Question Paper Solutions

Unit 1

1. **Define a) Young's Modulus, b) Shear Modulus and c) Poisson's Ratio and write the relationship between them (June 2014)**

Solution:

i) **Young's Modulus or Modulus of Elasticity:**
It states that "Within the elastic limit the stress is directly proportional to strain"

i,e. Stress α strain

Stress = Strain x constant

$$\frac{Stress}{strain} = constant$$

This constant is called Elastic modulus or Modulus of Elasticity or Young's Modulus and it is denoted as E

$$\therefore \mathbf{E} = \frac{\sigma}{\epsilon}$$

ii) **Shear Modulus or Modulus of Rigidity:**

It is defined as "the ratio of shear stress to the displacement per unit sample length (shear strain)" and denoted by G or C or N expressed in kN/m^2.

$$\tau = \frac{Shear\ Stress}{Shear\ strain}$$

iii) **Poisson's Ratio** :

Poisson's Ratio is defined as the ratio between lateral strain to the longitudinal strain and denoted by μ or 1/m. It is dimensionless quantity.

$$\mu \text{ or } 1/m = \frac{\epsilon_b}{\epsilon_l} = \frac{\delta b/_b}{\delta l/_l}$$

Relation between them is $\boldsymbol{E = 2G(1 + \mu)}$

2. **Define Bulk Modulus and Volumetric Strain** (**June 2013 /Jan2015/June2015**)

Bulk Modulus: The ratio of direct stress to volumetric strain.

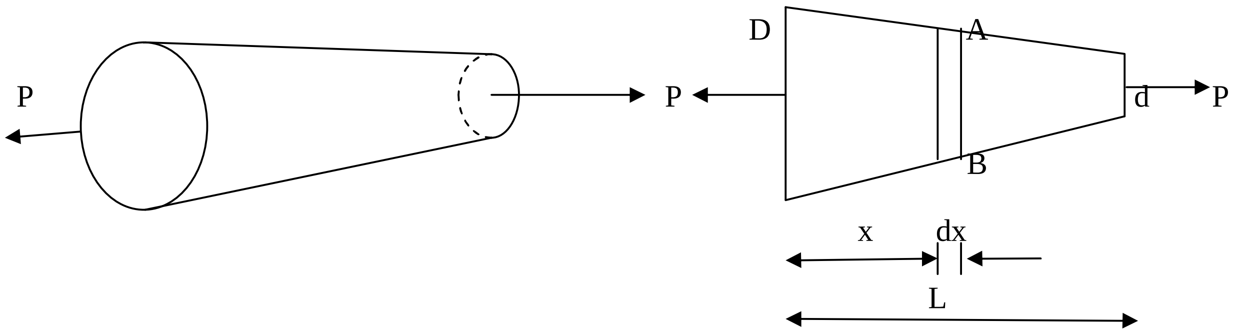
$$\mathbf{K} = \frac{Stress}{Volumetric\ Strain} = \frac{p}{\epsilon_v}$$

Volumetric Strain : It is defined as the ratio between change in volume to the actual volume

$$\epsilon_V = \frac{\delta V}{V} = \frac{dV}{V}$$

3. **Derive an equation for deformation in tapering circular bar subjected to an axial load P**

(**June 2014**)



Consider a tapering bar as shown in the fig. The diameter of the bar varies from D to d over a length of L

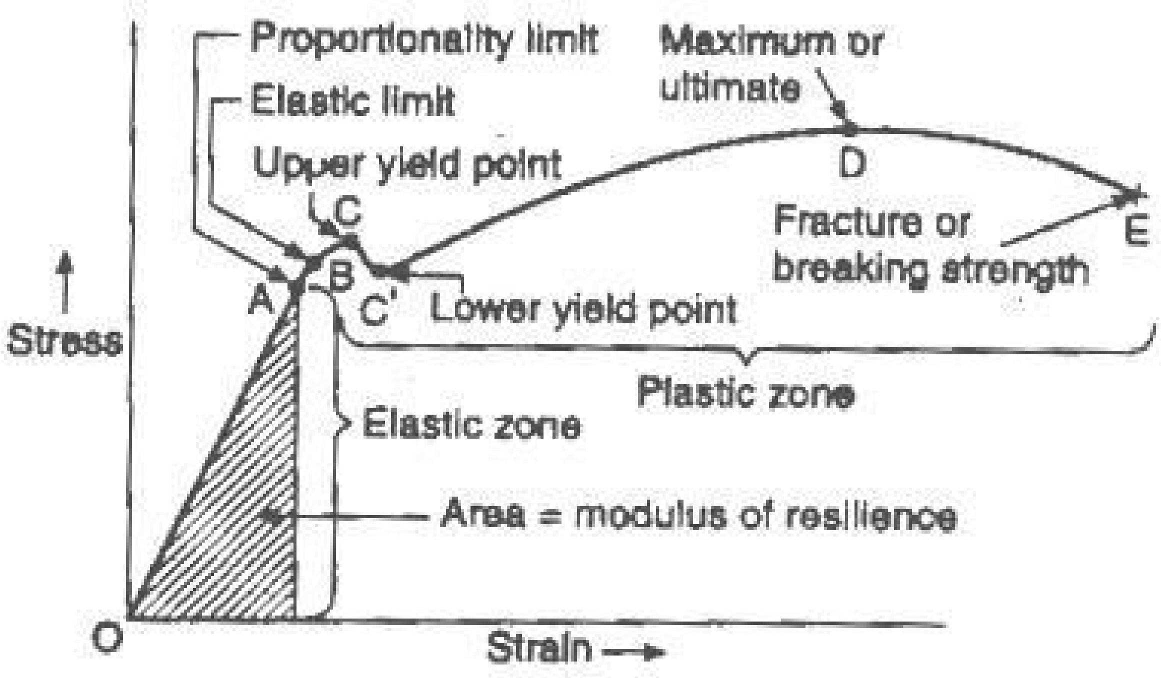
The diameter at the section AB $= D - \frac{D-d}{L}x = D - kx$ …….(1)

The change in length over the element dx = $\frac{PL}{AE} = \frac{4P\,dx}{\pi(D-kx)^2 E}$

The total change in length dl = $\int_0^L \frac{4P\,dx}{\pi(D-kx)^2 E} = \frac{4P}{k\pi E}\left[\frac{1}{D-kx}\right]_0^L = \frac{4PL}{\pi E(D-d)}\left(\frac{1}{d} - \frac{1}{D}\right) = \frac{4PL}{\pi EDd}$

4. **Breifly explain the behavior of ductile material under gradually increasing tensile load**

(**July2014/Jan2015**)



A typical tensile test specimen on mild steel is as shown above. The ends of the specimen are gripped into a universal testing machine. The specimen has lager diameter at the ends to see that the specimen at the ends to see that the specimen does not fail in the end regions. The specimen should fail in the gauged portion. The deformation is recorded as the load is applied(increases).

