VTU QUESTIONS PAPER SOLUTIONS

PART-A

UNIT-1: BASIC PROPERTIES OF FLUIDS

Q.1 Define the following fluid properties with units: (July 2014,2015)

i) Mass Density
ii) Specific Gravity
iii) Dynamic Viscosity
iv) Vapour Pressure
v) Capillarity

Ans:

(i) Mass density or Specific mass (°): Units: kg/m^3

Mass density or specific mass is the mass per unit volume of the fluid.

$$\therefore \rho = \frac{Mass}{Volume}$$

$$\rho = \frac{M}{V} \text{ or } \frac{dM}{dV}$$

(ii) **Specific gravity or Relative density (S):** Units: No-unit

It is the ratio of density of the fluid to the density of a standard fluid.

$$S = \frac{\rho_{fluid}}{\rho_{standard\ fluid}}$$

(iii) **Dynamic Viscosity:** Units: « $\frac{N\text{-sec}}{m^2}$ ***or*** « ***Pa - S***

Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it. 'F' is the force required to move the plate with a velocity 'U' According to Newton's law shear stress is proportional to shear strain.

$$F \sim A$$

$$F \sim \frac{1}{Y}$$

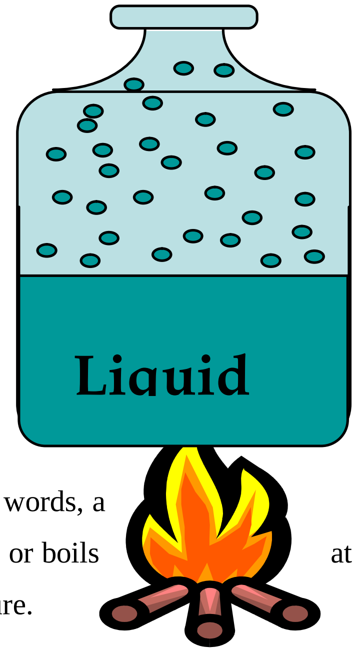
$$F \sim U$$

$$\therefore F \sim \frac{AU}{Y}$$

$$F = \mu . \frac{AU}{Y}$$

'μ' is the constant of proportionality called Dynamic Viscosity or Absolute Viscosity or Coefficient of Viscosity or Viscosity of the fluid.

(iv) Vapour Pressure: Units: N/m^2 or Pascal



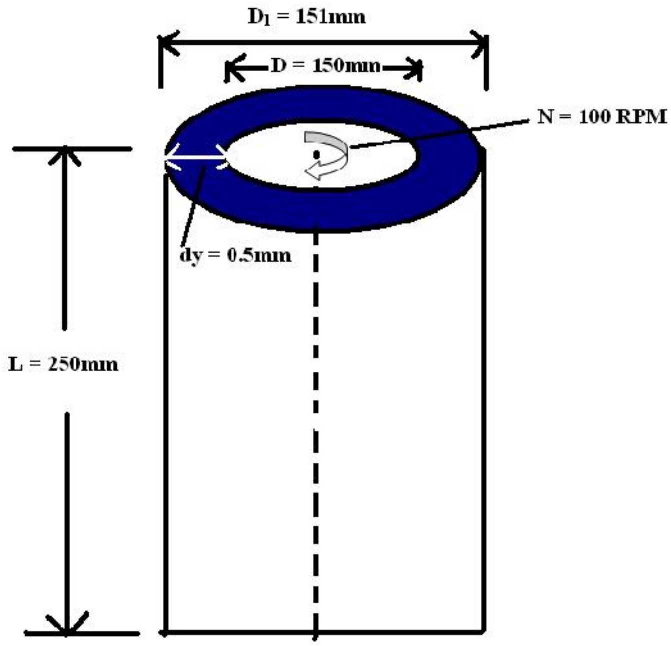
Vapor pressure is defined as the pressure at which a liquid will boil (vaporize) and is in equilibrium with its own vapor (Refer figure). Vapor pressure is important to fluid flows because, in general, pressure in a flow decreases as velocity increases. This can lead to ***cavitation***, which is generally destructive and undesirable. In particular, at high speeds the local pressure of a liquid sometimes drops below the vapor pressure of the liquid. In such a case, ***cavitation*** occurs. In other words, a "cavity" or bubble of vapor appears because the liquid vaporizes or boils at the location where the pressure dips below the local vapor pressure.

(v) Capillarity: Units: Capillary rise is measured in 'm' or 'cm'

Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them. Capillarity is due to cohesion adhesion and surface tension of liquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion is greater than adhesion then will be capillary fall or depression. The surface tensile force supports capillary rise or depression.

Q.2. A 150 mm diameter vertical cylinder rotates concentrically with inside another cylinder of diameter 151.0mm. Both cylinders are 250mm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque 12 N-m is required to rotate the inner cylinder with at 100rpm. Determine the viscosity of the fluid. (Dec2013,July 2014)

Ans:



Given N = 100 RPM

$$\boldsymbol{u} = \frac{® \times \boldsymbol{D} \times \boldsymbol{N}}{60} = \frac{® \times 0.15 \times 100}{60} = 0.786\boldsymbol{m/s}$$

D_1 = 151mm

D = 150 mm

L = 250mm

Torque T = 12 N-m

Torque T = F × r

$$12 = F \times \left(\frac{150}{2 \times 1000}\right)$$

Shear Force F = 160 N

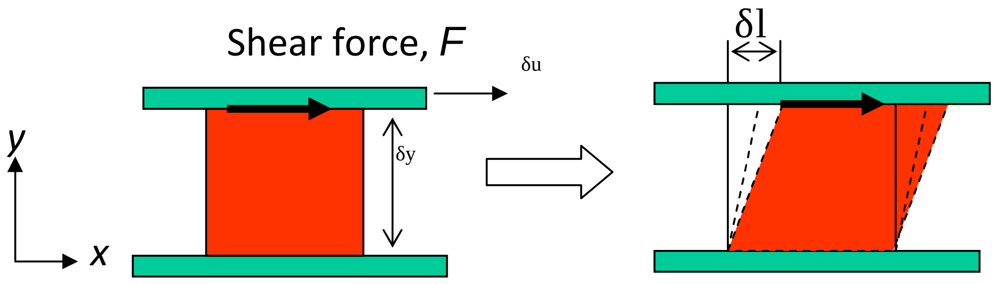
$$\boldsymbol{F} = ² \times \boldsymbol{A} = « \times \frac{\boldsymbol{du}}{\boldsymbol{dy}} \times ® \times \boldsymbol{D} \times \boldsymbol{L}$$

$$160 = « \times \frac{0.786}{0.0005} \times ® \times 0.15 \times 0.25$$

« **= 0.8635 N-sec/m²**

Q.3 Define fluid. Distinguish between liquids and gases. (Dec 2013,Jan 2014)

Ans:-Fluid: A *fluid* is a substance which deforms continuously under the action of shearing forces, however small they may be. Conversely, it follows that: If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.

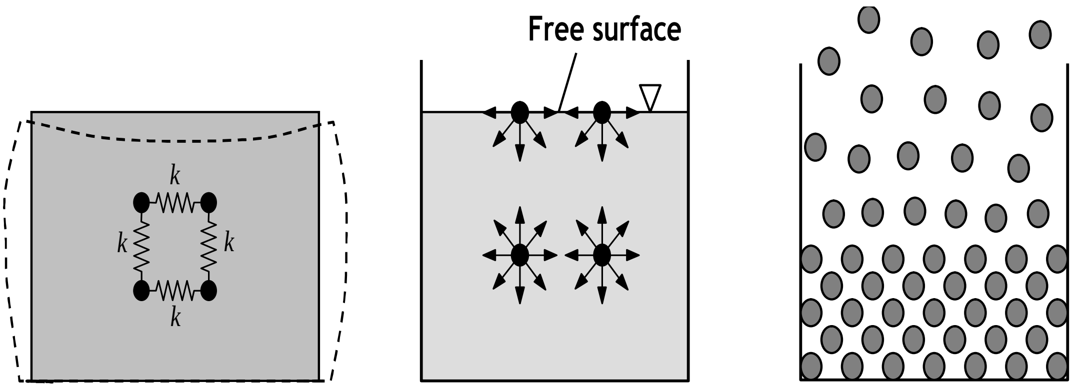


Fluid deforms continuously under the action of a shear

$$\tau_{yx} = \frac{dF_x}{dA_y} = f\left(\text{Deformation Rate}\right)$$

Liquids: It exhibits a free surface, Takes the shape of the container, practically incompressible

Gases: It occupies the full space, compressible and lighter than the liquids

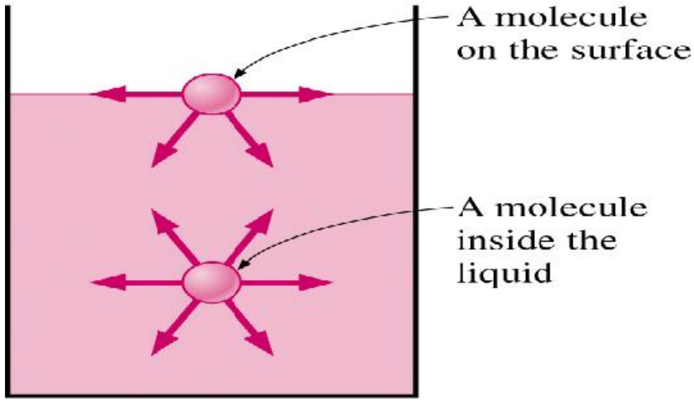


(a) Solid (b) Liquid (c) Gas

Q.4 Explain the phenomenon of surface tension. Derive an expression for pressure inside a liquid droplet (Dec 13, Jan 2015)

Ans:

The phenomenon of Surface tension: It is a force per unit length experienced by liquids at free surface due to imbalance of molecular forces at the interface of liquid –gas interface. The liquid surface at an interface appears to acts as a stretched elastic membrane.

