cdots (4)$$
$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(2\theta_C + 0 - 3\frac{\Delta}{L}\right)$$
$$= EI\theta_C - \frac{3EI\Delta}{8} \qquad \cdots\cdots (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - 3\frac{\Delta}{L}\right]$$
$$= 0 + \frac{2EI}{4}\left(0 + \theta_C - 3\frac{\Delta}{4}\right) = \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta \qquad \cdots\cdots (6)$$

In the above equation there are three unknowns, θ_B, θ_C& Δ,accordingly the boundary conditions are,joint conditions , $M_{BA} + M_{BC} = 0, M_{CB} + M_{CD} = 0$
shear condition, $H_A + H_D + \sum P_H = 0, \quad \frac{M_{AB}+M_{BA}}{4}+\frac{M_{CD}+M_{DC}}{4} = 0$
$$\therefore M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$$

Now, $M_{BA} + M_{BC} = EI\theta_B - \frac{3EI}{8}\Delta - 56.25 + \frac{EI}{2}\theta_B + \frac{EI}{4}\theta_C$ =-56.25+$\frac{3EI}{2}\theta_B$+$\frac{EI}{4}\theta_C - \frac{3EI}{8}\Delta$= 0 $\cdots\cdots (7)$
$M_{CB} + M_{CD} = 93.75 + \frac{EI}{2}\theta_C + \frac{EI}{4}\theta_B$+ $EI\theta_C - \frac{3EI\Delta}{8} = 93.75 + \frac{EI}{4}\theta_B + \frac{3EI}{2}\theta_C - \frac{3EI\Delta}{8} = 0$ $\cdots\cdots (8)$

$M_{AB} + M_{BA} + M_{CD} + M_{DC} = \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} + EI\theta_B - \frac{3EI}{8}\Delta + EI\theta_C - \frac{3EI\Delta}{8}$ +$\frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta$= $\frac{3EI}{2}\theta_B + \frac{3EI}{2}\theta_C - \frac{3EI}{2}\Delta$= 0

$$\therefore EI\Delta = EI\theta_B + EI\theta_C \qquad \cdots\cdots (9)$$

Substitute in (7) & (8), equation (9),

-56.25+$\frac{3EI}{2}\theta_B$+$\frac{EI}{4}\theta_C - \frac{3EI}{8}(\theta_B + \theta_C) = 0$

-56.25+$\frac{9EI}{8}\theta_B$-$\frac{EI}{8}\theta_C$=0 $\cdots\cdots (10)$

93.75+$\frac{EI}{4}\theta_B + \frac{3EI}{2}\theta_C - \frac{3EI}{8}(\theta_B + \theta_C) = 0$

$93.75-\frac{EI}{8}\theta_B+\frac{9EI}{8}\theta_C = 0$ $\cdots\cdots(11)$

Solving equations (10) & (11), we get

$$\theta_B = \frac{41.25}{EI}$$

By equation (10),

$$EI\theta_C = 8\left[-56.25 + \frac{9EI}{8}\theta_B\right] = 8\left[-56.25 + \frac{9}{8}(41.25)\right] = -78.75$$

$$\theta_C = \frac{-78.75}{EI}$$

$EI\Delta = EI\theta_B + EI\theta_C = 41.25 - 78.75 = -37.5$

$$\Delta = \frac{-37.5}{EI}$$

Substituting these values in slope deflection equation, we have,

$$M_{AB} = \frac{1}{2}(41.25) - \frac{3}{8}(-37.5) = 34.69KNm$$

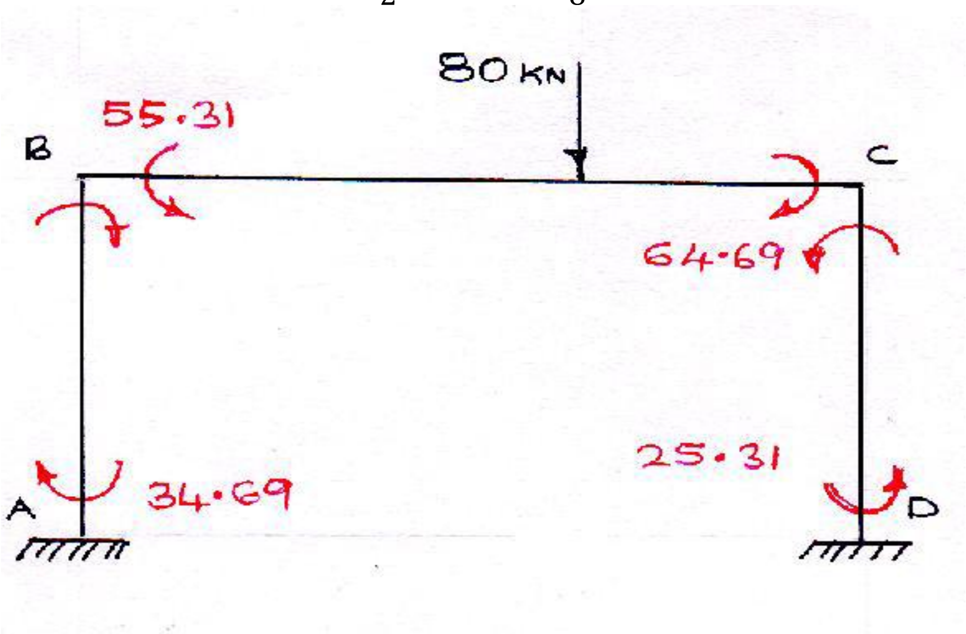
$$M_{BA} = 41.25 - \frac{3}{8}(-37.5) = 55.31KNm$$

$$M_{BC} = -56.25 + \frac{1}{2}(41.25) + \frac{1}{4}(-78.75) = -55.31KNm$$

$$M_{CB} = 93.75 + \frac{1}{2}(-78.75) + \frac{1}{4}(41.25) = 64.69KNm$$

$$M_{CD} = -78.75 - \frac{3}{8}(-37.5) = -64.69KNm$$

$$M_{DC} = \frac{1}{2}(-78.75) - \frac{3}{8}(-37.5) = -25.31KNm$$



Reactions: consider the free body diagram of beam and columns

Column AB:

$$H_A = \frac{34.69 + 55.31}{4} = 22.5KN$$

Span BC:

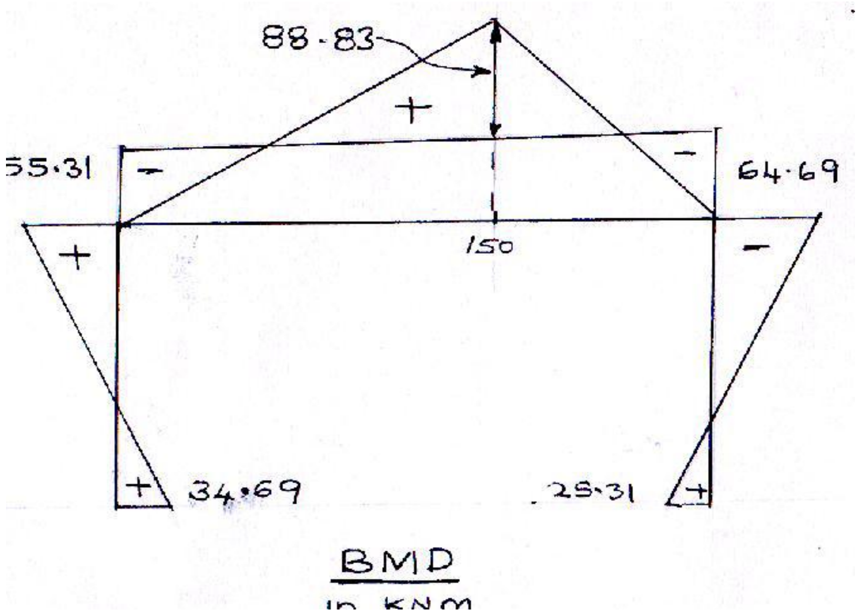
$$R_B = \frac{55.31 - 64.69 + 80 \times 3}{8} = 28.83KN$$

$$R_C = 80 - R_B = 51.17KN$$

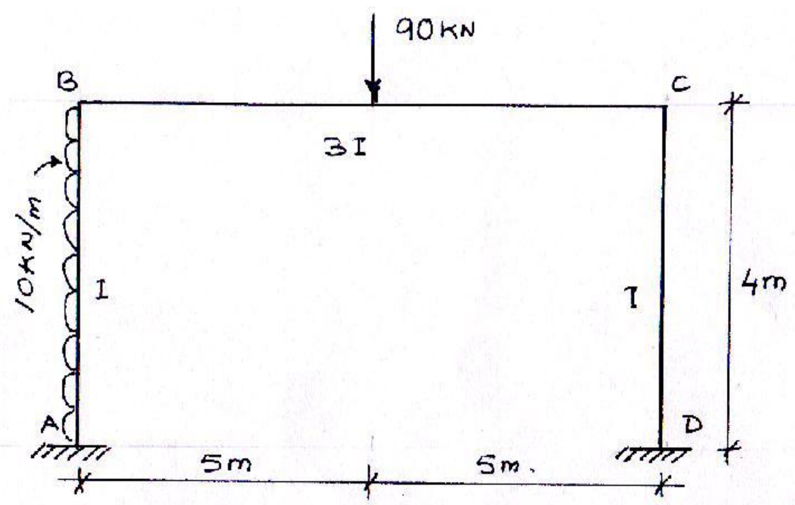
Column CD:

$$H_D = \frac{64.69 + 25.31}{4} = 22.5KN$$

Check:
ΣH = 0, $H_A + H_D = 0$, 22.5 – 22.5 = 0
Hence okay



6. Q. Frame ABCD is subjected to a horizontal force of 20 KN at joint C as shown in figure. Analyse and draw bending moment diagram.