The elongation is recorded with the help of strain gauges. The loading is done till the specimen fails. A graph of stress verses strain is plotted and a typical stress strain curves for mild steel is as follows

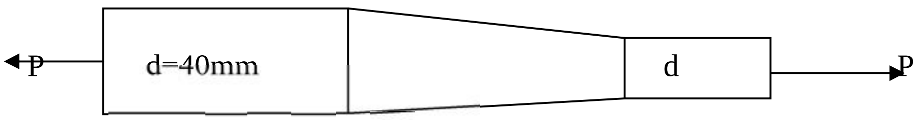
Point A -Proportionality Limit
Point B- Elastic Limit
Point C- Lower Yield point
Point C'-Upper yield Point
Point D- Ultimate stress
Point E- Failure or Rupture or Breaking stress

From the graph the plot from O-A is linear i,e the stress is proportional to the strain within this limit as such A is called proportionality limit. "Hooks Law" is valid in this region. On further increase in load, the curve takes a fall as represented from A to C. Between A and C there exists a point B. The material behaves like an elastic material till this limit and as such B is called Elastic Limit. Loading the material beyond C causes permanent deformation of the specimen i,e the cross sectional area reduces and length increases till the upper yield point C is reached. Loading the specimen beyond C, the material regains some strength and this continues till ultimate stress (D) is reached. On further increase in load the specimen finally fails at failure stress(E).

1) Proportional Limit / Limit of proportionality :-
   It is the limiting or maximum stress value upto which the stress is proportional to the strain.
2) Elastic Limit :-This is the maximum or limiting stress value such that there is mo permanent deformation in the material.
3) Yield Point /Yield Strength:- These are the lower stresses at which the extension of the specimen is rapid without much increase in the strain.
4) Ultimate stress or yield strength:- this is the maximum stress, the material can resist.
5) Breaking stress/ Breaking strength:- this is stress at which the specimen fails or breaks or ruptures.

5. **A member is of total length of 2m its diameter is 40mm for the first 1m length. In the next 0.5m it is gradually decreases to a diameter 'd'. for the remaining length the diameter 'd' remains same. The member is subjected to an axial pull of 150 kN and its total elongation is 2.39mm. Determine the diameter 'd', assume E = 2 X 10^5N/mm^2 (Jan2015)**

Solution:



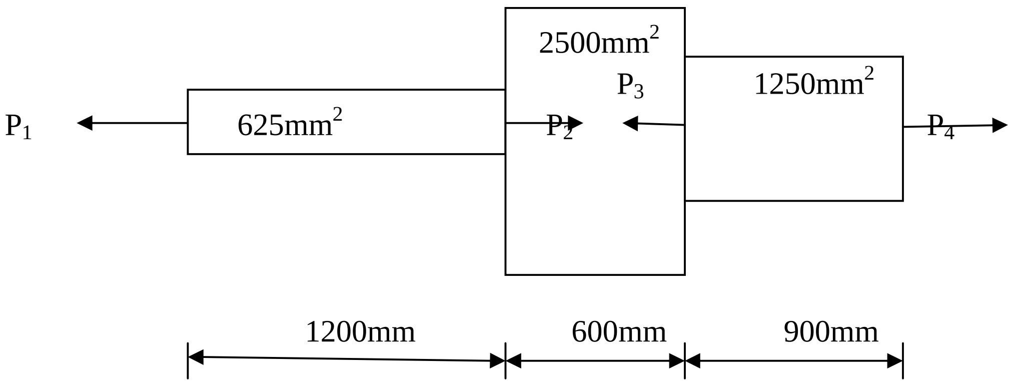
$$dl = dl_1 + dl_2 + dl_3$$

$$2.39 = \frac{150\times10^3}{2\times10^5}\left[\frac{4\times1000}{\pi\times40^2} + \frac{4\times500}{\pi\times40\times d} + \frac{4\times500}{\pi\times d^2}\right]$$

Solving d =20mm

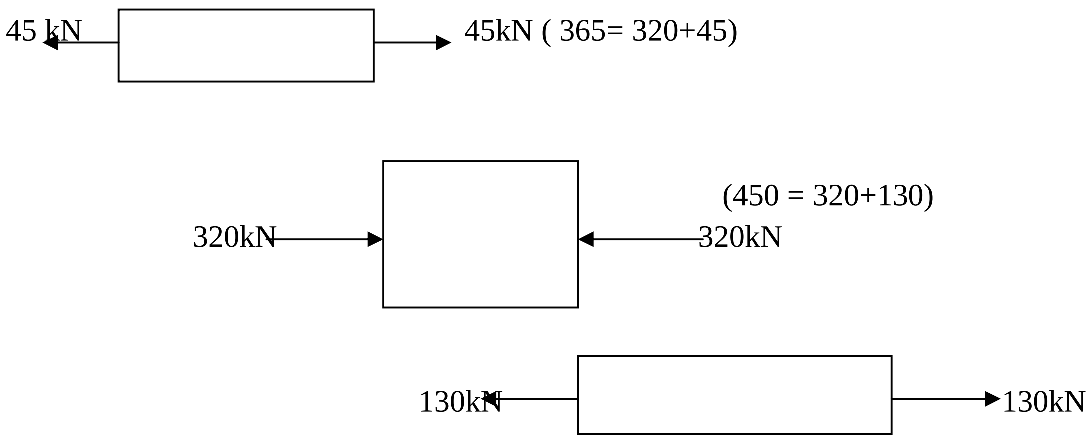
6. **A member ABCD subjected to point loads P_1, P_2, P_3 and P_4 as shown in fig 1. Calculate P_2 for equilibrium if P_1 = 45 kN, P_3 = 450kN and P_4 = 130kN, if E = 2.1X 10^5 N/mm^2. Determine the total elongation of the member.**

(June 2013)



P_1 = 45 kN, P_3 = 450 kN and P_4 = 130 kN

From Figure $P_1 + P_3 = P_2 + P_4$ $\therefore$ P_2 = 365



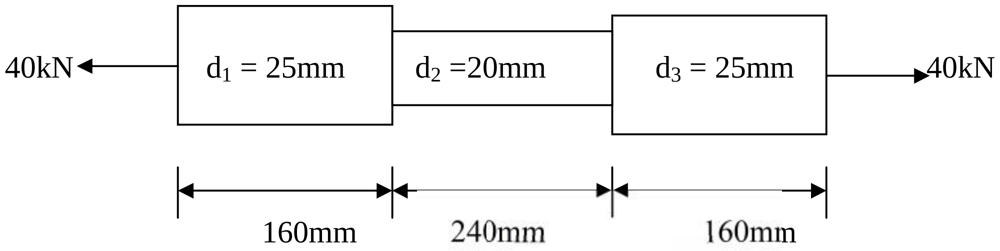
$$dl = dl_1 - dl_2 + dl_3$$

$$dl = = \frac{1}{2.1 \times 10^5}\left[\frac{45 \times 10^3 \times 1200}{625} - \frac{320 \times 10^3 \times 600}{625} + \frac{130 \times 10^3 \times 1200}{900}\right]$$

dl =

7. **The bar shown in fig 2 is tested in universal testing machine. It is observed that at a load of 40kN the total extension is 0.285mm. Determine the Young's modulus of the material**

(**Dec 2014**)



$$dl = dl_1 + dl_2 + dl_3$$

$$0.285 = \frac{40 \times 10^3}{E}\left[\frac{4 \times 160}{\pi \times 625} + \frac{4 \times 240}{\pi \times 400} + \frac{4 \times 160}{\pi \times 625}\right]$$

$$E = 56.63X\ 10^3\ N/mm^2$$

8. **A mild steel rod 2.5 m long having a cross sectional area of 50 mm2 is subjected to a tensile force of 1.5 kN. Determine the stress, strain, and the elongation of the rod. Take E = 2 × 105 N/mm^2**

(**June 2015**)

Solution:

Data Given
Length of the rod 'L' = 2.5 m = 250 mm
Area of cross-section 'A' = 50 mm^2
Tensile force 'P' = 1.5 kN = 1.5 × 10^3 N
Young's Modulus 'E' = 2 × 105 N/mm^2
Stress σ = P/A= 1.5×10^3 / 50 =30N /mm^2
Since, E = Stress / Strain Strain = Stress / E = 30 / 2 × 105 = 0.0015
Also, Elongation = Strain x Original length = 0.0015 × 2500 = 0.375 mm.