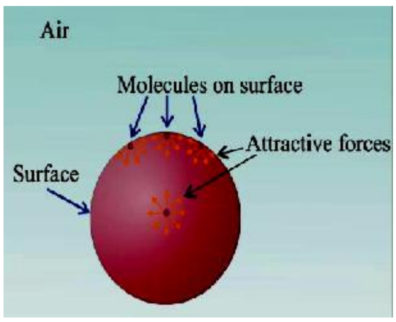


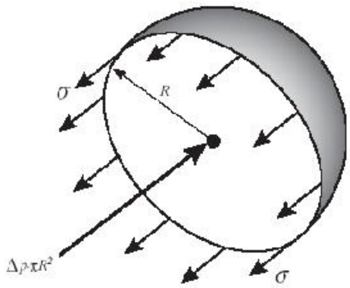
Pressure inside a Liquid droplet: Liquid droplets tend to assume a spherical shape since a sphere has the smallest surface area per unit volume.

The pressure inside a drop of fluid can be calculated using a free-body diagram of a spherical shape of radius R cut in half, as shown in Figure below and the force developed around the edge of the cut sphere is 2πRσ. This force must be balance with the difference between the internal pressure pi and the external pressure Δp acting on the circular area of the cut. Thus,

$$2\pi R\sigma = \Delta p \pi R^2$$

$$\Delta p = \left(p_{\text{int}\,ernal} - p_{external}\right) = \frac{2\times\pm}{R} = \frac{4\times\pm}{D}$$





Q.5 A cylinder of 120mm diameter rotates concentrically inside a fixed cylinder of diameter 125mm. Both the cylinders are 300mm long. Find the viscosity of the liquid that fills the space between the cylinders if a torque of 0.9 N—m required to maintaining speed of 60 rpm. (Dec 2013)

Solution: Given: D_1 = 120mm, D_2 = 125 mm, L= 300mm, N= 60 RPM, Torque = 0.9 N-m

$$y = \frac{(125-120)}{2\times 1000} = 2.5\times 10^{-3}\,m$$

$$Torque \;\; T = F \times \frac{d}{2} \Rightarrow 0.9 = F \times \frac{120}{2 \times 1000}$$

$$F = 15N$$

Area A = $\pi \times D \times L = \pi \times \frac{120}{1000} \times 0.3 = 0.1131 m^2$

$$V = \frac{® \, D \, N}{60} = \frac{® \times 0.12 \times 60}{60} = 0.377 m/s$$

$$² = « \frac{V}{y} \Rightarrow \frac{F}{A} = « \left(\frac{0.377}{2.5 \times 10^{-3}} \right)$$

$$\frac{15}{0.1131} = « \left(\frac{0.377}{2.5 \times 10^{-3}} \right)$$

«=0.88 N-sec/m^2

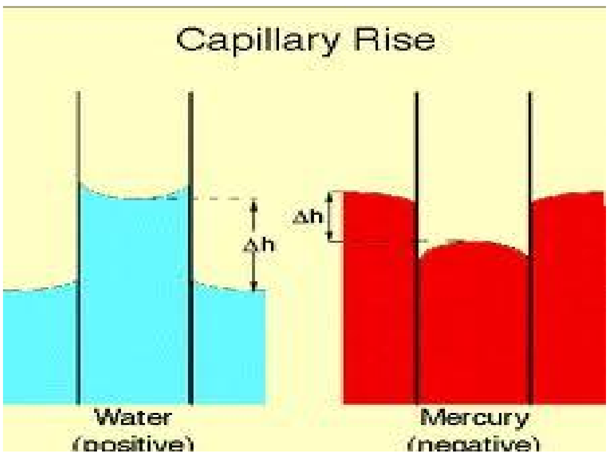
Q.6. Write units of (i) Surface tension (ii) Dynamic viscosity (iii) Power (iv) Momentum and (v) Pressure (July 2013, Jan 2014)

Ans:

i)	Surface tension	- N/m
ii)	Dynamic viscosity	- N-s/m^2 or Pa-sec
iii)	Power	- Watts or Joule/sec
iv)	Momentum and	- Kg-m/sec
v)	Pressure.	- Pa or N/m^2

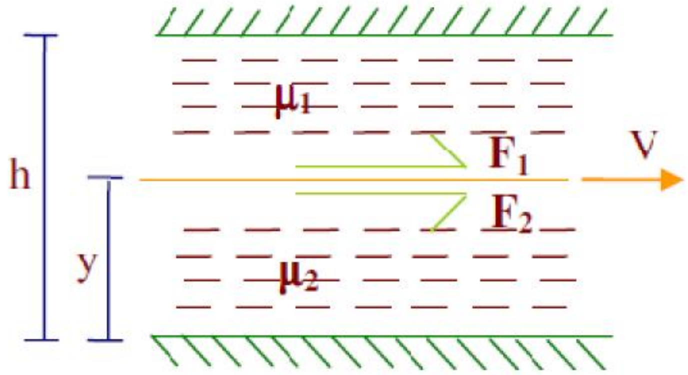
Q.7 Illustrate capillary rise and drop with appropriate sketches clearly indicating the fluids involved in each case. (July 2013, July 2015)

Ans:



Q. 8 A thin plate is placed between two flat surfaces 'h' cm apart such that the viscosity of the liquids on the top and bottom of the plate are μ_1 and μ_2 respectively. Determine the position of the thin plate such that the viscous resistance to uniform motion of the thin plate is minimum. Assume 'h' to be very small. (July 2013)

Ans. The pull required, to drag the plate is minimum i.e. $\left(\frac{dF}{dy}\right)_{minimum}$



To determine y = ? , when (F1 + F2) is minimum. As per Newton's Law of Viscosity, the sheer force F is given by,

$$Shear\ Force\ 'F' = « \left(\frac{du}{dy}\right) \times A$$

Shear force on bottom plate

$$F_1 = \frac{A\mu_1 V}{h\ \ y}$$

Shear force on bottom plate

$$F_2 = \frac{A\mu_2 V}{y}$$

y **Total force F = (F_1 + F_2)**

$$F = \frac{A\mu_1 V}{h - y} + \frac{A\mu_2 V}{y}$$

For F to be min . $\frac{dF}{dy} = 0$

$$\frac{dF}{dy} = 0$$

$$= \frac{V\mu_1 A}{(h - y)^2} - \frac{V\mu_2 A}{y^2}$$

$$\frac{(h - y)^2}{y^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{h - y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$(h - y) = y\sqrt{\frac{\mu_1}{\mu_2}}$$

$$h = y\sqrt{\frac{\mu_1}{\mu_2}} + y$$

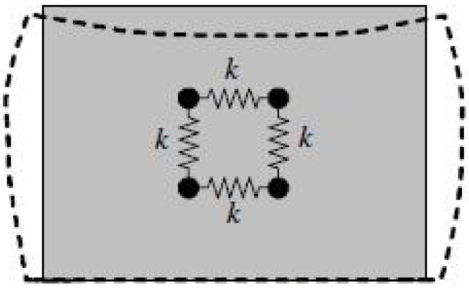
$$h = y\left[1 + \sqrt{\frac{\mu_1}{\mu_2}}\right]$$

$$\therefore \quad y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}}$$

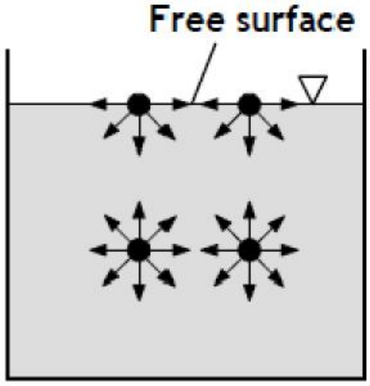
Q.9 List the differences between liquids and gases. (July 2013 , July 2015)

Ans:Differences between Liquids and Solids: The differences between thebehaviour of Liquids and solidsunder an applied force are as follows:

- For a solid, the strain is a function of the applied stress, providing that the elastic limit is not exceeded. For a liquid, the rate of strain is proportional to the applied stress.
- The liquid has free surface and acquires the shape of container while solid has defined boundaries
- The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow as long as the force is applied and will not recover its original form when the force is removed.



(a) Solid



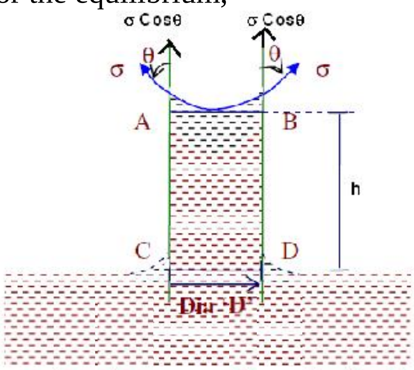
(b) Liquid

Q.10.What is capillarity? Obtain an expression for capillary rise of a liquid in a glass tube. (July 2013)

Ans:

Capillarity: Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them. Capillarity is due to cohesion adhesion and surface tension of liquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion is greater than adhesion then will be capillary fall or depression. The surface tensile force supports capillary rise or depression.