A. The frame is symmetrical but loading is unsymmetrical. Hence there is a sway, assume sway towards right. In this problem

$$\theta_A = 0, \theta_D = 0, \theta_B \neq 0, \theta_C \neq 0$$

FEMS:

$$M_{FAB=} \ -\frac{wL^2}{12} = -\frac{10 \times 4^2}{12} = -13.33\text{KNm}$$

$$M_{FBA=} \ \frac{wL^2}{12} = \frac{10 \times 4^2}{12} = 13.33 \text{ KNm}$$

$$M_{FBC=} \ -\frac{wL}{8} = -\frac{90 \times 10}{8} = -112.5\text{KNm}$$

$$M_{FCB=} \ \frac{wL}{8} = \frac{90\times10}{8} = 112.5\text{KNm}$$

Slope deflection equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\frac{\Delta}{L}\right] = -13.33 + \frac{2EI}{4}\left(0 + \theta_B - 3\frac{\Delta}{4}\right)$$
$$= -13.33 + \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} \qquad \cdots\cdots (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\frac{\Delta}{L}\right]$$
$$= 13.33 + \frac{2EI}{4}\left(2\theta_B + 0 - 3\frac{\Delta}{4}\right) = 13.33 + EI\theta_B - \frac{3EI}{8}\Delta \qquad \cdots\cdots (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -112.5 + \frac{2E3I}{10}(2\theta_B + \theta_C)$$
$$= -112.5 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C \qquad \cdots\cdots (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 112.5 + \frac{2E3I}{10}[2\theta_C + \theta_B]$$
$$= 112.5 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B \qquad \cdots\cdots (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(2\theta_C + 0 - 3\frac{\Delta}{4}\right)$$
$$= EI\theta_C - \frac{3EI\Delta}{8} \qquad \cdots\cdots (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - 3\frac{\Delta}{L}\right]$$
$$= 0 + \frac{2EI}{4}\left(0 + \theta_C - 3\frac{\Delta}{4}\right) = \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta \qquad \cdots\cdots (6)$$

In the above equation there are three unknowns, $\theta_B, \theta_C \& \Delta$, accordingly the boundary conditions are,

Joint conditions , $M_{BA} + M_{BC} = 0, M_{CB} + M_{CD} = 0$

Shear condition, $H_A + H_D + \sum P_H = 0, \quad H_A + H_D + 40 = 0$

$$H_A \times 4 = M_{AB} + M_{BA} - 10 \times 4 \times \frac{4}{2}$$

$$H_A = \frac{M_{AB} + M_{BA} - 80}{4}$$

$$H_D \times 4 = M_{CD} + M_{DC}$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$\therefore \frac{M_{AB} + M_{BA} - 80}{4} + \frac{M_{CD} + M_{DC}}{4} + 40 = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} + 80 = 0$$

Now, $M_{BA} + M_{BC} = 0$

$$13.33 + EI\theta_B - \frac{3EI}{8}\Delta - 112.5 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C = 0$$

$2.2EI\theta_B + 0.6EI\theta_C - 0.375\, EI\Delta - 99.17 = 0 \quad \cdots\cdots (7)$

$$M_{CB} + M_{CD} = 0$$

$$112.5 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B + EI\theta_C - \frac{3EI\Delta}{8} = 0$$

$112.5 + 2.2\, EI\theta_C + 0.6EI\theta_B - 0.375\, EI\Delta = 0 \quad \cdots\cdots (8)$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} + 80 = 0$$

$$-13.33 + \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} + 13.33 + EI\theta_B - \frac{3EI}{8}\Delta + EI\theta_C - \frac{3EI\Delta}{8} + \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta + 80 = 0$$

$1.5\, EI\theta_B + 1.5\, EI\theta_C - 1.5\, EI\Delta + 80 = 0 \quad \cdots\cdots (9)$

Solving equations (7), (8)& (9)

$$\theta_B = \frac{72.65}{EI}$$

$$\theta_C = -\frac{59.64}{EI}$$

$$\Delta = \frac{66.34}{EI}$$

Substituting these values in slope deflection equation, we have,

$$M_{AB} = -13.33 + \frac{1}{2}(72.65) - \frac{3}{8}(66.34) = -1.88KNm$$

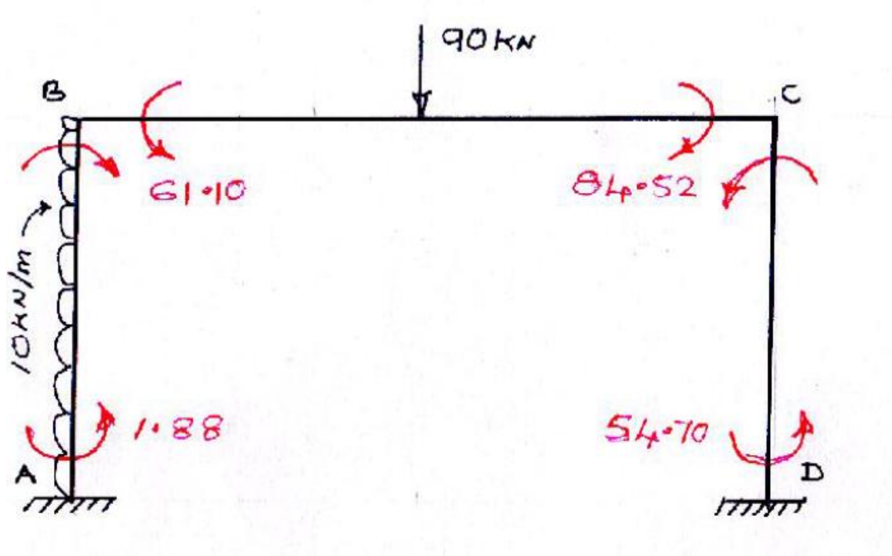
$$M_{BA} = 72.65 - \frac{3}{8}(66.34) = 61.10KNm$$

$$M_{BC} = -112.5 + \frac{6}{5}(72.65) + \frac{3}{5}(-59.64) = -61.10KNm$$

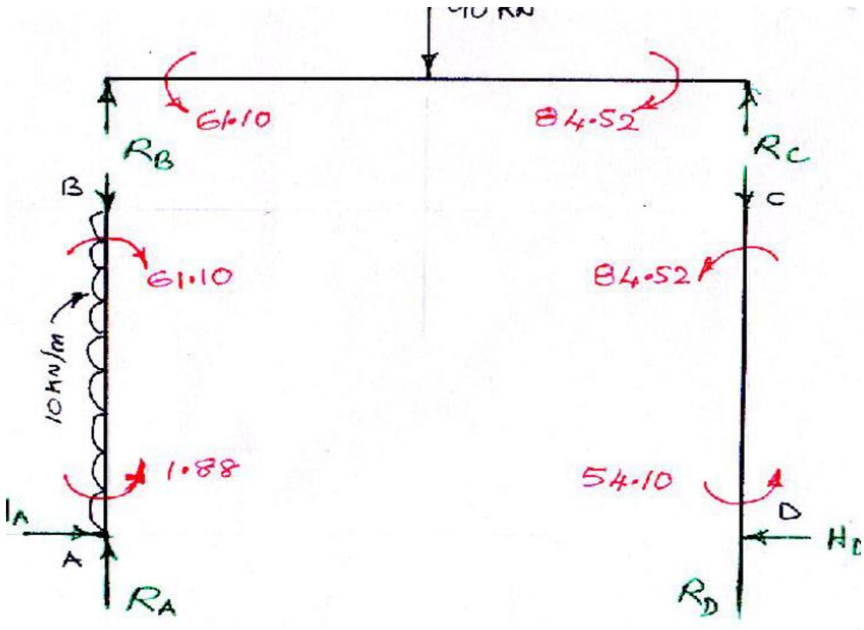
$$M_{CB} = 112.5 + \frac{6}{5}(-59.64) + \frac{3}{5}(72.65) = 84.52KNm$$

$$M_{CD} = -59.64 - \frac{3}{8}(66.34) = -84.52KNm$$

$$M_{DC} = \frac{1}{2}(-59.64) - \frac{3}{8}(66.34) = -54.70KNm$$



Reactions: Consider the free body diagram of various members



Member AB:

$$H_A = \frac{61.10 - 1.88 - 10 \times 4 \times 2}{4} = -5.195 \text{ KN}$$

Span BC:

$$R_C = \frac{84.52 - 61.10 + 90 \times 5}{10} = 47.34 \text{ KN}$$

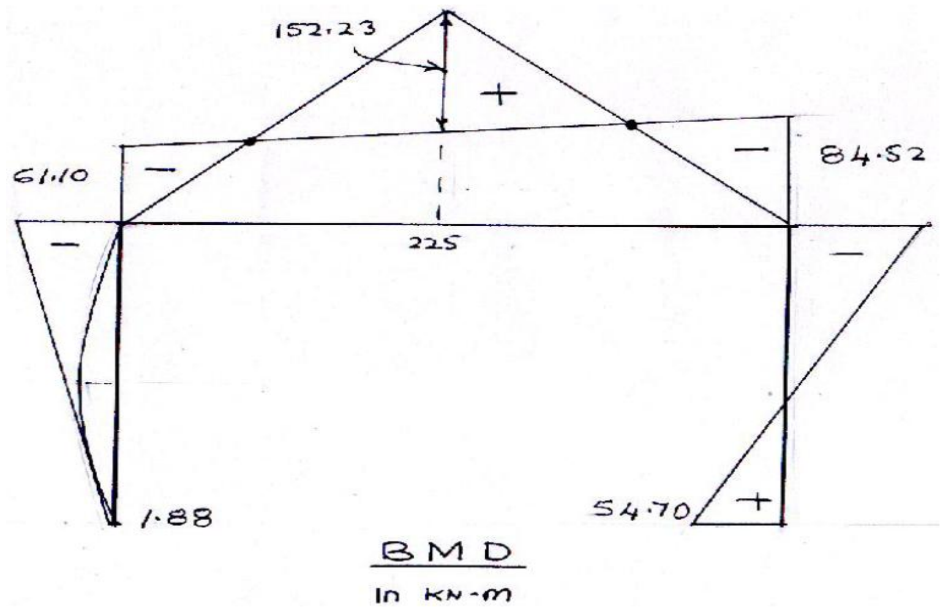
$$R_B = 90 - R_C = 38.34\text{KN}$$

Column CD:

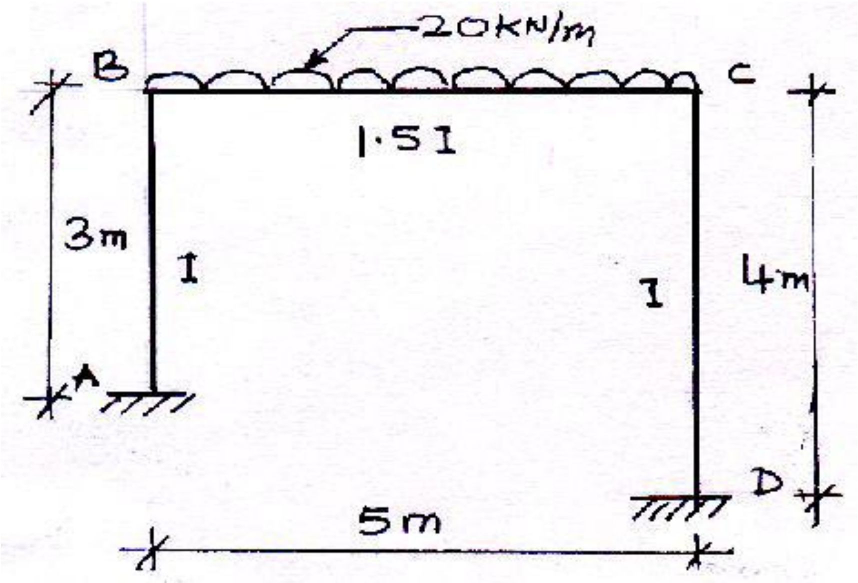
$$H_D = \frac{84.52 + 54.7}{4} = 34.81\text{KN}$$

Check:

$\Sigma H = 0$, $H_A + H_D + 10\times 4 = 0$, $-5.2 + 34.81 + 10\times 4 = 0$



7.Q.Analyse the portal frame and draw the B.M.D.



A. It is an unsymmetrical problem, hence there is a sway be towards right.

$$\theta_A = 0, \theta_D = 0, \theta_B \neq 0, \theta_C \neq 0$$

FEMS:

$$M_{FAB} = M_{FBA} = 0 = M_{FCD} = M_{FDC}$$

$$M_{FBC} = -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67KNm$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67KNm$$

Slope deflection equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\frac{\Delta}{L}\right] = \frac{2EI}{3}\left(0 + \theta_B - 3\frac{\Delta}{3}\right)$$

$$= \frac{2EI}{3}\theta_B - \frac{2EI\Delta}{3} \qquad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\frac{\Delta}{L}\right]$$

$$= \frac{2EI}{3}\left(2\theta_B + 0 - 3\frac{\Delta}{3}\right) = \frac{4EI}{3}\theta_B - \frac{2EI}{3}\Delta \qquad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{2E(1.5)I}{5}(2\theta_B + \theta_C)$$

$$= -41.67 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{2E(1.5)I}{5}[2\theta_C + \theta_B]$$
$$= 41.67 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B \qquad \cdots\cdots(4)$$
$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(2\theta_C + 0 - 3\frac{\Delta}{4}\right)$$
$$= EI\theta_C - \frac{3EI\Delta}{8} \qquad \cdots\cdots(5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - 3\frac{\Delta}{L}\right]$$
$$= 0 + \frac{2EI}{4}\left(0 + \theta_C - 3\frac{\Delta}{4}\right) = \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta \qquad \cdots\cdots(6)$$

In the above equation there are three unknowns, θ_B, θ_C& Δ, accordingly the boundary conditions are,
Joint conditions , $M_{BA} + M_{BC} = 0, M_{CB} + M_{CD} = 0$
Shear condition, $H_A + H_D = 0, \quad \therefore \frac{M_{AB} + M_{BA}}{3} + \frac{M_{CD} + M_{DC}}{4} + 40 = 0$

$$4(M_{AB} + M_{BA}) + 3(M_{CD} + M_{DC}) = 0$$

Now, $\qquad M_{BA} + M_{BC} = 0$

$$\frac{4EI}{3}\theta_B - \frac{2EI}{3}\Delta - 41.67 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C = 0$$

$$2.53EI\theta_B + 0.6EI\theta_C - \frac{2EI}{3}\Delta - 41.67 = 0 \qquad \cdots\cdots(7)$$

$$M_{CB} + M_{CD} = 0$$

$$41.67 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B + EI\theta_C - \frac{3EI\Delta}{8} = 0$$

$41.67+2.2\ EI\theta_C+0.6EI\theta_B - 0.375\ EI\Delta = 0 \qquad \cdots\cdots(8)$

$$4(M_{AB} + M_{BA}) + 3(M_{CD} + M_{DC}) = 0$$

$$4\left(\frac{2EI}{3}\theta_B - \frac{2EI\Delta}{3} + \frac{4EI}{3}\theta_B - \frac{2EI}{3}\Delta\right) + 3(EI\theta_C - \frac{3EI\Delta}{8} + \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta) = 0$$

$8\ EI\theta_B + 4.5\ EI\theta_C - 7.53\ EI\Delta = 0 \qquad \cdots\cdots(9)$

Solving equations (7), (8)& (9)

$$\theta_B = \frac{25.46}{EI}$$

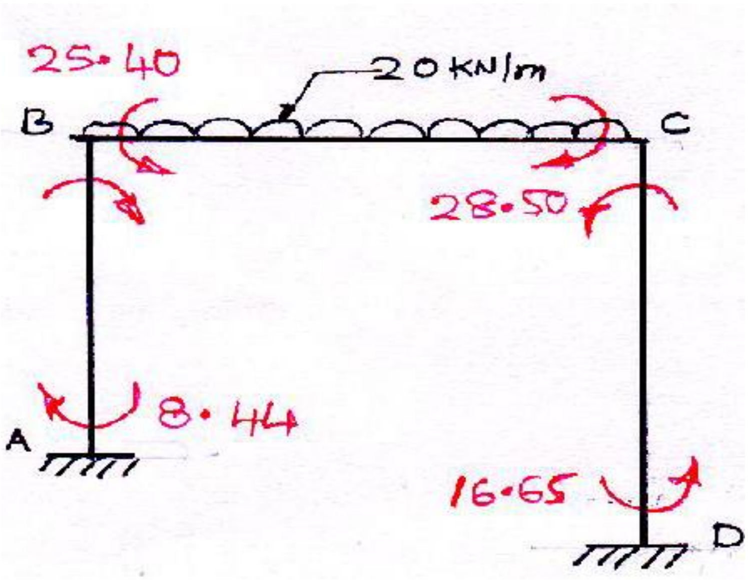
$$\theta_C = \frac{-23.17}{EI}$$

$\Delta = \frac{12.8}{EI}$

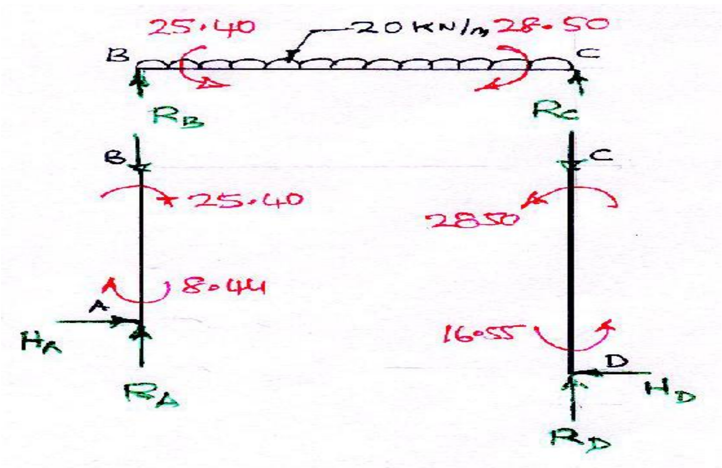
Substituting these values in slope deflection equation, we have,

$$M_{AB} = \frac{2}{3}(25.46) - \frac{2}{3}(12.8) = 8.44KNm$$
$$M_{BA} = \frac{4}{3}(25.46) - \frac{2}{3}(12.8) = 25.4KNm$$
$$M_{BC} = -41.67 + \frac{6}{5}(25.46) + \frac{3}{5}(-23.17) = -25.4KNm$$
$$M_{CB} = 41.67 + \frac{6}{5}(-23.17) + \frac{3}{5}(20.46) = 28.5KNm$$

$$M_{CD} = -23.17 - \frac{3}{8}(12.8) = -28.5KNm$$
$$M_{DC} = \frac{1}{2}(-23.17) - \frac{3}{8}(12.8) = -16.65KNm$$



Reactions: Consider the free body diagram



Member AB:

$$H_A = \frac{25.4 + 8.44}{3} = 11.28 \text{ KN}$$

Span BC:

$$R_C = \frac{28.5 - 25.4 + 20 \times 5 \times \frac{5}{2}}{5} = 51.64 \text{ KN}$$