Unit 2

1. Define composite member, temperature stresses **(Dec 2013)**

A section made up of more than one material designed to resist the applied load is called 'COMPOSITE SECTION'

When a body is subjected to change in temperature its dimensions will also be changed (since the bodies are subjected to thermal expansion or contraction). For metals when the temperature of a body is increased there is a corresponding increase in its dimensions. When the body is allowed to expand (without restraining) no stress develops. But, in case the body is restrained prevents the expansion, then the stresses in the body will develop. These stresses are called as thermal stress

2. Define Modular Ratio, Volumetric strain **(Jun 2013/June2015)**

Modular Ratio: Modular ratio between two materials is defined as the ratio of Young's Modulus of Elasticity of two materials.

Volumetric Strain : It is defined as the ratio between change in volume to the actual volume

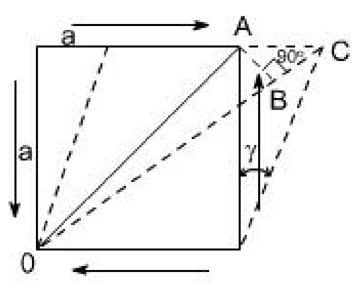
$$\epsilon_V = \frac{\delta V}{V} = \frac{dV}{V}$$

3. Derive the relation between modulus of rigidity, young's Modulus and poisson's ratio **(Dec 2013)**

Relation between E, G and :

Let us establish a relation among the elastic constants E,G and u. Consider a cube of material of side 'a' subjected to the action of the shear and complementary shear stresses as shown in the figure and producing the strained shape as shown in the figure below.

Assuming that the strains are small and the angle A C B may be taken as 45^0.



Therefore strain on the diagonal OA

= Change in length / original length

Since angle between OA and OB is very small hence OA @ OB therefore BC, is the change in the length of the diagonal OA

$$\text{Thus, strain on diagonal OA} = \frac{BC}{OA}$$

$$= \frac{AC\cos 45^0}{OA}$$

$$OA = \frac{a}{\sin 45^0} = a.\sqrt{2}$$

hence

$$\text{strain} = \frac{AC}{a\sqrt{2}}.\frac{1}{\sqrt{2}}$$

$$= \frac{AC}{2a}$$

but $AC = a\gamma$

where γ = shear strain

$$\text{Thus, the strain on diagonal} = \frac{a\gamma}{2a} = \frac{\gamma}{2}$$

From the definition

$$G = \frac{\tau}{\gamma} \text{ or } \gamma = \frac{\tau}{G}$$

$$\text{thus, the strain on diagonal} = \frac{\gamma}{2} = \frac{\tau}{2G}$$

Now this shear stress system is equivalent or can be replaced by a system of direct stresses at 45^0 as shown below. One set will be compressive, the other tensile, and both will be equal in value to the applied shear strain.

thus, for the direct state of stress system which applies along the diagonals:

$$\text{strain on diagonal} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{\tau}{E} - \mu\frac{(-\tau)}{E}$$

$$= \frac{\tau}{E}(1+\mu)$$

equating the two strains one may get

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

or

$$\boxed{E = 2G(1+\mu)}$$

4. Define principle of superposition **(June 2014)**

The homogeneity and additivity properties together are called the superposition principle. A linear function is one that satisfies the properties of superposition. Which is defined as

$$F(x_1 + x_2 + \cdots) = F(x_1) + F(x_2) + \cdots$$ **Additivity**

$$F(ax) = aF(x)$$ **Homogeneity**

for scalar *a*.

5. A steel rod is 18m long at a temperature at 25˚C. Find the free expansion when the temperature is raised to 85˚C. Also find the temperature stress produced when
   i) the expansion is completely prevented
   ii) The rod is permitted to expand by 4.5mm
   Given E_s = 200KPa, α_s= 12X 10^{-6} /˚C **(June 2015)**

Solution: i) When the expansion is completely prevented

The free expansion Δl= L.α.Δt = 18 (85-25)12×10^{-6} = 0.01296m

The thermal stress is = σ_t = E ϵ = E Lα Δt/L = E α Δt = 200x10^3x12x10^{-6}(85-25)

= 144 N/mm^2

ii) when the rod is permitted to expand by 4.5mm

the expansion = 0.01296X1000-4.5 = 8.46mm

the thermal stresses = E (ΔL- δ)α Δt/L = 6.78N/mm^2

6. A bar of brass of 25 mm diameter is enclosed in a steel tube of 25mm internal diameter and 50mm external diameter. The bar & the tube are rigidly connected at both ends. Find the stresses in both the materials of the system, where the temperature is raised from 15˚C to 95˚C. Assume E_s = 2X 10^5MPa, α_s= 11.6 X 10^{-6} /˚C, E_b= 1X 10^5MPa, α_b= 18.7 X 10^{-6} /˚C **(Dec 2013)**

Solution: i) When the expansion is completely prevented

The free expansion Δl= L.α.Δt = 18 (85-25)12×10^{-6} = 0.01296m

The thermal stress is = σ_t = E ϵ = E Lα Δt/L = E α Δt = 200x10^3x12x10^{-6}(85-25)

= 144 N/mm^2

ii) when the rod is permitted to expand by 4.5mm

the expansion = 0.01296X1000-4.5 = 8.46mm

the thermal stresses = E (ΔL- δ)α Δt/L = 6.78N/mm^2

7. A load of 2MN is applied on a column 500mm X 500mm. The column is reinforced with four steel bars 10mm diameter, one in each corner. Find the stresses in concrete and steel bar. E_s = 2.1X 10^5N/mm^2, Ec= 1.4X 10^4 N/mm^2

Solution : $\epsilon_l = \frac{dl}{l} = \frac{0.3}{150} = 2 \times 10^{-3}$

$$\epsilon_d = \frac{dd}{d} = \frac{0.079}{12} = 6.58 \times 10^{-3}$$

$\mu = \frac{\epsilon_d}{\epsilon_l} = 0.303$

$E = \frac{\sigma}{\epsilon} = \frac{20\times1000\times4}{2\times10^{-3}\times\pi\times12^2} = 88.42 \times 10^3 N/mm^2$

$E = 3K(1-2\mu)$

$K = \frac{E}{3(1-2\mu)} = 74.80 \times 10^3 N/mm^2$

$E = 2G(1+\mu)$

$G = \frac{E}{2(1+\mu)} = 33.93 \times 10^3 N/mm^2$

8. A 12mm diameter specimen is subjected to a tensile force of 20kN and deformation is 0.3mm, observed over a gauge length of 150mm. The reduction in diameter is 0.0079mm. Determine the elastic constants **(June2014)**

Solution : $\epsilon_l = \frac{dl}{l} = \frac{0.3}{150} = 2 \times 10^{-3}$

$\epsilon_d = \frac{dd}{d} = \frac{0.079}{12} = 6.58 \times 10^{-3}$

$\mu = \frac{\epsilon_d}{\epsilon_l} = 0.303$

$E = \frac{\sigma}{\epsilon} = \frac{20\times1000\times4}{2\times10^{-3}\times\pi\times12^2} = 88.42 \times 10^3 N/mm^2$

$E = 3K(1-2\mu)$

$K = \frac{E}{3(1-2\mu)} = 74.80 \times 10^3 N/mm^2$

$E = 2G(1+\mu)$

$G = \frac{E}{2(1+\mu)} = 33.93 \times 10^3 N/mm^2$

Unit 3

1. Define principal stresses and principal planes. **(Jan2015)**

For a given compound stress system, there exists a maximum normal stress and a minimum normal stress which are called the Principal stresses. The planes on which these Principal stresses act are called Principal planes. In a general 2-D stress system, there are two Principal planes which are always mutually perpendicular to each other. Principal planes are free from shear stresses. In other words Principal planes carry only normal stresses.