Let us consider a small tube of diameter 'D' dipped in a liquid of specific weight g. 'h' is the capillary rise. For the equilibrium,



Vertical force due to surface tension = Weight of column of liquid ABCD

$$(\sigma Cos\theta)\pi D) = \gamma \times Volume$$

$$(\sigma Cos\theta)\pi D = \gamma \times \frac{\pi}{4} \times D^2 \times h$$

$$h = \frac{4\sigma Cos\theta}{\gamma D}$$

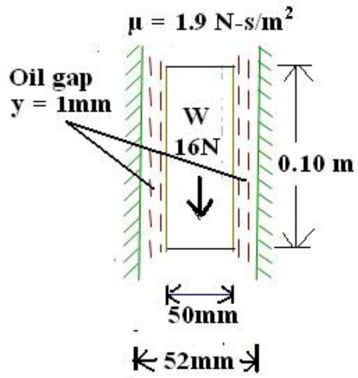
Q.11 A 50mm diameter and 0.10m long cylindrical body slides vertically down in a 52mm diameter cylindrical tube. The space between the cylindrical body and tube wall is filled with oil of viscosity 1.9 N-s/m^2. Determine its velocity of fall if its weight is 16N. (July 2013, July 2015)

Ans: Given μ = 1.9 N-s/m^2, D_1= 52mm, D_2 = 50mm, L = 0.1m, W = 16N

The oil gap is y = $\frac{(52-50)}{2}$ = 1mm = 0.001m

$$ShearForce = W = 16N = \mu\left(\frac{du}{dy}\right)\times A = 1.9\left(\frac{U}{0.001}\right)\times \pi\, D\, L = 1.9\left(\frac{U}{0.001}\right)\times \pi \times \frac{50}{1000}\times 0.1$$

$U = 0.536 m/s$



Q.12.Define and mention units of (i) Surface tension (ii) Kinematic viscosity (iii) Density (July 2013, Jan 2014, Jan 2015)

Ans:

(i)	Surface tension	- Unbalanced force at the air-liquid interface	N/m
(ii)	Kinematic viscosity	- Ratio of dynamic viscosity to density	m^2/s
(iii)	Density	- mass per unit volume	kg/m^3

Q.13 Calculate he capillary rise in a glass tube of 2.5mm when immersed in mercury. The temperature of the liquid is 20îC and the value of surface tension of mercury at 20îC in contact with air is 0.5 N/m. the contact angle for mercury ¯ = 135î (July 2013)

Ans: Given; d = 2.5mm = 0.0025m, σ = 0.5 N/m, θ = 135°

The capillary rise $h = \frac{4\times\sigma\times\cos\theta}{\gamma_w \times d} = \frac{4\times 0.5\times\cos 135^\circ}{13.6\times 9810\times 0.0025} = -0.00424m = -4.24mm$

h = - 4.24 mm (capillary depression)

Q.14 Two fixed parallel plates are 2.5cm apart. The space between the surfaces is filled with oil of viscosity 0.8 N-s/m^2. A flat thin late of 0.5m^2 area moves through the oil at a velocity of 0.6m/s. Calculate the drag force when,

(i) Plate is equidistant from both the planes

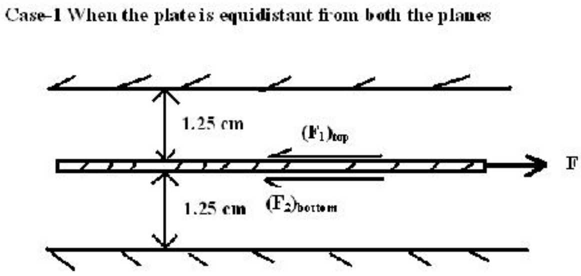
(ii) Plate is at a distance of 1cm from one of the plane surface

(July 2013, July2015)

Ans: Given: Oil Viscosity 0.8 N-s/m^2, A = 0.5m^2, V = 0.6m/s

Case-1 When the plate is equidistant from both the planes

Total force 'F' = $(F_1)_{top} + (F_2)_{bottom}$



for equidistant $F_1 = F_2$

Total force 'F' = $2\times(F_1)_{top}$ - Eq.1

$$(F_1)_{top} = {}^{2}{}_{1} \times A = « \times \left(\frac{du}{dy_1}\right)_{top}$$

$$(F_1)_{top} = 0.8\times\left(\frac{0.6}{0.0125}\right)\times 0.5$$

$(F_1)_{top}$ = 19.2 N

Force required to pull the plate F = 2×19.2 = 38.4 N

UNIT-2: PRESSURE AND ITS MEASUREMENTS

Q 1 State and prove Pascal's Law (July 2013, 2014, jan 2015, July 2015)

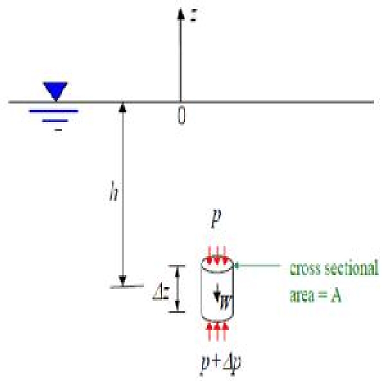
Ans: Consider a small vertical cylinder of fluid in equilibrium, where *positive z is pointing vertically upward*. Suppose the origin z = 0 is set at the free surface of the fluid. Then the pressure variation at a depth *z* = -*h* below the free surface is governed by

$$(p + \Delta p)A + W = pA$$

$$\Delta pA + ° gA\Delta z = 0$$

$$\Delta p = - ° g\Delta z$$

$$\frac{dp}{dz} = - ° g \quad \text{or} \quad \frac{dp}{dz} = -¥ \quad Eq.(1) \quad (\text{as } \Delta z \rightarrow 0)$$



Therefore, the hydrostatic pressure increases

Linearly with depth at the rate of the specific weight

$\gamma = \rho g$ of the fluid.

Homogeneous fluid: ρ **is constant**

By simply integrating the above equation-1:

$$\int dp = -\int ° g\, dz \qquad p = - ° gz + C$$

Where *C* is constant of integration

When *z* = 0 (on the free surface), $p = C = p_0$= (the atmospheric pressure).

Hence, $$p = - ° g z + p_0$$

Pressure given by this equation is called ABSOLUTE PRESSURE, i.e., measured above perfect vacuum.

However, for engineering purposes, it is more convenient to measure the pressure above a

p = ¥h

datum pressure at atmospheric pressure. By setting p_0= 0,

$$\mathbf{p = -°gz + 0 = -°gz = °gh}$$

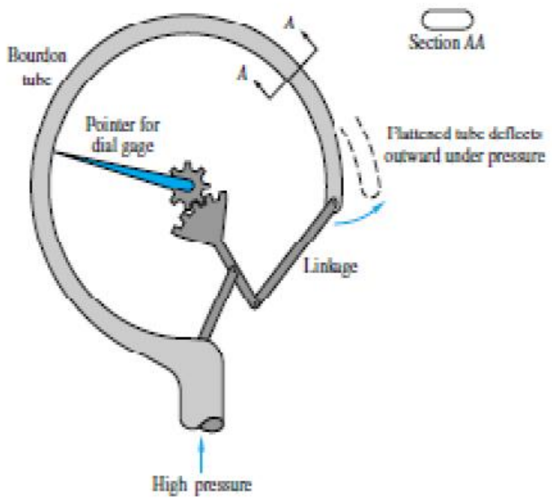
The equation derived above shows that when the density is constant, **the pressure in a liquid at rest increases linearly with depth from the free surface.**

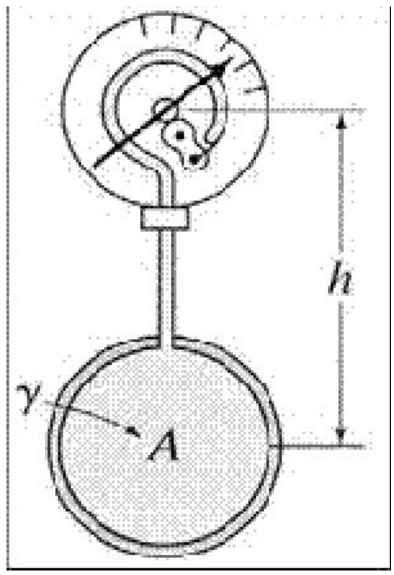
For a given pressure intensity ‘h’ will be different for different liquids since, ‘γ’ will be different for different liquids.

$$h = \frac{P}{¥}$$

Q.2 With neat sketch, explain Bourdon’s pressure gauge (July 2014, Jan 2015)

Ans: Is a device used for measuring gauge pressures the pressure element is a hollow curved metallic tube closed at one end the other end is connected to the pressure to be measured. When the internal pressure is increased the tube tends to straighten pulling on a linkage to which is attached a pointer and causing the pointer to move. When the tube is connected the pointer shows zero. The *bourdon tube,* sketched in figure.





It can be used for the measurement of liquid and gas pressures up to 100s of MPa.