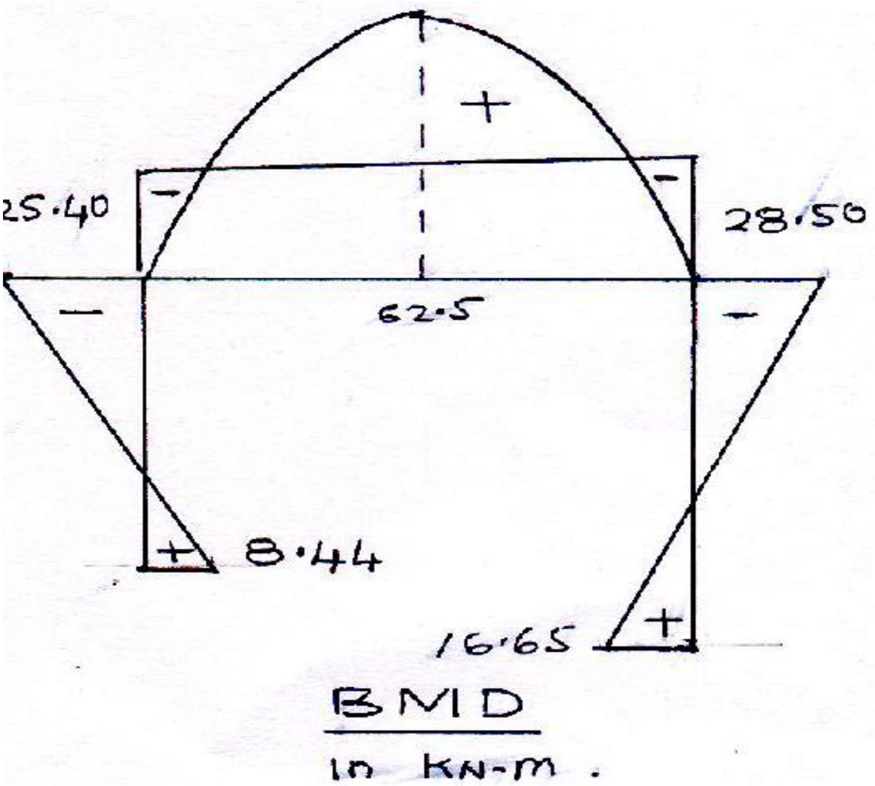
$$R_B = 20 \times 5 - 51.64 = 48.36\text{KN}$$

Column CD:

$$H_D = \frac{28.5 + 16.65}{4} = 11.28\text{KN}$$

Check:

$\Sigma H = 0$, $H_A + H_D = 0$ Satisfied, hence okay



MOMENT DISTRIBUTION METHOD

This method of analyzing beams and frames was developed by Hardy Cross in 1930. Moment distribution method is basically a displacement method of analysis. But this method side steps the calculation of the displacement and instead makes it possible to apply a series of converging corrections that allow direct calculation of the end moments. This method of consists of solving slope deflection equations by successive approximation that may be carried out to any desired degree of accuracy. Essentially, the method begins by assuming each joint of a structure is fixed. Then by unlocking and locking each joint in succession, the internal moments at the joints are distributed and balanced until the joints have rotated to their final or nearly final positions. This method of analysis is both repetitive and easy to apply. Before explaining the moment distribution method certain definitions and concepts must be understood.

Sign convention: In the moment distribution table clockwise moments will be treated +ve and anti clockwise moments will be treated –ve. But for drawing BMD moments causing concavity upwards (sagging) will be treated +ve and moments causing convexity upwards (hogging) will be treated –ve.

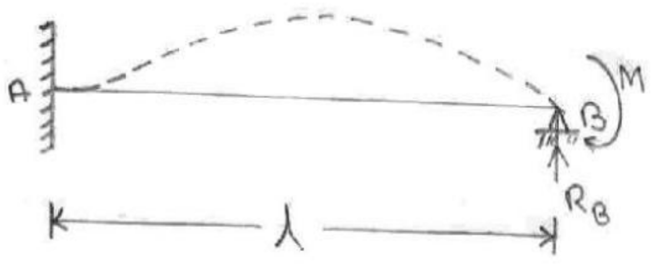
Fixed end moments: The moments at the fixed joints of loaded member are called fixed end moment. FEM for few standards cases are given in previous chapter.

Member stiffness factor:

a) Consider a beam fixed at one end and hinged at other as shown in figure subjected to a clockwise couple M at end B. The deflected shape is shown by dotted line.

BM at any section xx at a distance x from 'B' is given by

$$EI\frac{d^2y}{dx^2} = R_B x - M$$



Integrating $EI\frac{dy}{dx} = \frac{R_B x^2}{2} - Mx + C_1$

Using condition at x = L, $\frac{dy}{dx} = 0$

$$C_1 = ML - \frac{R_B L^2}{2}$$

$$\therefore\ EI\frac{dy}{dx} = \frac{R_B x^2}{2} - Mx + \left(ML - \frac{R_B L^2}{2}\right) \qquad \cdots (1)$$

Integrating again $EIy = \frac{R_B x^3}{6} - \frac{Mx^2}{2} + \left(ML - \frac{R_B L^2}{2}\right)x + C_2$

Using condition at x =0, y=0, $C_2 = 0$

$$\therefore \; EIy = \frac{R_B x^3}{6} - \frac{Mx^2}{2} + \left(ML - \frac{R_B L^2}{2}\right)x \qquad \cdots (2)$$

Using at x=L, y=0 in equation (2)

$$R_B = \frac{3M}{2L}$$

Substituting in equation (1)

$$EI\frac{dy}{dx} = \frac{3Mx^2}{4L} - Mx + \frac{ML}{4} \qquad \cdots (3)$$

Substituting at x =0, $\frac{dy}{dx} = \theta_B \; in \; equation$ (3)

$EI\theta_B = \frac{ML}{4}, \; M = \frac{4EI}{L}\theta_B$

The term in parenthesis

$\left\{K = \frac{4EI}{L}\right\}$ For far end fixed …………………. (4) is refered to as stiffness factor at B and can be defined as moment M required to rotate end B of the beam $\theta_B = 1$ radian

b) Consider freely supported beam as shown in figure subjected to a clockwise couple M at B

By using $\Sigma M_B = 0$

$$R_A = \frac{M}{L}(\downarrow)$$

And using $\Sigma V = 0$ $R_B = \frac{M}{L}(\uparrow)$

BM at a section xx at distance x from ‘B’ is given by

$$EI\frac{d^2y}{dx^2} = \frac{M}{L}x - M$$

Integrating $EI\frac{dy}{dx} = \frac{M}{L}\frac{x^2}{2} - Mx + C_1$

Integrating again $EIy = \frac{M}{L}\frac{x^3}{6} - \frac{Mx^2}{2} + C_1x + C_2$

$$\text{At } x = 0 \; y = 0, \quad C_2 = 0$$

$$\text{At } x = L \quad y = 0, C_1 = \frac{ML}{3}$$

$$EI\frac{dy}{dx} = \frac{M}{L}\frac{x^2}{2} - Mx + \frac{ML}{3}$$

Substituting at x =0, $\frac{dy}{dx} = \theta_B$ in the above equation $EI\theta_B = \frac{ML}{3}$, $M = \frac{3EI}{L}\theta_B$

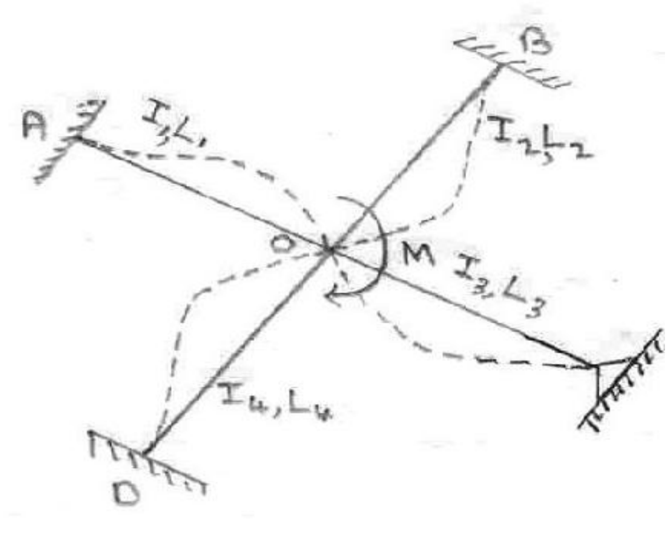
The term in parenthesis

$\left\{K = \frac{3EI}{L}\right\}$ is termed as stiffness factor at B when far end A is hinged …………………. (5)

Joint stiffness factor:

If several members are connected to a joint, then by the principle of superposition the total stiffness factor at the joint is the sum of the member stiffness factors at the joint i.e.,
$K_T = \Sigma K$

E.g. For joint '0', $K_T = K_{0A} + K_{OB} + K_{OC} + K_{OD}$



Distribution factors: If a moment 'M' is applied to a rigid joint 'o', as shown in figure, the connecting members will each supply a portion of the resisting moment necessary to satisfy moment equilibrium at the joint. Distribution factor is that fraction which when multiplied with applied moment 'M' gives resisting moment supplied by the members. To obtain its

value imagine the joint is rigid joint connected to different members. If applied moment M cause the joint to rotate an amount 'θ', Then each member rotates by same amount.
From equilibrium requirement

M = $M_1 + M_2 + M_3 + \ldots\ldots$
$= K_1\theta + K_2\theta + K_3\theta = \theta\sum K$

$DF_1 = \frac{M1}{M} = \frac{K_1\theta}{\theta\sum K} = \frac{K_1}{\sum K}$

In general DF = $\frac{K}{\sum K}$ ....(6)

Member relative stiffness factor: In majority of the cases continuous beams and frames will be made from the same material so that their modulus of electricity E will be same for all members. It will be easier to determine member stiffness factor by removing term 4E & 3E from equation (4) and (5) then will be called as relative stiffness factor.