2. In a 2-D stress system compressive stresses of magnitudes 100 MPa and 150 MPa act in two perpendicular directions. Shear stresses on these planes have magnitude of 80 MPa. Use Mohr's circle to find,

(i) Principal stresses and their planes
(ii) Maximum shears stress and their planes and
(iii) Normal and shear stresses on a plane inclined at 45^0 to 150 MPa stress. **(June2015)**

Given, $f_x = -150$ MPa
$f_y = -100$ MPa
q = 80 MPa

If Mohr's circle is drawn to scale, all the quantities can be obtained graphically. However, the present example has been solved analytically using Mohr's circle.

Construct Mohr's circle with earlier fig

From figure

$$OC = \frac{f_x + f_y}{2} = -125\,\text{MPa}$$

To find Radius of Circle

$$CH = \frac{f_x - f_y}{2} = 25\,MPa$$

$$CA = \sqrt{CH^2 + HA^2} = 83.82$$

$$\therefore \text{ Radius} = CD = CE = CF = CG = CA = 83.82 \text{ Units}$$

To find Principal Stress and Principal Planes

f_{n-max} = OC + CD
= – 125 – 83.82
= – 208.82 MPa

$f_{n\ min}$ = OC – CE
= – 125 – (–83.82)

$$= -41.18 \text{ MPa}$$

From figure

$$\alpha = \tan^{-1}\left(\frac{AH}{MC}\right) = 72^0.65$$

$$\text{But } 2\theta_{P1} = \llcorner ACH = \alpha = 72^0.65$$

$$\text{Hence, } \theta_{P1} = 36^0.32$$

$$\text{Further, } 2\theta_{P2} = \llcorner ACE = 180 + \alpha = 252^0.65$$

$$\text{Hence, } \theta_{P2} = 126^0.32$$

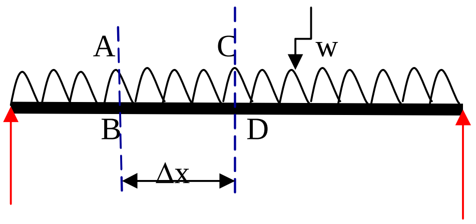
Unit 4

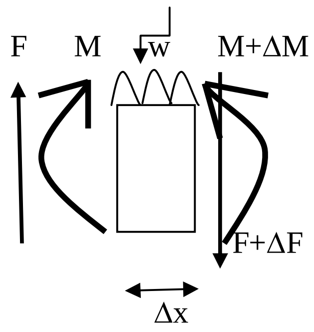
1. Derive a relationship between BM, SF and intensity of load

(June 2014/Jan2015)

RELATIONS BETWEEN LOAD, SHEAR AND MOMENT

Consider a simply supported beam subjected to a Uniformly Distributed Load w/m. Let us assume that a portion PQRS of length Δx is cut and taken out. Consider the equilibrium of this portion





$$\sum V = 0$$

$$F - (F + \Delta F) - w\,\Delta x = 0$$

$$\frac{\Delta F}{\Delta x} = -w$$

Limit $\Delta x \to 0$, then $\dfrac{dF}{dx} = -w$ or $F = \int w\,dx$

Taking moments about section CD for equilibrium

$$M-(M+\Delta M)+F\,\Delta x-(w(\Delta x)^2/2)=0$$

$$F=\frac{\Delta M}{\Delta x}$$

Limit $\Delta x \to 0$ then $F = \frac{M}{x}$ or $M = \int F\,dx$

Rate of change of Shear Force or slope of SFD at any point on the beam is equal to the intensity of load at that point.

2. Define Shear Force, Bending moment, SFD and BMD **(June 2013)**

Shear Force

The imbalance shear force which shears the beam in a section is called as Shear Force It is a single vertical force developed internally at any point on the beam to balance the external vertical forces and keep the point in equilibrium. It is therefore equal to algebraic sum of all external forces acting to either left or right of the section.

Bending Moment

The imbalance moment, which bends into a circular arc is called as bending moment. It is a moment developed internally at each point in a beam that balances the external moments due to forces and keeps the point in equilibrium. It is the algebraic sum of moments to section of all forces either on left or on right of the section

Shear Forces Diagram (SFD)

The SFD is one which shows the variation of shear force from section to section along the length of the beam. Thus the ordinate of the diagram at any section gives the Shear Force at that section.

Bending Moment Diagram (BMD)

The BMD is one which shows the variation of Bending Moment from section to section along the length of the beam. The ordinate of the diagram at any section gives the Bending Moment at that section.

3. Explain the terms : i)Hogging bending moment ii) Sagging bending moment and iii) Point of contra-flexure **(June2015)**

Sagging bending moment

The top fibers are in compression and bottom fibers are in tension.

Hogging bending moment

The top fibers are in tension and bottom fibers are in compression.

Point of Contraflexure

When there is an overhang portion, the beam is subjected to a combination of Sagging and Hogging moment. The point on the BMD where the nature of bending moment changes from hogging to sagging or sagging to hogging is known as point of contraflexure. Hence, at point of contraflexure BM is zero. The point corresponding to point of contraflexure on the beam is called as point of inflection

4. Draw SFD, BMD for the beam shown in the fig 3. Also locate the point of contra-flexure.

(Jan 2015)