Q.3 An open tank contains water up to a depth of 2m and above it, an oil of specific gravity 0.9 for a depth of 1m. Find the pressure intensity (i) At the interface of the two liquids (ii) At the bottom of the tank (July 2013,Dec 2014, July 2015)

Ans:

- Pressure Intensity at 'A'

$p_A = 0$

- Pressure Intensity at 'B'

$p_B = \gamma_{oil} \times H_{oil}$

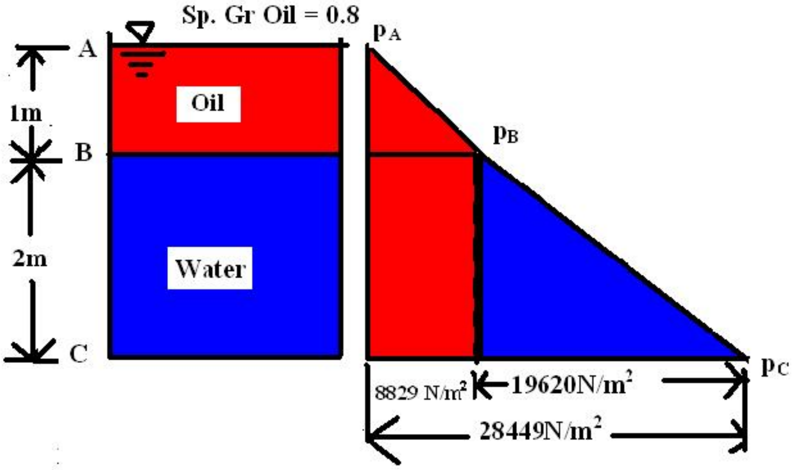
$p_B = 0.8 \times 9810 \times 1$

$p_B = 8829 \text{ N/m}^2$

- Pressure Intensity at 'B'

$p_C = \gamma_{oil} \times H_{oil} + \gamma_{water} \times H_{water}$

$\mathbf{p_C = 8829 + 19620 = 28449 \text{ N/m}^2}$



Q.4 What is manometer? Distinguish between U-tube differential manometer inverted U-tube differential manometer. (Dec 2014,July 2015)

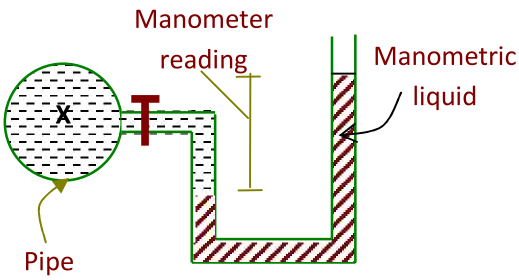
Ans

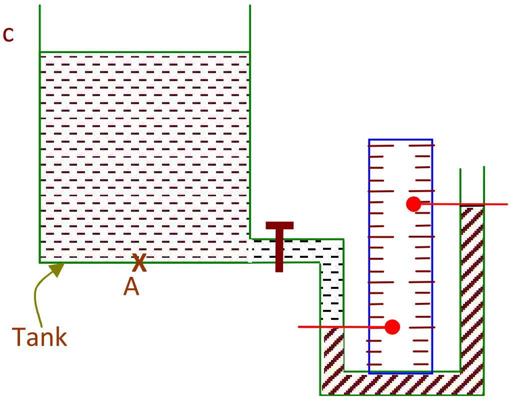
Monometers are used to measure intensity of pressure at a point. They are connected to the point at which the intensity of pressure is required. Such a point is called gauge point.

Common types of simple manometers are:

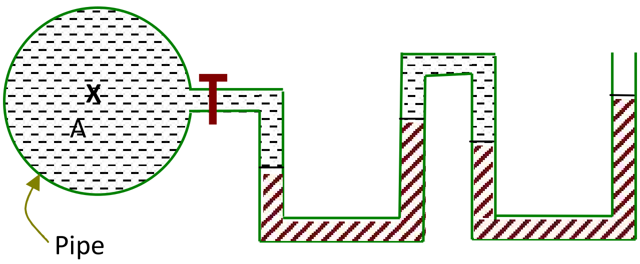
a) Piezometers
b) U-tube manometers
c) Single tube manometers
d) Inclined tube manometers

A U-tube manometers consists of a glass tube bent in U-Shape, one end of which is connected to gauge point and the other end is exposed to atmosphere. U-tube consists of a liquid of specific of gravity other than that of fluid whose pressure intensity is to be measured and is called monometric liquid.





An inverted manometer consist of glass but in the shape of inverted U. It is normally used for measurement of differential pressure when the flowing liquid is heavier than manometric liquid. Example water as main liquid and the Air as manometric liquid



Q. 5Find manometer reading 'h' for the Fig.Q2 (c) shown below: (Dec2013)

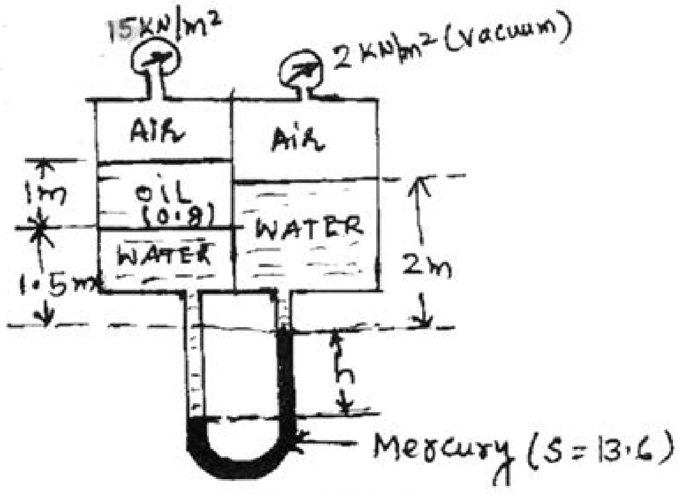


Fig. Q2 (c)

Ans: The equivalent pressure head at 'A' and 'B' is given by,

$$h_A = \frac{p_A}{¥} = \frac{15}{9.81} = 1.53m\ of\ \ water$$

$$h_B = \frac{p_B}{¥} = \frac{(-2)}{9.81} = -0.204m\ of\ \ water$$

Taking line x-x as datum

(Using pressure head and S1H1)

for liquid columns

$h_C = h_D$

1.53+ 1×0.8+1.5+h = -0.204+2×1+13.6h

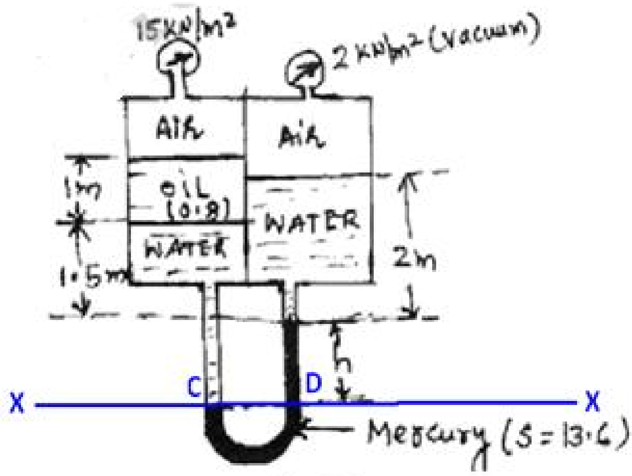


Fig. Q2 (c)

h = 2.034 m

Q. 6 Fig. shows a glass funnel fitted to a U tube-manometer. The manometer reading is 0.25m when the tunnel is empty what is the manometer reading when the tunnel is completely fitted with water? Take funnel height = 2m.(July 2013, Jan 2015)

Ans: Given: h_1 = 0.25m (mercury deflection),
Funnel Height = 2m
Consider the vessel to be completely filled with water.
Equating the pressure heads about the datum line X – X, we get

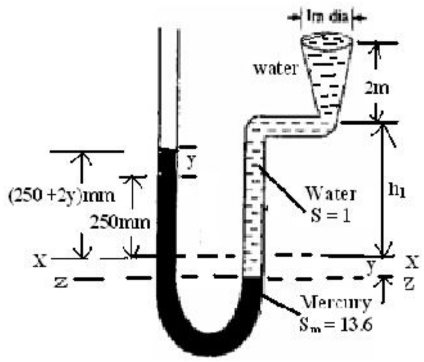
$h_1S_1 = h_2S_2$ or $h_1\times 1 = 250 \times 13.6$ or $h_1 = 3400$mm

Let the mercury level go down by 'y' mm in the right limb. Now the datum line is Z – Z.
Equating the pressure heads above the datum line Z – Z, we get,

$$(250 + 2y) \times 13.6 = (h_1 + y + 2000) \times 1$$
$$3400 + 27.2\,y = 3400 + y + 2000$$
$$y = 76.3 \text{ mm}$$

Thus the reading of the manometer when the vessel is completely filled with water = (250+2y) = (250+2×76.3) = 402.6mm

Hence the manometer reading is = 0.4026m



Vessel is Full of Water

Q. 2.7What are mechanical gauges? Give examples.(July 2013, july 2015)

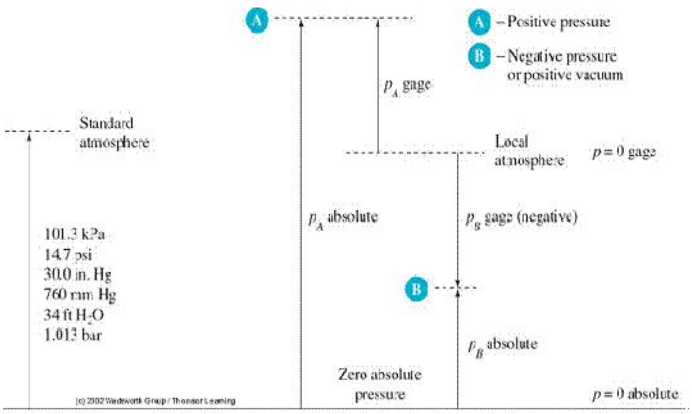
Ans: Mechanical gauges: Mechanical gauges are the devices used to measure pressure at a point. Mechanical gauge consists of an elastic element which deflects under the action of applied pressure and this deflection will move a pointer on a graduated dial leading to the measurement of pressure.

Examples: (i) Bourdon gauge (for gauge pressure)
(ii) Vacuum gauges (for –ve gauge Pressure)

Q.8 Explain gauge, absolute and vacuum pressures. How will you determine absolute pressure from the gauge and vacuum pressure? (July 2013,July14,jan 2015, July 2015)

Ans:

Absolute pressure at a point is the intensity of pressure ($(p_A)_{absolute}$ in Figure) at that point measured with reference to absolute vacuum or absolute zero pressure.



Gauge Pressure: If the intensity of pressure at a point is measurement with reference to atmosphere pressure, then it is called gauge pressure at that point ($(p_A)_{gauge}$ in Figure).

Gauge pressure at a point may be more than the atmospheric pressure or less than the atmospheric pressure.