$K_r = \frac{I}{L}$ for far end fixed

$K_r = \frac{3I}{4L}$ for far end hinged

Carry over factors: Consider the beam shown in figure

We have shown that

$$M = \frac{4EI}{L}\theta_A, R_B = \frac{3M}{2L}$$

BM at A $\left(EI\frac{d^2y}{dx^2}\right)_{at\ x=L} = [(3M/2L)x - M]_{x=L} = \frac{M}{2}$

+ve BM of $\frac{M}{2}$ at A indicates clockwise moment of $\frac{M}{2}$ at A. In other words the moment 'M' at the pin induces a moment of $\frac{M}{2}$ at the fixed end. The carry over factor represents the fraction of M that is carried over from hinge to fixed end. Hence the carry over factor for the case of far end fixed is +$\frac{1}{2}$. The plus sign indicates both moments are in the same direction.

Moment distribution method for beams:

Procedure for analysis:

(i) Fixed end moments for each loaded span are determined assuming both ends fixed.

(ii) The stiffness factors for each span at the joint should be calculated. Using these values the distribution factors can be determined from equation DF = $\frac{K}{\sum K}$

DF for a fixed end = 0 and DF = 1 for an end pin or roller support.

(iii) Moment distribution process: Assume that all joints at which the moments in the connecting spans must be determined are initially locked.

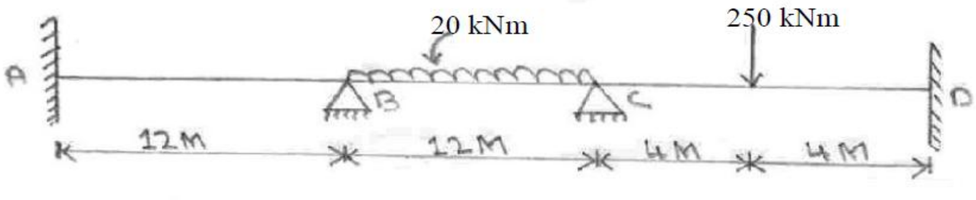
Then determine the moment that is needed to put each joint in equilibrium. Release or unlock the joints and distribute the counterbalancing moments into connecting span at each joint using distribution factors.

Carry these moments in each span over to its other end by multiplying each moment by carry over factor.

By repeating this cycle of locking and unlocking the joints, it will be found that the moment corrections will diminish since the beam tends to achieve its final deflected shape. When a small enough value for correction is obtained the process of cycling should be stopped with carry over only to the end supports. Each column of FEMs, distributed moments and carry over moment should then be added to get the final moments at the joints.

Then superimpose support moment diagram over free BMD (BMD of primary structure) final BMD for the beam is obtained.

1.Q. Analyse the beam shown in figure by moment distribution method and draw the BMD. Assume EI is constant



A. FEMS

$$M_{FAB=}\ M_{FBA=}\ 0$$

$$M_{FBC=}\ -\frac{wL^2}{12} = -\frac{20\ \times 12^2}{12} = -240KNm$$

$$M_{FCB=}\ \frac{wL^2}{12} = \frac{20\ \times 12^2}{12} = 240KNm$$

$$M_{FCD=}\ -\frac{wL}{8} = -\frac{250\times8}{8}=-250KNm$$

$$M_{FDC=}\ \frac{wL}{8} = \frac{250\times8}{8} =250KNm$$

(ii) Calculation of distribution factors

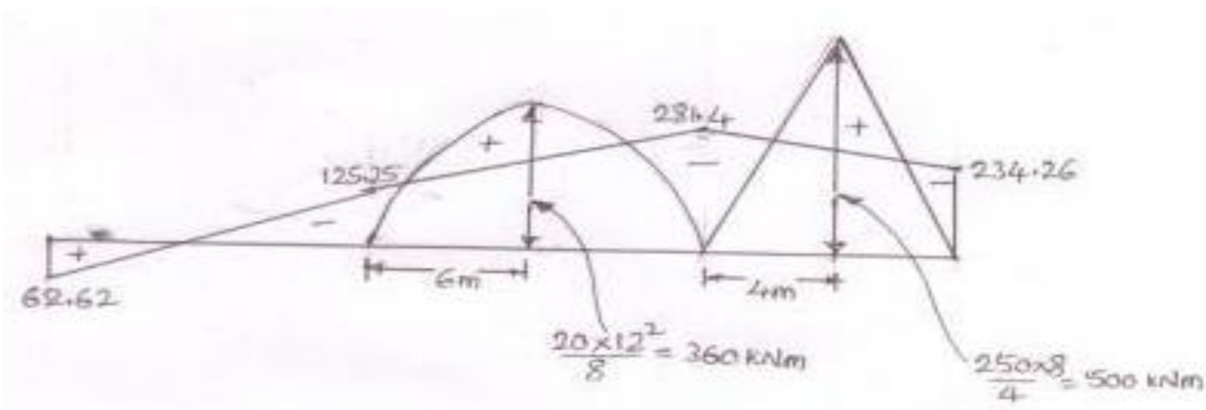
Jt.	Member	Relative stiffness (K)	ΣK	DF = $\frac{K}{\sum K}$
B	BA	I/12	I/6	0.5
	BC	I/12		0.5
C	CB	I/12	5I/24	0.4
	CD	I/8		0.6

(iii) The moment distribution is carried out in table below.

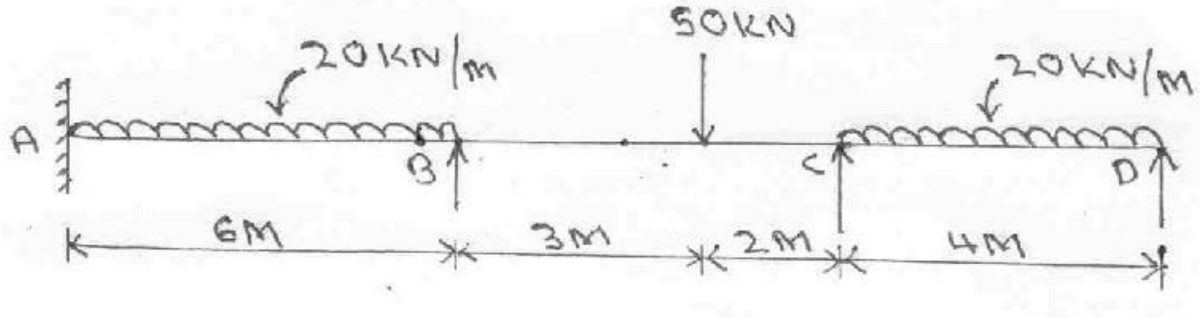
Jt	A	B		C		D
Member	AB	BA	BC	CB	CD	α
D.F	0	0.5	0.5	0.4	0.6	0
FEM	0	0	-240	+240	-250	+250
Balance		+120	+120	4	6	
C.O	60		2	60		3
Balance		-1	-1	-24	-36	
C.O	-0.5		-12	-0.5		-18
Balance		+6	+6	0.2	0.3	
C.O	3		0.1	3		0.15
Balance		-0.05	-0.05	-1.2	-1.8	
C.O	-0.03		-0.6	-0.03		-0.9
Balance		+0.3	+0.3	0.01	0.02	
C.O	0.15					0.01
Final moments	**62.62**	**125.25**	**-125.25**	**281.48**	**-281.48**	**234.26**

After writing FEMs we can see that there is a unbalancing moment of –240 KNm at B & -10 KNm at joint C. Hence in the next step balancing moment of +240 KNM & +10 KNm are applied at B & C Simultaneously and distributed in the connecting members after multiply with D.F. In the next step distributed moments are carried over to the far ends. This process is continued until the resulting moments are diminished an appropriate amount. The final moments are obtained by summing up all the moment values in each column.

Drawing of BMD is shown below in figure.



2. Q. Analyse the continuos beam as shown in figure by moment distribution method and draw the B.M. diagrams



Support B sinks by 10mm

E = 2 x 10^5 N/mm², I = 1.2 x 10^{-4} m^4

A. FEMS

$$M_{FAB} = -\frac{wL^2}{12} - \left[\frac{6EI}{L^2}\Delta\right] = -\frac{20 \times 6^2}{12} - \left[\frac{6 \times 2 \times 10^5 \times 1.2 \times 10^{-4} \times 10^{12}}{(6000)^2 \times 10^6} \times 10\right]$$

$$= -100 \text{ KNm}$$

$$M_{FBA} = \frac{wL^2}{12} - \left[\frac{6EI}{L^2}\Delta\right] = \frac{20 \times 6^2}{12} - \left[\frac{6 \times 2 \times 10^5 \times 1.2 \times 10^{-4} \times 10^{12}}{(6000)^2 \times 10^6} \times 10\right] = 20 \text{ KNm}$$

$$M_{FBC} = -\frac{wab^2}{L^2} + \left[\frac{6EI}{L^2}\Delta\right] = -\frac{50 \times 3 \times 2^2}{5^2} + \left[\frac{6 \times 2 \times 10^5 \times 1.2 \times 10^{-4} \times 10^{12}}{(5000)^2 \times 10^6} \times 10\right]$$

$$= 33.6 \text{KNm}$$

$$M_{FCB} = \frac{wa^2b}{L^2} + \left[\frac{6EI}{L^2}\Delta\right] = \frac{50 \times 2 \times 3^2}{5^2} + \left[\frac{6 \times 2 \times 10^5 \times 1.2 \times 10^{-4} \times 10^{12}}{(5000)^2 \times 10^6} \times 10\right]$$