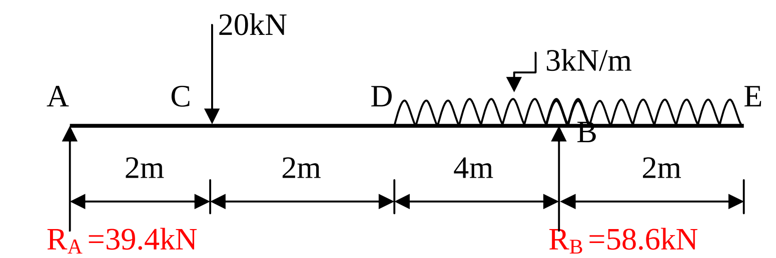
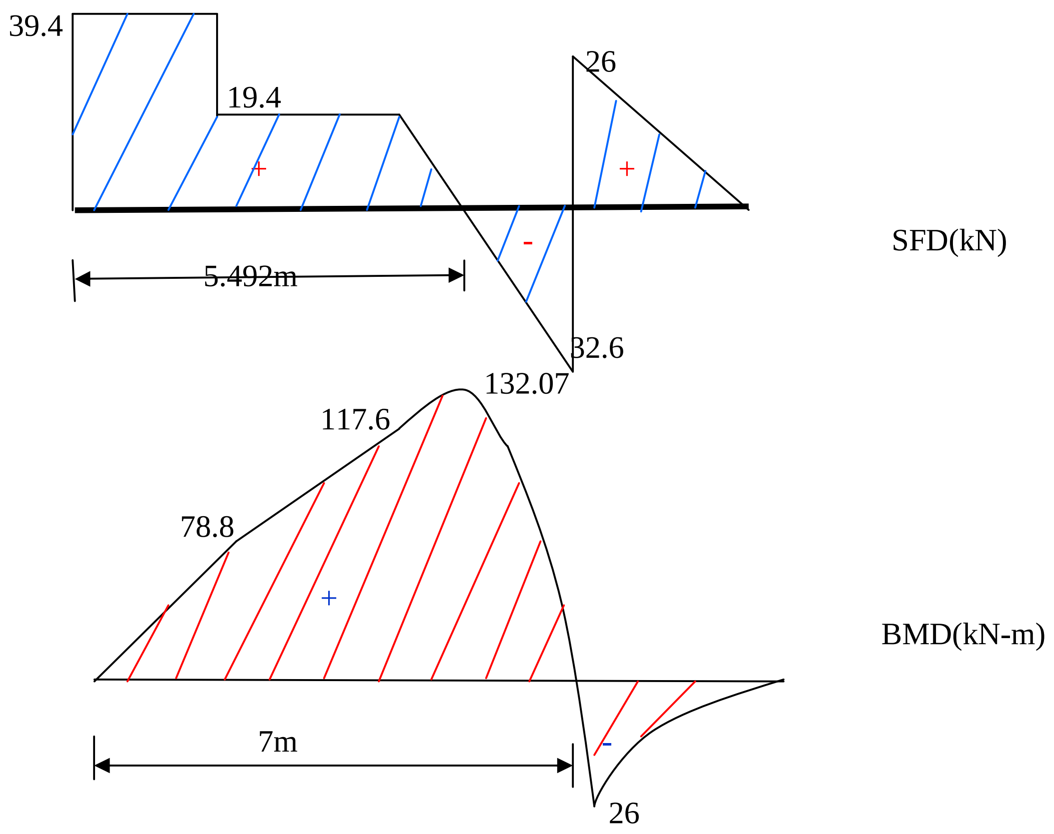
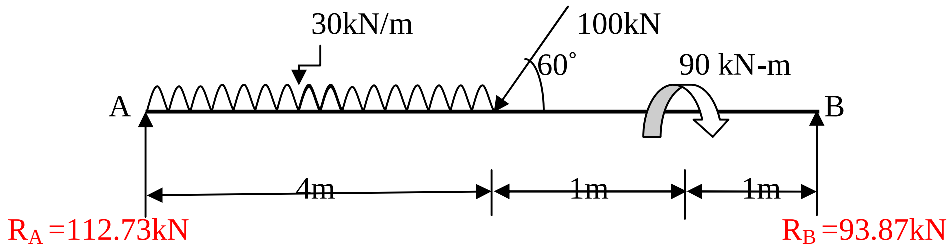


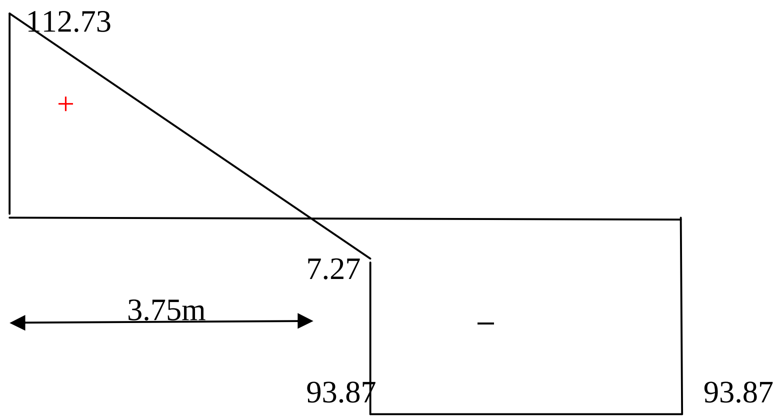
Fig 3

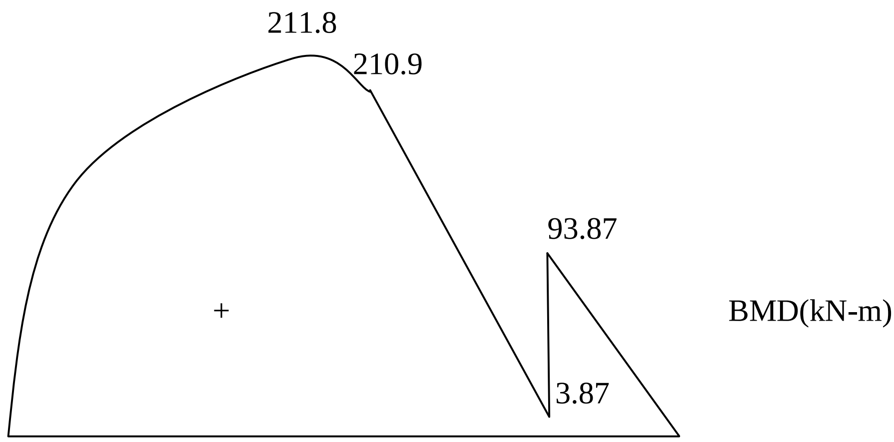


5. Calculate the SF and BM at all the salient points and draw the SFD and BMD for the beam shown in the fig 4

(June 2013)







Unit 5

1. Define Neutral axis, Section Modulus and Moment of Resistance

(June 2013/Jan2015)

Netural Axis

The layer neither subjected to tension nor to compression. Such a layer is called "Neutral Layer". The projection of Neutral Layer over the cross section of the beam is called "Neutral Axis".

Section Modulus: Section modulus of a beam is the ratio of moment of inertia of the cross section of the beam about the neutral axis to the distance of the farthest fiber from neutral axis.

$$\text{Therefore,} \quad Z = \frac{I}{y_{max}} \quad \text{unit} = mm^3$$

Moment of Resistance:

The tensile and compressive stresses result in a turning effect about the neutral axis. These are called moment $\mathbf{M}_T$ and $\mathbf{M}_C$ respectively. The chosen beam must be able to resist these moments with $\mathbf{M}_R$ (internal moment of resistance) if it is to remain in equilibrium.

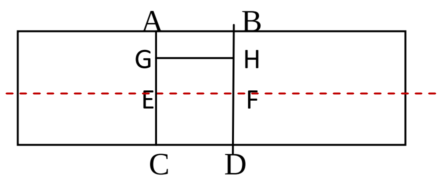
2. State the assumptions made in the pure bending **(Dec 2013)**

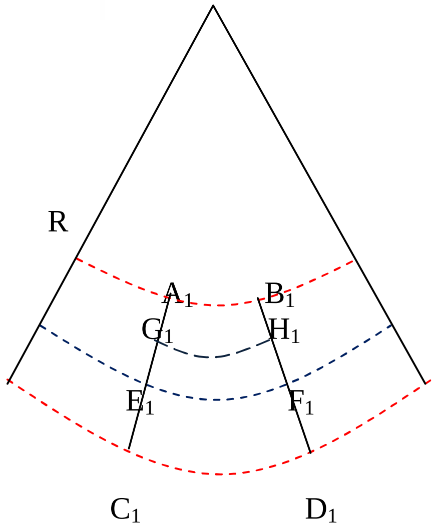
- The material is isotropic and homogenous.
- The material is perfectly elastic and obeys Hooke's Law i.e., the stresses are within the limit of proportionality.
- Initially the beam is straight and stress free.
- Beam is made up of number of layers and they undergo bending independently.
- Bending takes place over an arc of a circle and the radius of curvature is very large when compared to the dimensions of the beam.
- Normal plane sections before bending remain normal and plane even after bending.

3. Derive the equation of theory of simple bending with usual notation or

Derive a general bending equation $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ with usual notation **(Dec 2013/June2015)**

Euler- Bernoulli's Equation





Consider two section very close together (AB and CD). After bending the sections will be at A_1 B_1 and C_1 D_1 and are no longer parallel. AC will have extended to A_1 C1 and B_1 D_1 will have compressed to B_1D_1. The line EF will be located such that it will not change in length. This surface is called neutral surface and its intersection with Z-Z is called the neutral axis.