Vacuum Pressure: It is the pressure measured below the gauge pressure (point 'B' in Diagram)

For Gauge pressure:

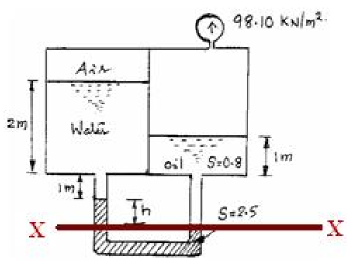
Absolute pressure at 'A'= (Gauge pressure at 'A'+ Standard Atmospheric pressure)

For Vacuum pressure:

Absolute pressure at 'B'= (Standard Atmospheric pressure - Vacuum pressure at 'B')

Q.9 Find the value of 'h' in metres in the Figure. When the air pressure on the water surface is 3.5m of water? (July 2013, july 2014)

Ans: Given Air pressure on water = 3.5m of water



Equating the pressure above level X – X

Left Limb: $(3.5 + 2.0 + 1.0) \times 9810 + (h) \times 2.5 \times 9810$

Right Limb: $(h + 1.0 + 1.0) \times 0.8 \times 9810 + 98{,}100$

On solving h = 3.0m

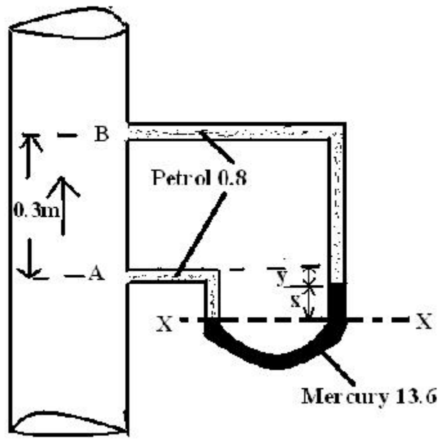
Q.10 A pipe contains petrol of specific gravity 0.8. The flow is upwards from the point 'A' to 'B' as shown in Fig. Q.(2c). A differential mercury manometer is connected at the two points 'A' and 'B' of the pipe which are 0.3m apart. Find the deflection in mercury column if the pressure difference between two points is 18kN/m^2 (July 2013,July 2014, jan 2015)

Ans: Given (p_A- p_B) = 18 kN/m^2, (Z_B - Z_A) = 0.3m, Sp.Gr Petrol = 0.8,

Sp.Gr. Mercury = 13.6

Equating the pressure about X-X on both limbs of U-tube manometer ($p = \gamma_w \times Sp.Gr \times H$)

$$p_A + (x+y)\times 0.8\times 9.81 = p_B + 0.3\times 0.8\times 9.81 + y\times 0.8\times 9.81 + x\times 13.6\times 9.81$$



(p_A – p_B) = 18 = 15.6456 + 125.86 x

x = 0.125m

UNIT-3 HYDROSTATIC PRESSURE ON SURFACES

Q1 Definei) Total pressure ii) Centre of pressure (Dec2013, June/July 2014, jan 2015)

Ans: (i) Total Pressure (P): This is that force exerted by the fluid on the contact surface (of the submerged surfaces), when the fluid comes in contact with the surface always acting normal to the contact surface. Units are N.

(ii)Centre of Pressure (C.P.): It is defined as the point of application of the total pressure on the contact surface.

Q. 2 Obtain an expression for total pressure and centre of pressure for inclined surface submerged in liquid (Dec 2013, July 2014)

Ans: **Hydrostatic Force on a Inclined submerged surface:**

The other important utility of the hydrostatic equation is in the determination of force acting upon submerged bodies. Among the innumerable applications of this is the force calculation in storage tanks, ships, dams etc.

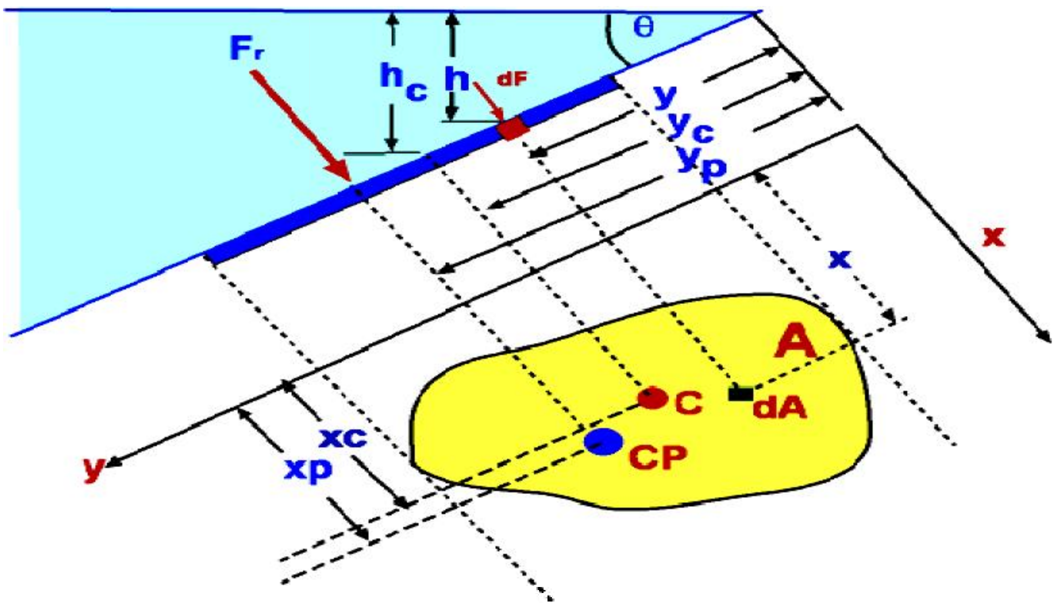


Figure 3.4 :Force upon a submerged object

First consider a planar arbitrary shape submerged in a liquid as shown in the figure. The plane makes an angle θ with the liquid surface, which is a free surface. The depth of water over the plane varies linearly. This configuration is efficiently handled by prescribing a coordinate frame such that the y-axis is aligned with the submerged plane. Consider an infinitesimally small area at a (x,y). Let this small area be located at a depth h from the free surface.dA = dx ×dy

Differential Force acting on the differential area dA of plane,

$$dF = (\text{Pressure}) \cdot (\text{Area}) = (¥h) \cdot (dA)$$ (Perpendicular to plane)

Then, Magnitude of total resultant force FR

$$F_R = \int_A ¥h\,dA = \int_A ¥\left(y \sin ¯\right) dA$$ Where $h = y \sin ¯$

$$= ¥ \sin ¯ \int_A y\,dA$$ ← 1st moment of the area - Related with the center of area

※ $\int_A y\,dA = y_c A$ Where y_c: y coordinate of the center of area (Centroid)

Centeror 1st moment

$$\int_M x\,dm = MX_C \quad \& \quad \int_M y\,dm = MY_C$$ (XC & YC: Center of Mass)

$\int_A x\,dA$ = xc & $\int_A y\,dA$ = yc (xc&yc: Center of Area)

Moment of inertia or 2nd moment

$$\int_M r^2\,dm = I$$ (2nd moment of Mass)

$$\int_A y^2\,dA = I_x \quad \& \quad \int_A x^2\,dA = I_y$$ (2nd moment of Area)

Then,

$$F_R = \gamma A y_c \sin\theta = (\gamma h_c)A$$

Where ¥ h_c : Pressure at the centroid = (Pressure at the centroid) × Area

- Magnitude of a force on an INCLINED plane
- Dependent on ¥ ,Area, and Depth of centroid
- Perpendicular to the surface (Direction)

i) Position of FR on y-axis ‘yR’ : y coordinate of the point of action of FR

Moment about x axis:

$$F_R y_R = (\text{¥} A y_c \sin^{-}) y_R = \int_A y\,dF = \int_A \text{¥} \sin^{-} y^2 dA = \text{¥} \sin^{-} \int_A y^2 dA$$

$$\therefore h_R = \frac{\int_A h^2 dA}{h_c A} = \frac{I_x}{h_c A} \text{ where } I_x = \int_A y^2 dA:$$

2nd moment of areaor, by using the parallel-axis theorem, $I_x = I_{xc} + A y_c^2$

$$\therefore h_{C.P.} = \bar{h} + \frac{I_G \text{Sin}^2\theta^\circ}{A\bar{h}}$$

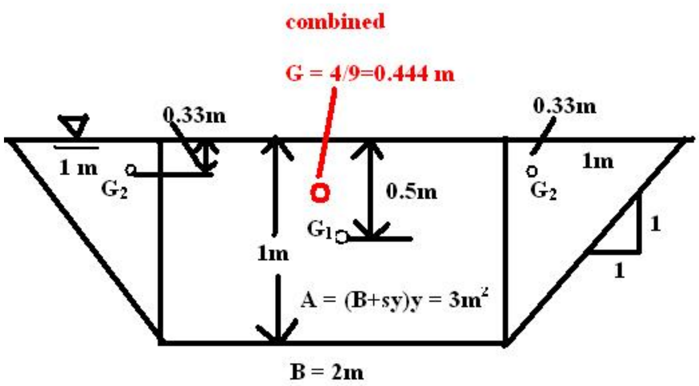
(The center of pressure below the centroid)

Q.3A trapezoidal channel 2m wide at the bottom and 1m deep has side slopes 1:1. Determine: i) Total pressure ii) Centre of pressure, when it is full of water (July 2014, Jan 2015)

Ans: Given B = 2m Area of flow A = (B+sy)y = 3m^2

The combined centroid will be located based on two triangular areas and one rectangle

(shown as G_1, G_2, G_2)



$$\bar{y} = \frac{A_1 \times h_1 + A_2 \times h_2 + A_3 \times h_3}{A_1 + A_2 + A_3}$$