$$= 93.6 \text{KNm}$$

$$M_{FCD} = -\frac{wL^2}{12} = -\frac{20 \times 4^2}{12} = -26.67 \text{KNm}$$

$$M_{FDC} = \frac{wL^2}{12} = \frac{20 \times 4^2}{12} = 26.67 \text{KNm}$$

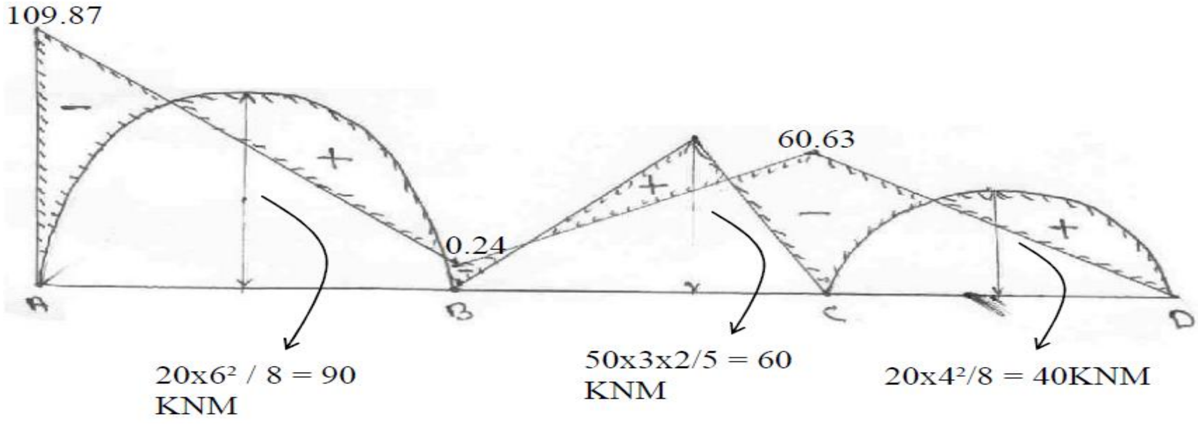
Distribution factor

Jt.	Member	Relative stiffness (K)	ΣK	DF = $\frac{K}{\sum K}$
B	BA BC	I/6 I/5	0.36I	0.46 0.54
C	CB CD	I/5 $\frac{3}{4} x \frac{I}{4} = 0.19I$	0.39I	0.51 0.49

MOMENT DISTRIBUTION

Jt	A		B		C		D
Member	AB	BA	BC	CB	CD	DC	
D.F		0.46	0.54	0.51	0.49		
FEM	-100	+20	+33.6	+93.6	-26.67	+26.67	
Release jt. 'D'						-26.67	
CO					-13.34		
Initial moments	-100	+20	+33.6	+93.6	-40.01	0	
Balance		-24.66	-28.94	-27.33	-26.26		
C.O	-12.33		-13.67	--14.47			
Balance		+6.29	+7.38	+7.38	+7.09		
C.O	+3.15		+3.69	+3.69			
Balance		-1.7	-1.99	-1.88	-1.81		
C.O	-0.85		-0.94	-1			
Balance		+0.43	+0.51	+0.51	+0.49		
C.O	+0.22		+0.26	+0.26			
Balance		-0.12	-0.14	-0.13	-0.13		

C.O	-0.06							
Final moments	**-109.87**	**+0.24**	**-0.24**	**+60.63**	**-60.63**			

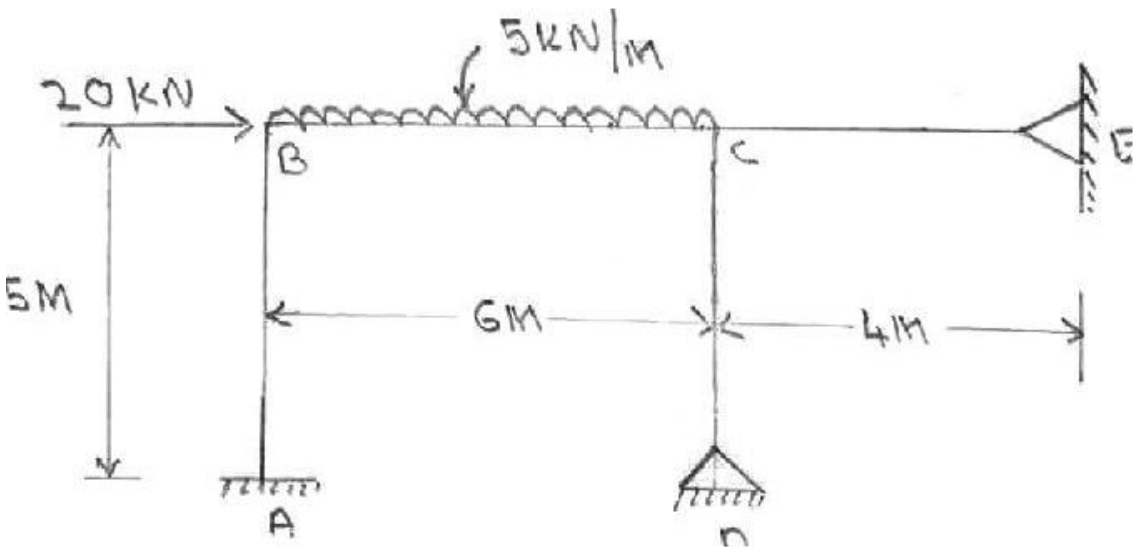


BMD

Moment distribution for frames: (No side sway)

The analysis of such a frame when the loading conditions and the geometry of the frame is such that there is no joint translation or sway, is similar to that given for beams.

3. Q. Analysis the frame shown in figure by moment distribution method and draw BMD assume EI is constant.



FEMS

$$M_{FAB=}\ M_{FBA=}\ M_{FCD=}\ M_{FDC=}\ M_{FCE=}\ M_{FEC=}\ 0$$

$$M_{FBC=}-\frac{5\times6^2}{12}=-15KNm$$

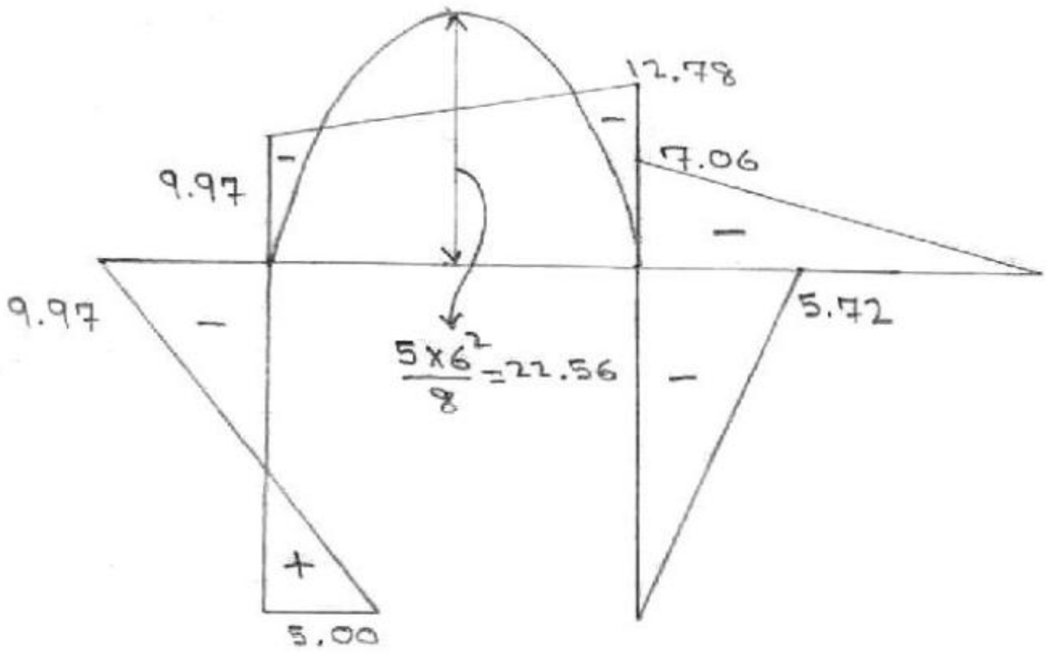
$$M_{FCB=}\ \frac{5\times6^2}{12}=15KNm$$

DISTRIBUTION FACTOR

Jt.	**Member**	**Relative stiffness (K)**	**ΣK**	**DF =** $\frac{K}{\sum K}$
B	BA	I/5	$\frac{11}{30}I$	0.55
	BC	I/6		0.45
C	CB	I/6 = 0.17 I	0.51 I	0.33
	CD	$\frac{3}{4}$ I/5 = 0.15 I		0.3
	CE	$\frac{3}{4}$ x $\frac{I}{4}$ = 0.19 I		0.37

MOMENT DISTRIBUTION

Jt	A	B		C			D	E
Member	AB	BA	BC	CB	CD	CE	DC	EC
D.F	0	0.55	0.45	0.33	0.3	0.37	1	1
FEM	0	0	-15	+15	0	0	0	0
Balance		8.25	6.75	-4.95	-4.5	-5.55		
C.O	4.13		-2.48	3.38				
Balance		1.36	1.12	-1.12	-1.01	-1.25		
C.O	0.68		0.56	0.56				
Balance		0.31	0.25	-0.18	-0.17	-0.21		
C.O	0.16		-0.09	0.13				
Balance		0.05	0.04	-0.04	-0.04	-0.05		
C.O	0.03							
Final moments	**5**	**9.97**	**-9.97**	**12.78**	**-5.72**	**-7.06**	**0**	**0**

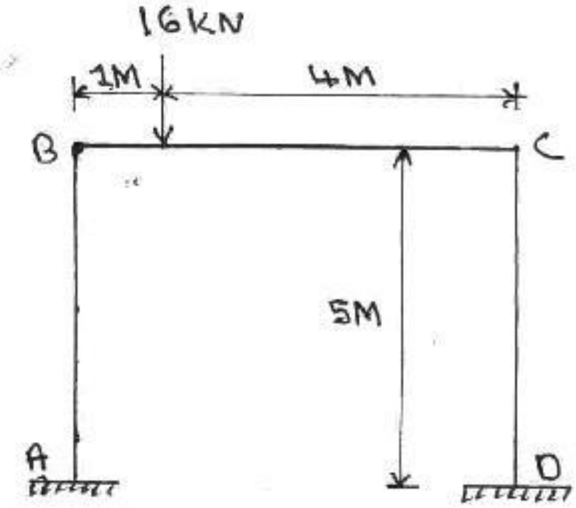


BMD

Moment distribution method for frames with side sway

Frames that are non symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

4.Q. Analyze the frame shown in figure by moment distribution method. Assume EI is constant.