The development lines of A'B' and C'D' intersect at a point 0 at an angle of θ radians and the radius of E_1F_1 = R.

Let y be the distance(E'G') of any layer H_1G_1 originally parallel to EF.

Then $H_1G_1 / E_1F_1 = (R+y)\theta / R\theta = (R+y)/R$

and the strain ϵ at layer $H_1G_1 = \epsilon = (H_1G_1' - HG) / HG = (H_1G_1 - HG) / EF$

$$= [(R+y)\theta - R\theta] / R\theta$$

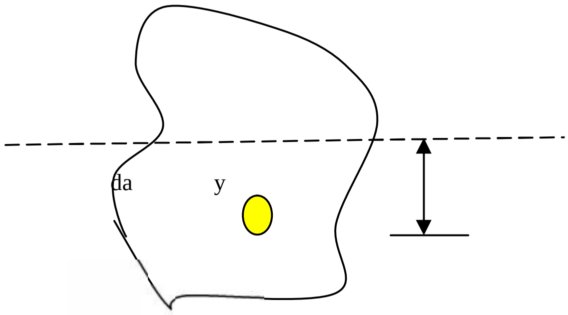
$$\epsilon = y / R.$$

The relation between stress and strain is $\sigma = E.\,\epsilon$ Therefore

$$\sigma = E.\,\epsilon = E.\,y / R$$

$$\sigma / E = y / R$$

Let us consider an elemental area ‘da ‘at a distance y, from the Neutral Axis.



stress developed over elemental area is $f = \frac{E}{R}\, y$

Force developed over elemental area = stress × area = $\frac{E}{R} y\, da$

Moment developed over elemental area about NA= Force × distance $= \frac{E}{R} y^2 da$

Total Moment developed from all the elements about the NA = $\int \frac{E}{R} da\, y^2$

$$M = \frac{E}{R} \int da\, y^2 = \frac{E}{R}\, I$$

$$\frac{M}{I} = \frac{E}{R} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

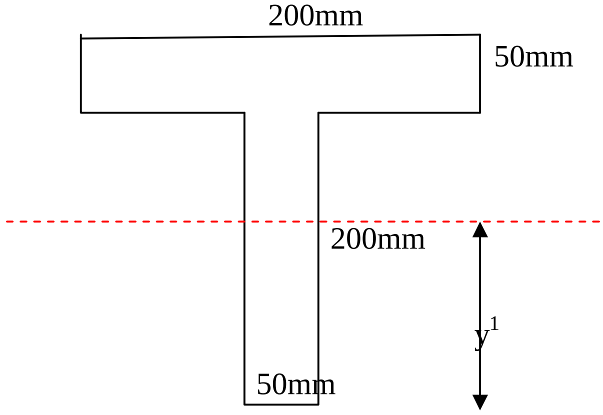
From Eqn (1) and (2), we get

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \quad \ldots\ldots\ldots\ldots\ldots(A)$$

4. Compare the flexural strength of the following three beams:
   i) I- section 320mm X160mm with 20mm thick flange and 13mm thick web
   ii) Rectangle section having depth twice the width
   iii) Solid circular section
   All the three beam sections have same cross-sectional area **(Jan 2015)**

5. A T section is having a flange of 200mm X 50mm. The web is also 200mm X 50mm. It is subjected to a bending moment 15kN-m. Draw the bending stress distributing across the section indicating the salient features

(Dec 2013/June2015)



$$y^1 = \frac{10000\times225+10000\times100}{20000} = 162.5mm$$

I =

$$\frac{200\times50^3}{12} + (200\times50\times(225-162.5)^2$$

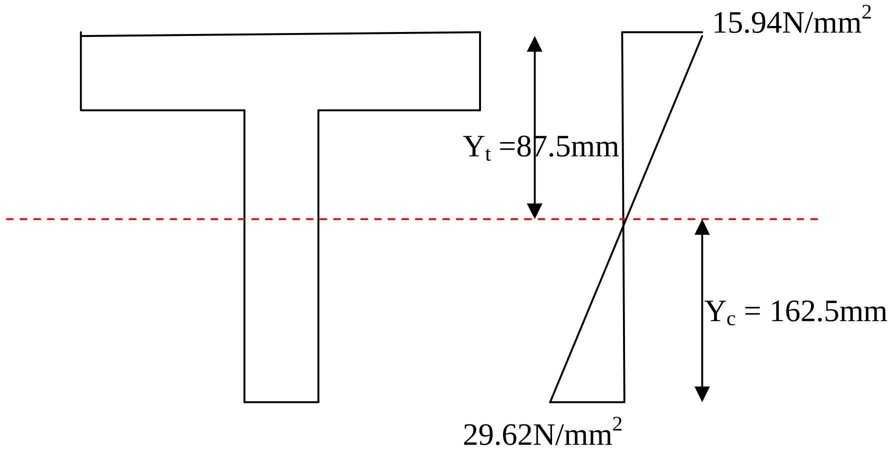
$$+\frac{200\times50^3}{12} + (200\times50\times(162.5-100)^2$$

$= 82.29X10^6\ mm^4$

The Bending moment equation is $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$

$$f = \frac{My}{I}$$

$f_t = \frac{My}{I} = 15.94\ N/mm^2$ $\quad$ $f_c = \frac{My}{I} = 29.62\ N/mm^2$



Unit 6

1. Define: 1) Slope 2) Deflection, 3) Elastic curve **(June 2013/June2014)**

Slope

Angle made by the tangent to the elastic curve with respect to horizontal

Deflection:

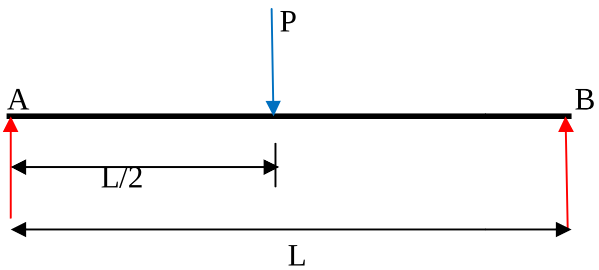
Vertical displacement measured from original neutral surface (refer to earlier chapter) to the neutral surface of the deformed beam.

Elastic Line:

It is a line which represents the deformed shape of the beam. Hence, it is the line along which the longitudinal axis of the beam bends

2. Derive an expression for the slope and deflection of a simply supported beam carrying a point load at the centre **(June2013)**

Concentrated load P at its Mid-span.



$$EI\,y'' = \frac{1}{2}Px - P\langle x - \frac{1}{2}L\rangle$$

$$EI\,y' = \frac{1}{4}Px^2 - \frac{1}{2}P\langle x - \frac{1}{2}L\rangle^2 + C_1 \text{ ....(1)}$$

$$EI\,y = \frac{1}{12}Px^3 - \frac{1}{6}P\langle x - \frac{1}{2}L\rangle^3 + C_1x + C_2 \text{ ....(2)}$$

At x =0; y =0 $\therefore$ C_2 =0

At x = L y =0

$$0 = \frac{1}{12}PL^3 - \frac{1}{48}PL^3 + C_1L$$

$$C_1 = -\frac{1}{16}PL^2$$

Maximum deflection occurs at x = L/2

Substituting the values of x and C_1 in equation…. (2)

$$EI\, y_{max} = \frac{1}{12}P(\tfrac{1}{2}L)^3 - \frac{1}{6}P(\tfrac{1}{2}L - \tfrac{1}{2}L)^3 - \frac{1}{16}PL^2(\tfrac{1}{2}L)$$