The total area A = 3m^2

Area of rectangle = 2 ×1 = 2m^2

Area of Triangle = $\frac{1}{2}$×1×1=0.5m^2

$$\bar{y} = \frac{\left((2\times1)\times0.5+\left[\frac{1}{2}\times(1\times1)\right]\times0.333+\left[\frac{1}{2}\times(1\times1)\right]\times0.333\right)}{3} = \frac{4}{9} = 0.444m$$

i) The total pressure P = γ_w×A×$\bar{y}$ = 9810×3×0.444 = 13080N
ii) Centre of pressure

The centroidal moment of Inertia of Rectangle and Triangle is

$$I_{G1} = \frac{2\times1^3}{12} = 0.1667m^4 \quad at \quad 0.5m \ \ from\ water - surface$$

$$I_{G1} = \frac{1\times1^3}{36} = 0.028m^4 \quad at \quad 0.333m \ \ from\ water - surface$$

$$\bar{h} = \bar{y} + \frac{I_g}{A\,\bar{y}} \cdots Eq.1$$

The moment of Inertia about combined centroid can be obtained by using parallel axis theorem

$I_G = \left((I_{G1}) + A_1 d_1^2\right) + \left((I_{G2}) + A_2 d_2^2\right) + \left((I_{G2}) + A_2 d_2^2\right)$ (as both the triangles are similar)

$$I_G = (0.1667 + 0.00618) + 2((I_{G2}) + A_2 d_2^2)$$

$$I_G = (0.1667 + 0.00618) + 2(0.028 + 0.0062) = 0.2408 m^4$$

Substituting in Eq.1, The centre of pressure from free surface of water

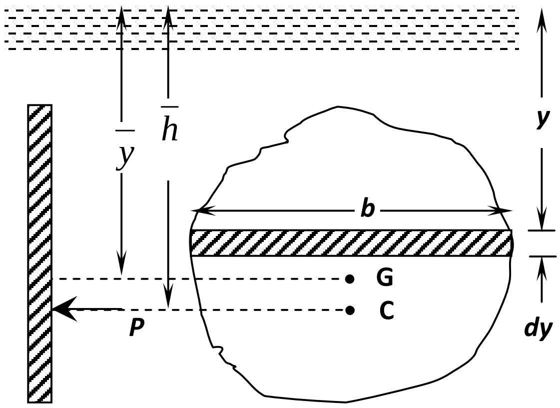
$$\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}} \cdots Eq.1$$

$$\bar{h} = 0.444 + \frac{0.2408}{3 \times 0.444} = 0.6252 m$$

Q 4Derive the expression for total pressure and centre of pressure on a vertical plate submerged in a static liquid (Dec 2013,July 2015)

Ans:*Vertical Plane surface submerged in liquid*

Consider a vertical plane surface of some arbitrary shape immersed in a liquid of mass density ρ as shown in Fig.



Let

A = Total area of the surface

$\bar{y}$ = Depth of Centroid of the surface from the free surface

G = Centroid of the immersed surface

C = Centre of pressure

$\bar{h}$ = Depth of centre of pressure

Consider a rectangular strip of breadth b and depth 'dy' at a depth 'y' from the free surface.

Total Pressure:

The pressure intensity at a depth y acting normal to the plane on the strip is $p = \rho g y$

Total pressure force on the strip = $dP = (\rho g y)\times dA$

$\therefore$ The Total pressure force on the entire area is given by integrating the above expression over the entire area along the plane

$P = \int dP = \int (\rho g y)\times dA = \rho g \int y\, dA$

But y and dA are on different planes and hence substituting for y from Eq. 1, we get

$P = \rho g \int y^* \sin\theta dA = \rho g \sin\theta \int y^* dA$ …(2)

But $\int y^* dA$ is the Moment of the entire area about the free surface of the liquid given by

$\int y^* dA = A\overline{y^*} \sin\theta = A\bar{y}$

Substituting in Eq. 2, we get $P = \rho g A\bar{y} = \gamma A\bar{y}$ … (3)

Where γ is the specific weight of water

For water, ρ=1000kg/m3 and g= 9.8m/s^2. The force will be expressed in Newton (N).

Centre of Pressure $(\bar{h})$**:** It is the location of total hydrostatic force P acting on the immersed plane. This is computed on the principle of Theorem of moments. The moment of the pressure force about the free surface is given by,

$M = P \times \bar{h} = \rho g A\bar{y}\,\bar{h}$ ...(4)

On solving $\bar{h} = \bar{y} + \frac{I_g}{A\bar{y}}$

Where $\bar{y}$ is the centroidal depth and $\bar{h}$ is the centre of pressure.

Q. 5Explain the procedure of finding hydrostatic force on a curved surface
(Dec 2013,Jan 2014)

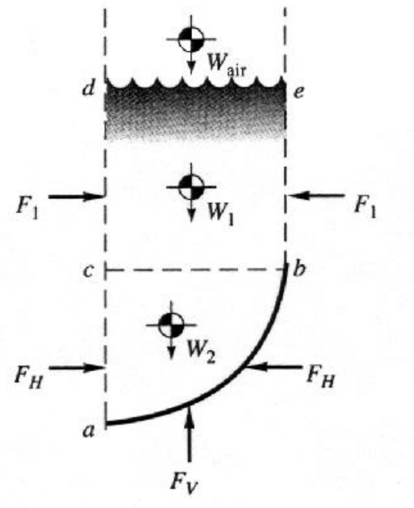
Ans: Hydrostatic Forces on Curved Surfaces

Since this class of surface is curved, the direction of the force is different at each location on the surface. Therefore, we will evaluate the x and y components of net hydrostatic force separately.

Consider curved surface, a-b. Force balances in x & y directions yield

$F_h = F_H$

$Fv = W_{air}+W_1 + W_2$



(b)

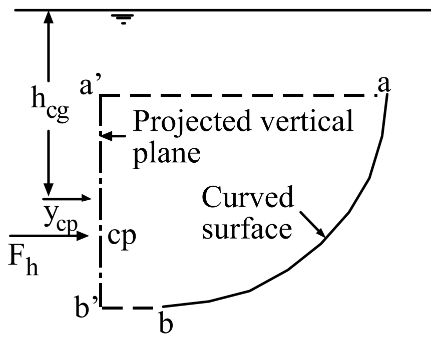
From this force balance, the basic rules for determining the horizontal and vertical component of forces on a curved surface in a static fluid can be summarized as follows:

Horizontal Component, F_h

1. The horizontal component of force on a curved surface equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.

The horizontal force will act through the c.p. (not the centroid) of the projected area.

from the Diagram:



All elements of the analysis are performed with the vertical plane. The original curved surface is important only as it is used to define the projected vertical plane.

Therefore, to determine the horizontal component of force on a curved surface in a hydrostatic fluid:

Vertical Component - Fv

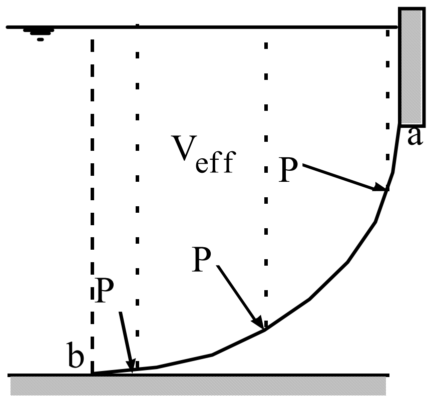
2. The vertical component of force on a curved surface equals the weight of the effective column of fluid necessary to cause **the pressure on the surface**.

The use of the words **effective column of fluid** is important in that there may not always actually be fluid directly above the surface. (See graphics below)

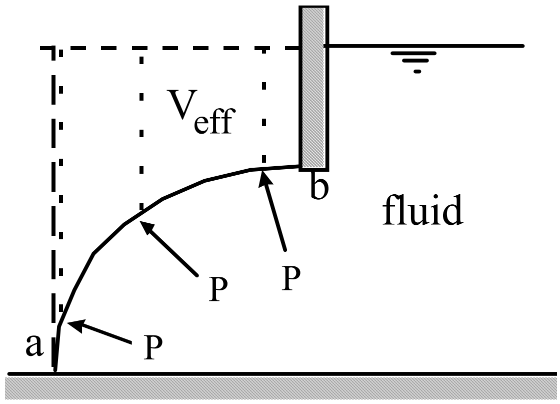
This effective column of fluid is specified by identifying the column of fluid that would be required to cause the pressure at each location on the surface.