A. Non Sway Analysis:
First consider the frame without side sway

$$M_{FAB} = M_{FBA} = M_{FCD} = 0$$

$$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24 KNm$$

$$M_{FCB} = \frac{16\times4\times1^2}{5^2} = 2.56 KNm$$

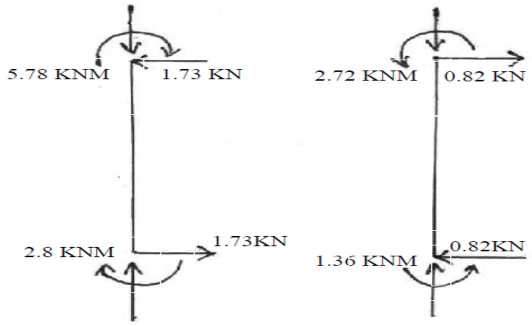
DISTRIBUTION FACTOR

Jt.	**Member**	**Relative stiffness K**	**ΣK**	**DF =** $\frac{K}{\sum K}$
B	BA BC	I/5 = 0.2 I I/5 = 0.2 I	0.4 I	0.5 0.5
C	CB CD	I/5 = 0.2I I/5 = 0.2I	0.4I	0.5 0.5

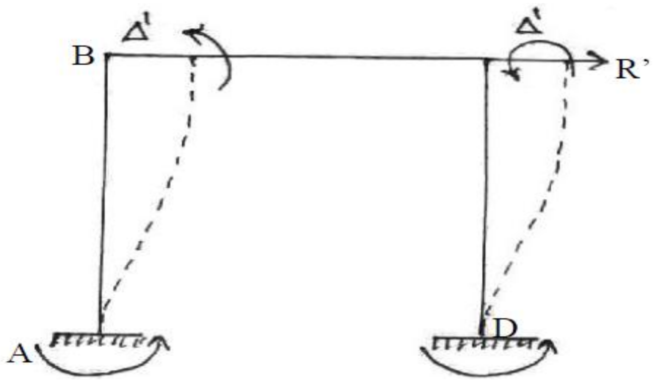
DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

Joint	A		B		C		D
Member	AB	BA	BC	CB	CD	DC	
D.F	0	0.5	0.5	0.5	0.5	0	
FEM	0		-10.24	2.56	0	0	
Balance		5.12	5.12	-1.28	-1.28		
CO	2.56		-0.64	2.56		-0.64	
Balance		0.32	0.32	-0.08	-0.08		
CO	0.16		-0.64	0.16		-0.64	
Balance		0.32	0.32	-0.08	-0.08		
C.O	0.16		-0.04	0.16		-0.04	
Balance		0.02	0.02	-0.08	-0.08		
C.O	0.01					-0.04	
Final moments	**2.89**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.36**	

FREE BODY DIAGRAM OF COLUMNS



By seeing of the FBD of columns R = 1.73 – 0.82
(Using ΣF_x =0 for entire frame) = 0.91 KN ⟵
Now apply R = 0.91 KN acting opposite as shown in the above figure for the sway analysis.
Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect Δ' as shown in the following figure.



Since both ends are fixed, columns are of same length & I and assuming joints B & C are temporarily restrained from rotating and resulting fixed end moment are

$$M'_{AB} = M'_{BA} = M'_{CD} = M'_{DC} = \frac{6EI}{L^2}\Delta$$

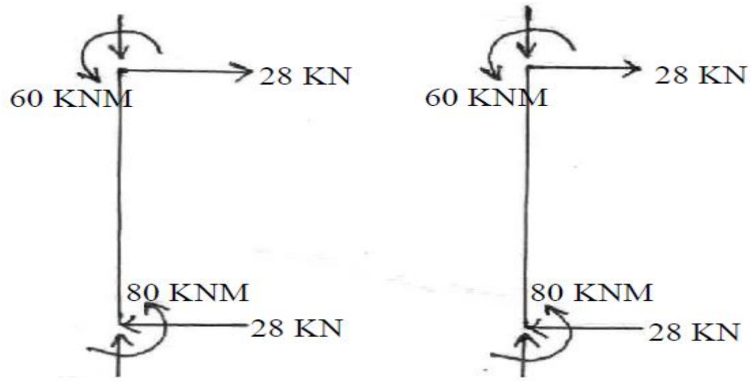
Assume $$M'_{BA} = -100KNm$$
$$M'_{AB} = M'_{CD} = M'_{DC} = -100KNm$$

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	01	0.5	0.5	0.5	0.5	0
FEM	−100	−100	0	0	−100	−100
Balance		50	50	50	50	
CO	25		25	25		25
Balance		−12.5	−12.5	−12.5	−12.5	
CO	-6.25		−6.25	−6.25		−6.25

Balance		3.125	3.125	3.125	3.125		
C.O	1.56		1.56	1.56		1.56	
Balance		−0.78	−0.78	−0.78	−0.78		
C.O	-0.39		−0.39	−0.39		0.39	
Balance		0.195	0.195	0.195	0.195		
C.O	0.1					0.1	
Final moments	**- 80**	**- 60**	**60**	**60**	**– 60**	**– 80**	

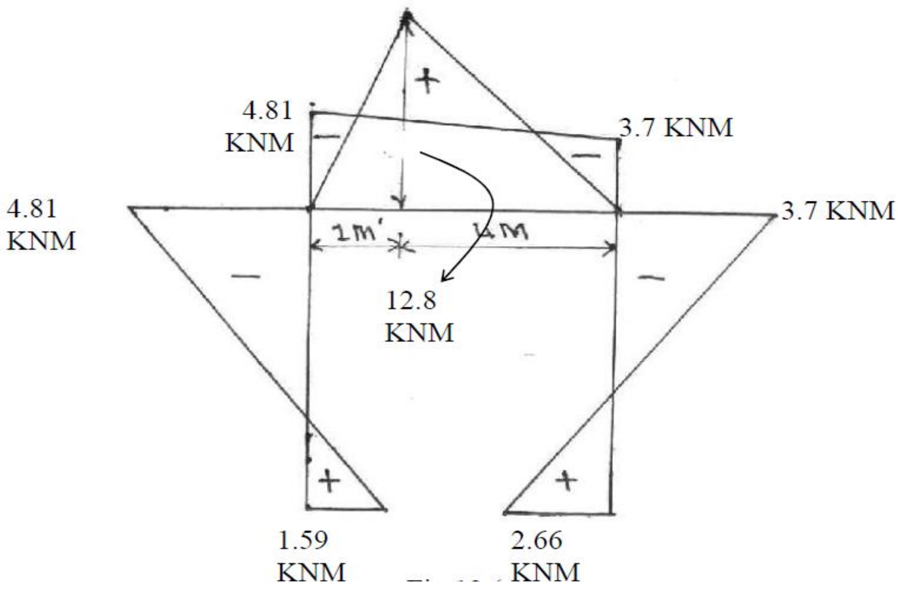
Free body diagram of columns



Using Σ F x = 0 for the entire frame
R' = 28 + 28 = 56 KN →
Hence R' = 56KN creates the sway moments shown in above moment distribution table.
Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.
Moments calculated for R = 0.91KN, as shown below.

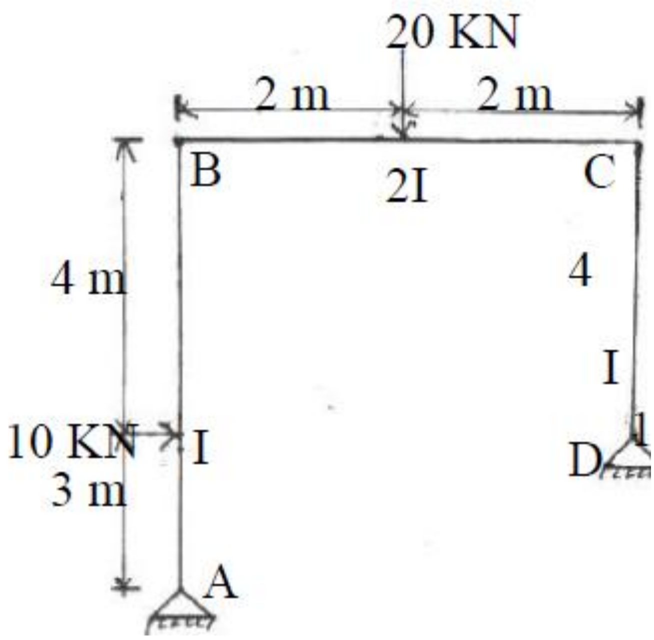
$$M_{AB} = 2.89 + \frac{0.91}{56}(-80) = 1.59KNm$$
$$M_{BA} = 5.78 + \frac{0.91}{56}(-60) = 4.81KNm$$

$$M_{BC} = -5.78 + \frac{0.91}{56}(60) = -4.81KNm$$
$$M_{CB} = 2.72 + \frac{0.91}{56}(60) = 3.7KNm$$
$$M_{CD} = -2.72 + \frac{0.91}{56}(-60) = -3.7KNm$$
$$M_{DC} = -1.36 + \frac{0.91}{56}(-80) = -2.66KNm$$