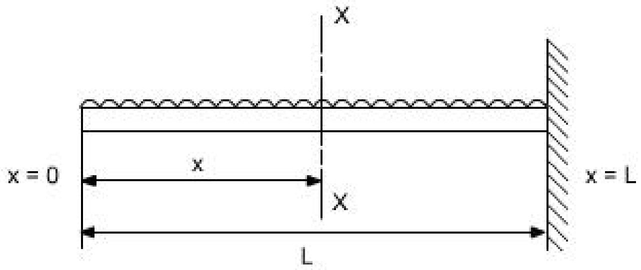
$$y_{max} = -\frac{PL^3}{48EI}$$

The negative sign indicates that the deflection is below the undeformed neural axis

$$\delta_{max} = \frac{PL^3}{48EI}$$

3. Establish the equation for slope and deflection for a cantilever beam of length L and carrying a UDL of w kN /m throughout. Also determine max slope and deflection. EI is constant

(June 2014/Jan2015)



$$S.F|_{x-x} = -w$$

$$B.M|_{x-x} = -w.x.\frac{x}{2} = w\left(\frac{x^2}{2}\right)$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = -\frac{wx^2}{2EI}$$

$$\int \frac{d^2y}{dx^2} = \int -\frac{wx^2}{2EI}\,dx$$

$$\frac{dy}{dx} = -\frac{wx^3}{6EI} + A$$

$$\int \frac{dy}{dx} = \int -\frac{wx^3}{6EI}\,dx + \int A\,dx$$

$$y = -\frac{wx^4}{24EI} + Ax + B$$

Boundary conditions relevant to the problem are as follows:

1. At x = L; y = 0

2. At x= L; dy/dx = 0

The second boundary conditions yields

$$A = +\frac{wx^3}{6EI}$$

whereas the first boundary conditions yields

$$B = \frac{wL^4}{24EI} - \frac{wL^4}{6EI}$$

$$B = -\frac{wL^4}{8EI}$$

$$\text{Thus, } y = \frac{1}{EI}\left[-\frac{wx^4}{24} + \frac{wL^3 x}{6} - \frac{wL^4}{8}\right]$$

So $y_{max}m$ will be at $x = 0$

$$\boxed{y_{max}m = -\frac{wL^4}{8EI}}$$

$$\boxed{\left(\frac{dy}{dx}\right)_{max}m = \frac{wL^3}{6EI}}$$

Unit 7

1. List the assumptions made in the theory of torsion **(Dec 2013/June2015)**

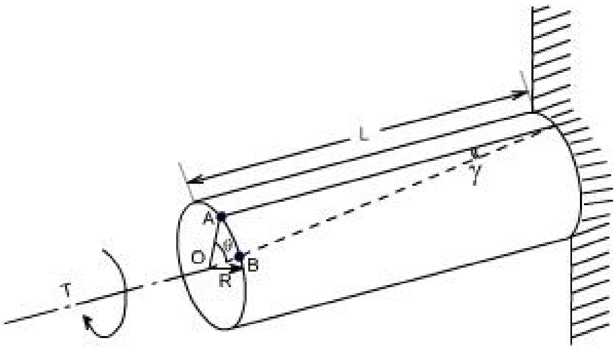
ASSUMPTIONS IN TORSION THEORY

1. Material is homogenous and isotropic
2. Plane section remain plane before and after twisting i.e., no warpage of planes.
3. Twist along the shaft is uniform.
4. Radii which are straight before twisting remain straight after twisting.
5. Stresses are within the proportional limit.

2. Derive the torsion equation for circular member $\frac{T}{I_p} = \frac{f_S}{R} = \frac{G\theta}{L}$ with usual notations

(June 2014/June 2013/Jan2015)

Consider now the solid circular shaft of radius R subjected to a torque T at one end, the other end being fixed Under the action of this torque a radial line at the free end of the shaft twists through an angle q , point A moves to B, and AB subtends an angle ' g ' at the fixed end. This is then the angle of distortion of the shaft i.e the shear strain.



Since angle in radius = arc / Radius

arc AB = Rq

= L g [since L and g also constitute the arc AB]

Thus, g = Rq / L (1)

From the definition of Modulus of rigidity or Modulus of elasticity in shear

$$G = \frac{\text{shear stress}(\tau)}{\text{shear strain}(\gamma)}$$

where γ is the shear stress set up at radius R.

Then $\frac{\tau}{G} = \gamma$

Equating the equations (1) and (2) we get $\frac{R\theta}{L} = \frac{\tau}{G}$

$\frac{\tau}{R} = \frac{G\theta}{L}\left(= \frac{\tau'}{r}\right)$ where τ' is the shear stress at any radius r.

Stresses: Let us consider a small strip of radius r and thickness dr which is subjected to shear stress t'.

The force set up on each element

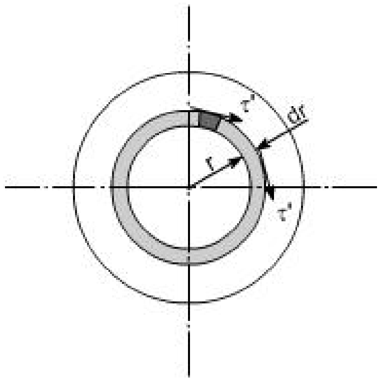
= stress x area

= τ' x 2p r dr (approximately)

This force will produce a moment or torque about the center axis of the shaft.

= τ' . 2 p r dr . r

= 2 p τ' . r^2. dr



The total torque T on the section, will be the sum of all the contributions. $T = \int_0^R 2\pi\tau' r^2 dr$

Since t' is a function of r, because it varies with radius so writing down t' in terms of r from the equation (1).

i.e $\tau' = \frac{G\theta . r}{L}$

we get $T = \int_0^R 2\pi \frac{G\theta}{L} . r^3 dr$

$$T = \frac{2\pi G\theta}{L}\int_0^R r^3 dr$$

$$= \frac{2\pi G\theta}{L} . \left[\frac{R^4}{4}\right]_0^R$$

$$= \frac{G\theta}{L} . \frac{2\pi R^4}{4}$$

$$= \frac{G\theta}{L} . \frac{\pi R^4}{2}$$

$$= \frac{G\theta}{L} . \left[\frac{\pi d^4}{32}\right]$$ now substituting R = d/2

$$= \frac{G\theta}{L} . J$$

since $\frac{\pi d^4}{32}$ = J the polar moment of inertia

or $\frac{T}{J} = \frac{G\theta}{L}$ ......(2)

if we combine the equation no.(1) and (2) we get $\boxed{\frac{T}{J} = \frac{\tau'}{r} = \frac{G.\theta}{L}}$

3. A 150 mm diameter solid steel shaft is transmitting 450 kW power at 90rpm. Compute the maximum shearing stress. Find the change that would occur in the shearing stress, if the speed were increased to 360rpm **(Dec 2013)**

Solution:

$$P = \frac{2\pi N T_{ave}}{60000} \quad \Rightarrow T_{ave} = \frac{60000\,P}{2\pi N} = 47746.48N - m$$

$$\frac{T}{I_p} = \frac{\tau_s}{r} \quad \Rightarrow \tau_s = 7.2N/mm^2$$

When the speed is increased to 360mm

T_{ave} = 11936.62N-m

$\tau_s = 1.801N/mm^2$

4. Determine the diameter of a solid circular shaft which will transmit 400 kilowatts at 300 rpm. The maximum shear stress should not exceed 32 N /mm^2.the twist should not be more than 1° in a length of 2 m, assume the modulus of rigidity as 90 kN/m^2.