Thus, to identify the Effective Volume - $V_{eff:}$

$$\mathbf{F_V = V_{eff}}$$



Fluid above the surface



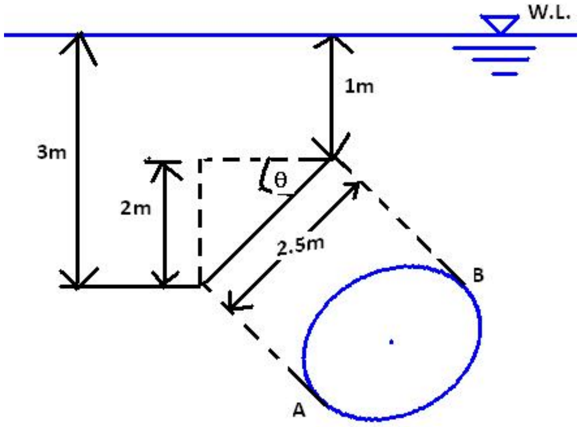
No fluid actually above surface

$$R = \sqrt{\left(\sum F_x^2\right) + \left(\sum F_y^2\right)} \qquad \text{‾} = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$

Q 6 A circular plate 2.5m diameter is immersed in water, its greatest and least depth below the free surface being 3m and 1m respectively. Find

(i) The total pressure on one face of the plate and(ii) Position of centre of pressure (Dec 2013, July 2015)

Ans: Given d = 2.5m,



$$- = Sin^{-1}\left(\frac{2}{2.5}\right)$$

$$- = 53.13^{\circ}$$

$$\bar{h} = 1 + 1 = 2m$$

$$A = \frac{®}{4}d^2 = \frac{®}{4} \times 2.5^2 = 4.909m^2$$

$$I_G = \frac{®}{64}d^4 = \frac{®}{64} \times (2.5)^4 = 1.917m^4$$

F = $\gamma_w A \bar{h}$ = 9.81×4.909×2 = 96.31 kN

$$\boldsymbol{h_{c.p.}} = \bar{\boldsymbol{h}} + \frac{\boldsymbol{I_G} \times \boldsymbol{Sin}^{2}-}{\boldsymbol{A} \times \bar{\boldsymbol{h}}} = 2 + \frac{1.917 \times \boldsymbol{Sin}^2 53.13^{\circ}}{4.909 \times 2}$$

$$\boldsymbol{h_{c.p.}} = 2.125\boldsymbol{m}$$

Q 7 Prove that for a plate submerged in horizontal position in water the center of pressure is same as centroid of the plate. (July 2013, July 2015)

Ans:

Consider a plane horizontal surface immersed in a static fluid. As every point of the horizontal surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal to p = σ g h, where 'h is the depth of surface (Refer Fig.).

Let 'dA' be the Elemental area and 'A' be the surface area

Then total force dP = p × dA

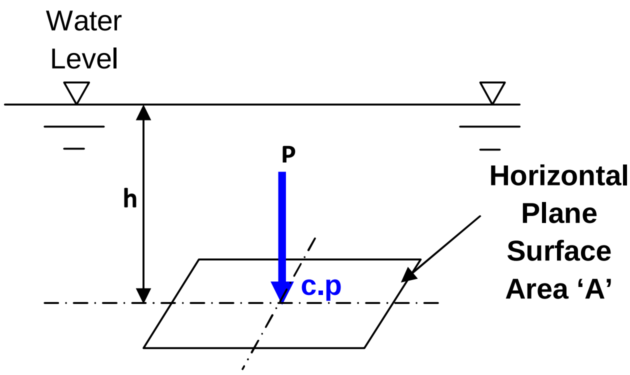


Fig. Total Pressure on a Horizontal Plane Surface

$$P = {}^{\circ}gA\bar{h}$$

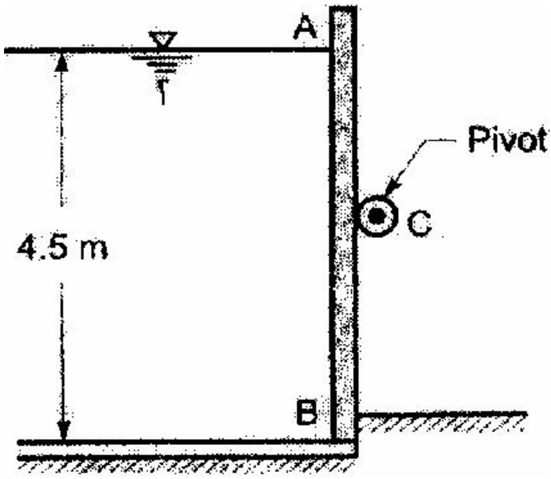
The centre of pressure

$$h_{cp} = \bar{h} + \frac{I_{...C} Sin^{2}-}{A\bar{h}} \cdots Eq.(1)$$ For Horizontal surface $- = 0^{\circ}$, substituting in Eq.(1)

$$h_{cp} = \bar{h}$$

Q. 8Figure shows a rectangular flash board AB which is 4.5m high and is pivoted at C. What must be the maximum height of C above B so that the flash board will be on the verge of tipping when water surface is at A? Also determine if the pivot of the flash board is at a height h =1.5m, the reactions at B and C when the water surface is 4m above B. (July2013,Dec2014)

Ans:



(i) The flash board would tip about the hinge point 'C' when the line of action of resultant 'R' pressure force 'F' lies from C to A anywhere on the board.

The limiting condition being the situation when the resultant force 'F' passes through 'C'

The resultant force'F' also passes through the centroid of the pressure diagram and the centre lies at $\frac{1}{3} \times AB = \frac{4.5}{3} = 1.5m$

Hence the maximum height of 'C' from 'B' = (4.5m-3.0m) =1.5m (from bottom)

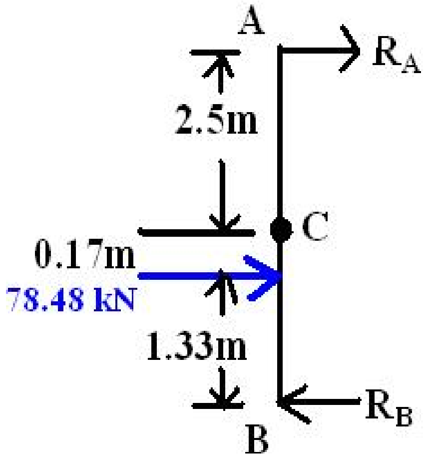
(ii) The pivot of the flash board is at a height h =1.5m from point B, the reactions at B and C when the water surface is 4m above B.

$$\bar{h} = \frac{4.0}{2} = 2.m$$

Hydrostatic force $P = \rho g A\bar{h}$ = 1000×9.81×(4.0×1.0)×2=78.48 kN acting at h_{cp}

$$h_{cp} = 2.0 + \frac{1\times(4.0)^3 Sin^2 90°}{4.0\times 2.0} = 2.67m \text{ from free water surfcae}$$

Or h = (4.0-2.67) = 1.33m from bottom



Let R_A and R_B be the reaction.

R_A+ 78.48 = R_B

by taking moment about pivot 'C'

R_A ×2.5 + 78.48×0.17 = R_B×1.5

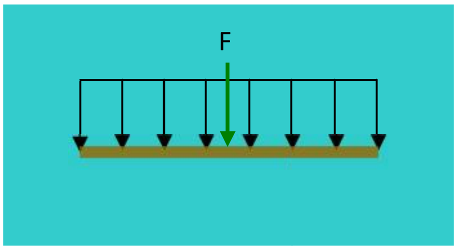
On solving R_A =104.38kN R_B = 182.86 kN

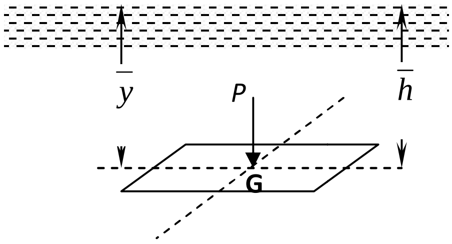
Q. 9Draw the water pressure diagrams for wholly submerged Horizontal, Vertical and Inclined plane surfaces. Write the expressions for total pressure and depth of centre of pressure for vertical plane surface. (July 2013,Jan 2014)

Ans:

(i) Pressure Diagram for wholly submerged Horizontal surface:

The resultant force acts at the centroid of the plane.

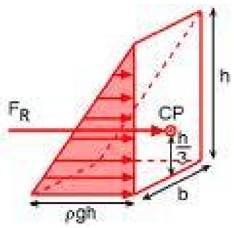




Total Pressure : $P = {}^{\circ}gA\,\bar{y}$

Centre of pressure : $\bar{h} = \bar{y}$

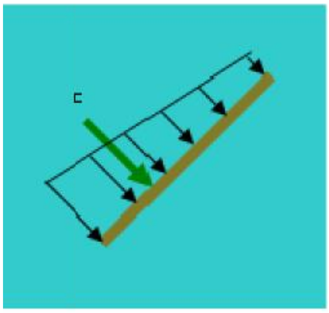
(ii) Pressure Diagram for wholly submerged Vertical surface:



Total Pressure : $P = \rho g A\bar{h} = \gamma A\bar{h}$

Centre of pressure : $\bar{h} = \frac{2h}{3}$

(iii) Pressure Diagram for wholly submerged Inclined surface:



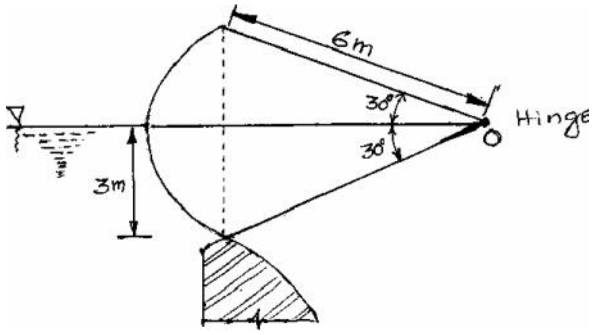
Total Pressure : $P = \rho g A\bar{h} = \gamma A\bar{h}$

Centre of pressure : $h_{C.P.} = \bar{h} + \frac{I_G \sin^2\theta^{\circ}}{A\bar{h}}$

Q. 10 Length of a Tainter gate perpendicular to paper is 0.50m. Find:

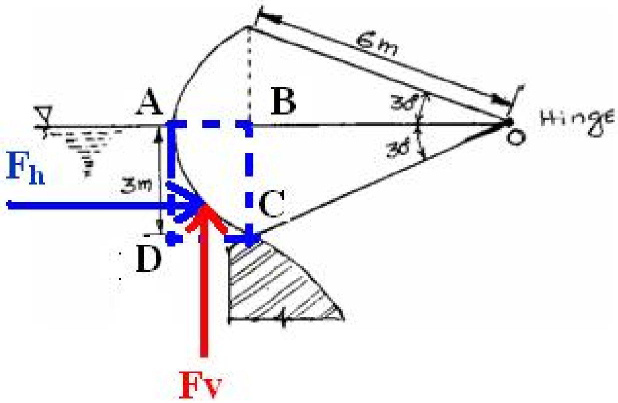
i) Total horizontal thrust of water on gate.