BMD

5.Q. Analysis the rigid frame shown in figure by moment distribution method and draw BMD



A. **Non Sway Analysis:**

First consider the frame held from side sway

FEMS

$$M_{FAB=} -\frac{10 \times 3 \times 4^2}{7^2} = -9.8\text{KNm}$$

$$M_{FBA=} \frac{10 \times 4 \times 3^2}{7^2} = 7.3\text{KNm}$$

$$M_{FBC} = -\frac{20\times4}{8} = -10KNm$$

$$M_{FCB} = \frac{20\times4}{8} = 10KNm$$

$$M_{FCD} = M_{FDC} = 0$$

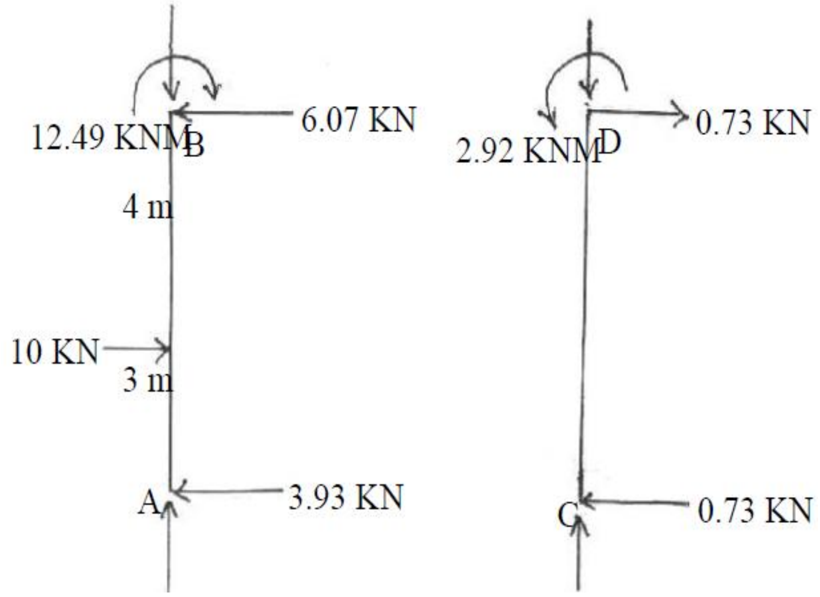
DISTRIBUTION FACTOR

Joint	**Member**	**Relative stiffness k**	**Σk**	**DF = $\frac{K}{\sum K}$**
B	BA	$\frac{3}{4}\times\frac{I}{7} = 0.11I$	0.61 I	0.18
	BC	2I/4 = 0.5I		0.82
C	CB	2I/4 = 0.5I	0.69 I	0.72
	CD	$\frac{3}{4}\times\frac{I}{4} = 0.19\ I$		0.28

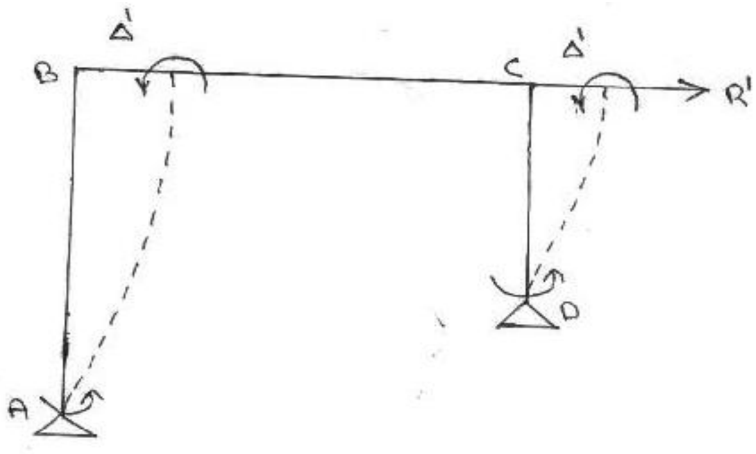
DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	-9.8	7.3	-10	10	0	0
Release jt. 'D'	+9.8					
CO		4.9				
Initial moments	0	12.2	-10	10	0	0
Balance		-0.4	-1.8	-7.2	-2.8	
CO			-3.6	-0.9		
Balance		0.65	2.95	0.65	0.25	
C.O			0.33	1.48		
Balance		-0.06	-0.27	-1.07	-0.41	
C.O			-0.54	-0.14		
Balance		0.1	0.44	0.1	0.04	
Final moments	**0**	**12.49**	**-12.49**	**2.92**	**-2.92**	**0**

FREE BODY DIAGRAM OF COLUMNS



Applying ΣFx = 0 for frame as a
Whole, R = 10 – 3.93 – 0.73
= 5.34 KN←
Now apply R = 5.34KN acting opposite
Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect Δ' as shown in figure



Since ends A & D are hinged and columns AB & CD are of different lengths

$M'_{BA} = -\frac{3EI}{L_1^2}\Delta', \quad M'_{CD} = -\frac{3EI}{L_2^2}\Delta'$,

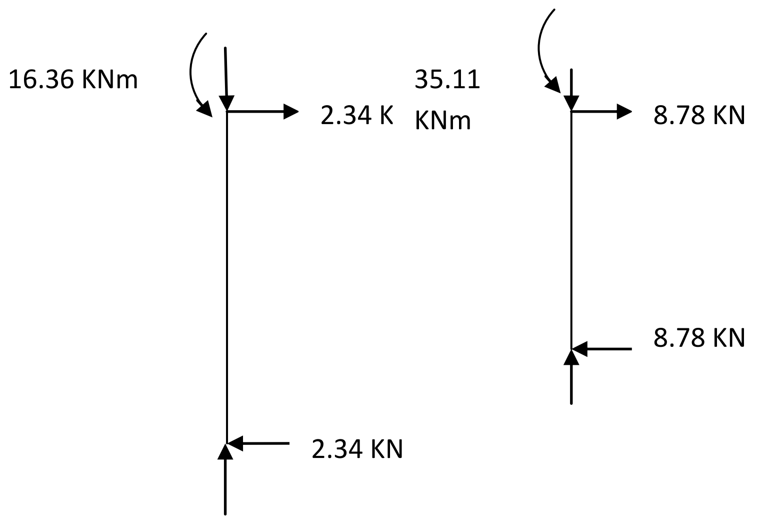
$$\frac{M'_{BA}}{M'_{CD}} = \frac{\frac{3EI}{L_1^2}\Delta'}{\frac{3EI}{L_2^2}\Delta'} = \frac{L_2^2}{L_1^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

Assume $M'_{BA} = -16KNm, M'_{AB} = 0$
$M'_{CD} = -49KNm, \ M'_{DC} = 0$

MOMENT DISTRIBUTION FOR SWAY ANALYSIS

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	0	-16	0	0	-49	0
Balance		2.88	13.12	35.28	13.72	
CO			17.64	6.56		
Balance		-3.18	-14.46	-4.72	-1.84	
CO			-2.36	-7.23		
Balance		0.42	1.94	5.21	2.02	
C.O			2.61	0.97		
Balance		-0.47	-2.14	-0.7	-0.27	
C.O			0.35	-1.07		
Balance		0.06	0.29	0.77	0.3	
C.O			0.39	0.15		
Balance		-0.07	-0.32	-0.11	-0.04	
Final moments	**0**	**-16.36**	**16.36**	**35.11**	**-35.11**	**0**

FREE BODY DIAGRAMS OF COLUMNS AB &CD



Using ΣFx = 0 for the entire frame

R' = 11.12 kN ⟶

Hence R' = 11.12 KN creates the sway moments shown in the above moment distribution table. Corresponding moments caused by R = 5.34 kN can be determined by proportion. Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.

$$M_{AB} = 0$$

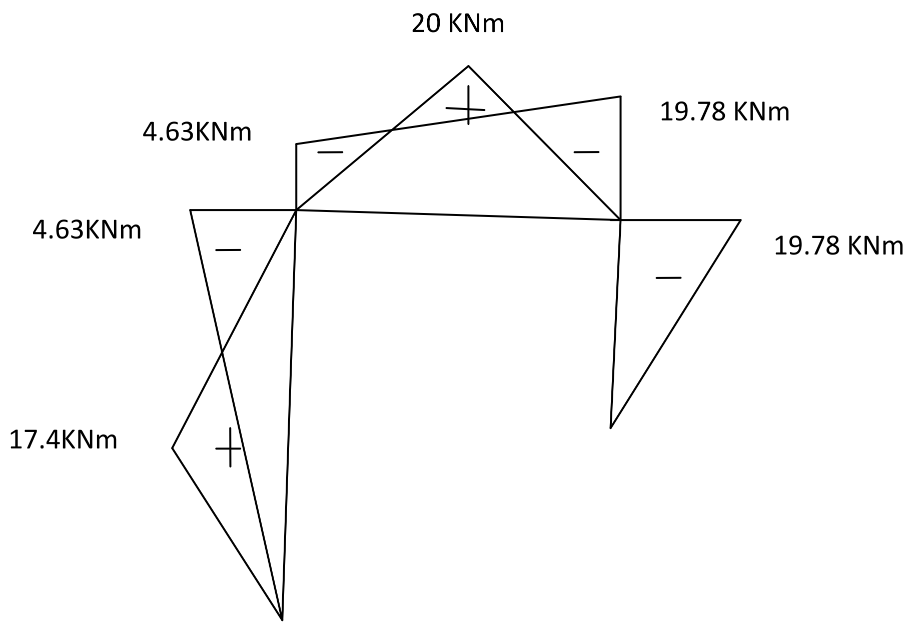
$$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63\text{KNm}$$

$$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63\text{KNm}$$

$$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78\text{KNm}$$

$$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78\text{KNm}$$

$$M_{DC} = 0$$



B.M.D