(June 2014/June2015)

Solution

$$P = \frac{2\pi N T_{ave}}{60000} \quad \Rightarrow T_{ave} = \frac{60000\,P}{2\pi N} = 12732.40N - m$$

The diameter due shear stress

$$\frac{T}{I_p} = \frac{\tau_s}{r} \quad \Rightarrow \frac{12732.4 \times 1000}{\pi d^4/_{32}} = \frac{32}{d/_2}$$

d = 126.54 mm

The diameter due to angle of twist

$$\frac{T}{I_p} = \frac{C\theta}{L} \quad \Rightarrow \frac{12732.4 \times 1000}{\pi d^4/_{32}} = \frac{90 \times 10^{-3} \times 0.0174}{2000}$$

d = 325 mm

Unit 8

1. List the assumptions made in Euler's theory of long columns **(June 2014/Jan2015)**

Solution:

The assumptions made in the analysis are as follows:

- The column is long and fails by buckling.
- The column is axially loaded.
- The column is perfectly straight and the cross sections are uniform (prismatic).
- The column is initially free from stress.
- The column is perfectly elastic, homogeneous and isotropic.

2. Distinguish short column and long column, Define "slenderness ratio" of a column. **(June 2013/Dec 2013/Jan2015/June2015)**

Short Column :

A short column essentially fails by crushing and not by buckling. A column is said to be short, if $l_e/b \leq 15$ or $l_e/r_{min} \leq 50$, where l_e = effective length, b = least lateral dimension and r_{min}= minimum radius of gyration.

Long Column :

A long column essentially fails by buckling and not by crushing. In long columns, the stress at failure is less than the yield stress. A column is said to be long $l_e/b > 15$ or $l_e/r_{min} > 50$.

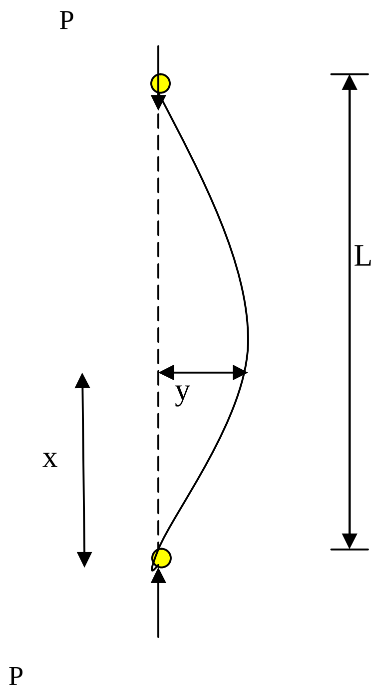
Slenderness ratio is defined as the ratio of effective length (l_e) of the column to the minimum radius of gyration (r_{min}) of the cross section. $\lambda = \frac{l_e}{r_{min}}$

Since an axially loaded column tends to buckle about the axis of minimum moment of inertia (I_{min}), the minimum radius of gyration is used to calculate slenderness ratio. Further, $r_{min} = \sqrt{\frac{I_{min}}{A}}$, where A is the cross sectional area of column.

3. Using Euler's theory, derive an equation for the crippling load of a long column Pinned at both ends **(June 2013)**

Both ends hinged (pinned)

Consider a long column with both ends hinged subjected to critical load P as shown.



Consider a section at a distance x from the origin. Let y be the deflection of the column at this section. Bending moment in terms of load P and deflection y is given by

$$M = -P\,y \qquad \text{------------ (1)}$$

We can also write that for beams/columns the bending moment is proportional to the curvature of the beam, which, for small deflection can be expressed as

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \quad \text{or } M = EI\,\frac{d^2y}{dx^2} \ldots\ldots\ldots\ldots\ldots(2)$$

where E is the Young's modulus and I is the moment of Inertia.

Substituting eq.(1) in eq.(2)

$$-P\,y = EI\,\frac{d^2y}{dx^2}$$

$$\text{or} \qquad \frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right) y = 0$$

This is a second order differential equation, which has a general solution form of

$$y = C_1 \sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right) \qquad \text{------(3)}$$

where C_1 and C_2 are constants. The values of constants can be obtained by applying the boundary conditions:

(i) y = 0 at x = 0. That is, the deflection of the column must be zero at each end since it is pinned at each end. Applying these conditions (putting these values into the eq. (3)) gives us the following results: For y to be zero at x =0, the value of C_2 must be zero (since cos (0) = 1).

(i) Substituting y = 0 at x = L in eq. (3) lead to the following.

$$0 = C_1 \sin\left(L\sqrt{\frac{P}{EI}}\right)$$

While for y to be zero at x = L, then either C_1 must be zero (which leaves us with no equation at all, if C_1 and C_2 are both zero), or

$$\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0$$

which results in the fact that

$$\left(L\sqrt{\frac{P}{EI}}\right) = n\pi$$

$$\text{or} \qquad L\sqrt{\frac{P}{EI}} = n\pi \qquad \text{where } n = 0, 1, 2, 2\ldots$$

$$\text{or} \qquad P = \frac{n^2\pi^2 EI}{L^2}$$

Taking least significant value of n, i.e. n = 1

We have $$P = \frac{\pi^2 EI}{L^2}$$

or $$P_E = \frac{\pi^2 EI}{l_e^2}$$

where l_e =L.

4. A simply supported beam of length 4 m is subjected to a uniformly distributed load of 30 kN/m over the whole span and deflects 15mm at the centre. Determine the crippling load when this beam is used as a column with both the ends hinged **(June 2013)**

$$\delta = \frac{5}{384}\frac{wl^4}{EI}$$

EI = 6.67 X 10^9 N-mm^2

$$P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 6.67\times10^9}{4000^2} = 4114.39 \text{ N}$$

5. A solid round bar 4m long and 50mm in diameter was found to extend by 4.6 mm long under a tensile load of 50kN. This bar is used a strut with both ends hinged {pinned}. Determine Euler's crippling load for the bar and also safe load taking factor of safety as 4. **(Dec 2014)**

Solution $E = \frac{P\,L}{A\,dl} = \frac{50\times1000\times4\times4000}{\pi\times50\times50\times4.6} = 22143.30\ N/mm^2$

$$P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2\times22143.30\times306796.15}{4000^2} = 4190.60 \text{ N}$$

$P_{safe} = P_E/FS = 4190.6/4 = 1047.64$ N

6. Find the Euler's critical load for a hollow cylindrical cast iron column 150 mm external diameter, 20 mm wall thickness if it is 6 m long with hinges at both ends. Assume Young's modulus of cast iron as 80 kN/mm^2. Compare this load with given by Rankine's formula using Rankine's constant a = 1/1,600 and f_c = 567 N/mm^2. **(June 2015)**

Given Data: External diameter = 150 mm, thickness = 20 mm, length = 6 m, E = 80 kN/mm^2, a = 1/1,600 and f_c = 567 N/mm^2.

$$A_0 = 8167.41 \text{ mm}^2, \; I = 17.66 \times 10^6 \text{ mm}^4$$

From Euler's Condition $l_e = l$ $P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 80 \times 10^3 \times 17.66 \times 10^6}{6000^2} = 387.33kN$

From Rankine Formulae $P_R = \frac{f_c A}{1 + a\left(\frac{l_e}{r_{min}}\right)^2} = \frac{567 \times 8167.41}{1 + \frac{1}{1600}\left(\frac{6000}{46.5}\right)^2} = 406.01kN$