ii) Total vertical component of water pressure against gate.
iii) Resultant water pressure on gate and its inclination with horizontal.
(July 2013 , July 2015)



Ans: Given L = 0.5m,
AD = BC = 3m, $\gamma_W = 9.81\ kN/m^3$

(i) Total horizontal thrust of water on gate



$F_h = \gamma_W \times A \times \bar{h}$

$F_h = 9.81 \times (3.0 \times 0.5) \times \frac{3}{2}$

F_h= 22.07 kN ì **Rightward**

Acting at

$$\overline{h_{c.p.}} = h_c + \frac{I_G \times \sin^2 90°}{A \times h_c}$$

$$\overline{h_{c.p.}} = 1.5 + \frac{\left(\frac{0.5 \times 3^3}{12}\right) \times \sin^2 90°}{(3.0 \times 0.5) \times 1.5} = 1.5 + 0.5 = 2.0\mathbf{m}$$

(ii) Total vertical component of water pressure against gate = upward thrust due area ABC

Upward thrust due area ABC = Area AOC - ΔOBC

Area ABC = $\frac{\pi \times R^2}{12} - \frac{1}{2} \times OB \times BC$

Area ABC = $\frac{\pi \times 6^2}{12} - \frac{1}{2} \times 3\cos 30° \times 3$

Area ABC = **1.636 m²**

Fv= γ_W × Area ABC × L

Fv= 9.81 × 1.636 × 0.5 = 8.024 kN‐ë **upward**

(iii) Resultant water pressure on gate and its inclination with horizontal

$$\mathbf{R} = \sqrt{\mathbf{F}_h^2 + \mathbf{F}_v^2} = \sqrt{(22.07)^2 + (8.024)^2} = 23.48 \quad \mathbf{kN}$$

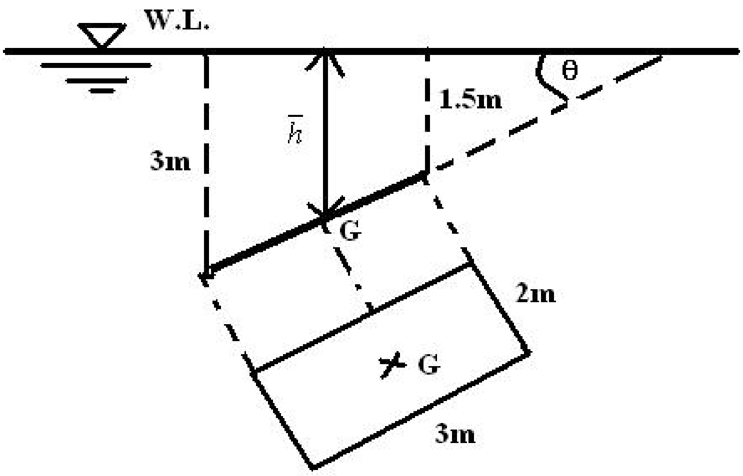
Inclination $\theta = \tan^{-1}\left(\frac{8.024}{22.07}\right) = 0.3637$

$\theta = 20°$

Q. 11.A 2m wide and 3m deep rectangular plane surface lies in water in such a way the top of and bottom edges are at a distance of 1.5m and 3m respectively from the surface. Determine the hydrostatic force and centre of pressure (Dec 2013, Jan 2015)

Ans: Given A = 3m × 2m = 6m²,

$$I_G = \frac{2 \times 3^3}{12} = 4.5m^4$$



Hydrostatic force

$$P = ¥_w \times A \times \bar{h}$$

$$P = 9.81 \times 6 \times \left(\frac{3 + 1.5}{2}\right)$$

P = 132.435 kN

$$\sin ¯ = \frac{(3.0 - 1.5)}{3} = 0.5$$

$$¯ = 30°$$

The centre of pressure

$$h_{C.P} = \bar{h} + \frac{I_G \times Sin^2 \text{—}}{A\,\bar{h}}$$

$$h_{C.P} = 2.25 + \frac{4.5 \times \left(1/4\right)}{6 \times 2.25} = 2.33m$$

UNIT-4 KINEMATICS OF FLOW

Q.1.Distinguish between: i) Laminar flow and Turbulent flow based on Reynolds Number ii) Uniform and non-uniform flow(Dec 2013, July 2014)

Ans:i) Laminar flow and Turbulent flow:When the flow occurs like sheets or laminates and the fluid elements flowing in a layer does not mix with other layers, then the flow is said to be laminar. The Reynolds number (Re) for the flow will be less than 2000.

$$R_e = \frac{\text{°vD}}{\text{«}}$$

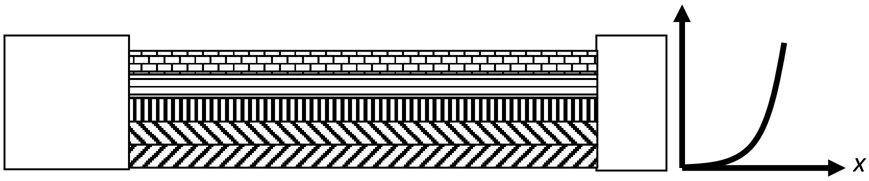


Fig. 5 Laminar flow

When the flow velocity increases, the sheet like flow gets mixes with other layer and the flow of fluid elements become random causing turbulence. There will be eddy currents generated and flow reversal takes place. This flow is said to be Turbulent.

For pipe flow:

Re < 2000 – Laminar flow

Re 2000 – 4000 – Transition

Re > 4000 Turbulent flow

ii) Uniform and non-uniform flow:A flow is said to be uniform if the properties (P) of the fluid and flow **do not change (with direction) over a length of flow** considered along the flow at any instant.

$$\frac{\partial}{\partial x}(P) = 0$$

A flow is said to be non-uniform if the properties (P) of the fluid and **flow change (with direction) over a length of flow** considered along the flow at any instant.

$$\frac{\partial}{\partial x}(P) \neq 0$$

Q. 2.Obtain an expression for continuity equation for three dimensional flows
(Dec 2013, July 2014)

Ans: Consider a parallelepiped ABCDEFGH in a fluid flow of density γ as shown in Fig. Let the dimensions of the parallelepiped be dx, dy and dz along x, y and z directions respectively. Let the velocity components along x, y and z be u, v andw respectively.

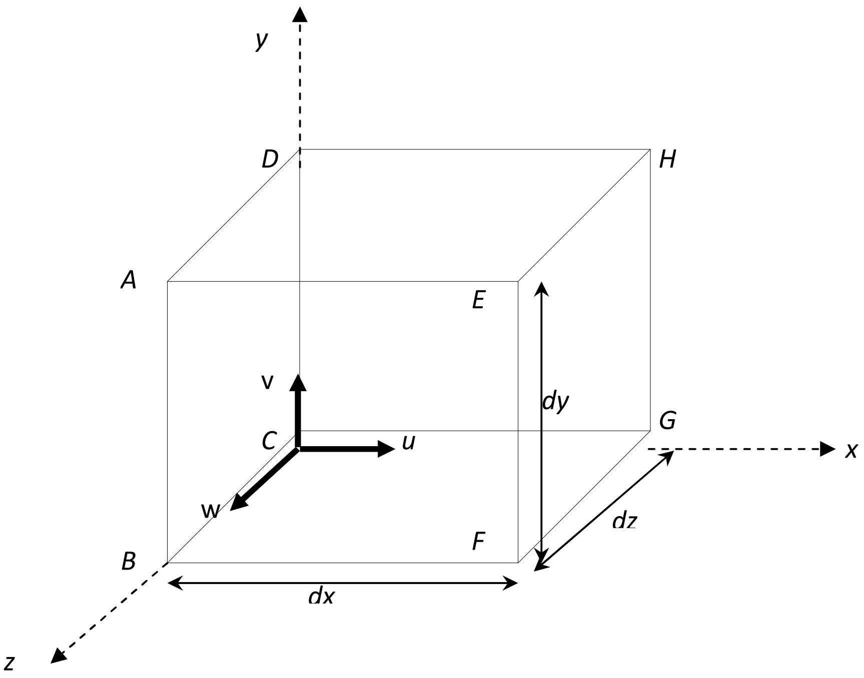


Fig. Parallelepiped in a fluid flow

Mass rate of fluid flow entering the section ABCD along x direction is given by ρ× Area x-flow velocity

$$M_{x1} = \circ\, u\, dy\, dz \qquad \ldots(01)$$

Similarly mass rate of fluid flow leaving the section EFGH along x- direction is given by

$$M_{x2} = \left[\circ\, u + \frac{\partial}{\partial x}\left(\circ\, u \right) dx \right] dy\, dz \qquad \ldots(02)$$

Net gain in mass rate of the fluid along the x- axis is given by the difference between the mass rate of flow entering and leaving the control volume. i.e. Eq. 1 – Eq. 2

$$dM_x = \circ\, u\, dy\, dz - \left[\circ\, u + \frac{\partial}{\partial x}\left(\circ\, u \right) dx \right] dy\, dz$$

$$dM_x = -\frac{\partial}{\partial x}\left(\circ\, u \right) dx\, dy\, dz \qquad \ldots(03)$$

Similarly net gain in mass rate of the fluid along the y and z- axes are given by

$$dM_y = -\frac{\partial}{\partial y}\left(\circ \text{ v}\right) dx\, dy\, dz \qquad \ldots(04)$$

$$dM_z = -\frac{\partial}{\partial z}\left(\circ \text{ w}\right) dx\, dy\, dz \qquad \ldots(05)$$

Net gain in mass rate of the fluid from all the three axes are given by,

$$dM = -\frac{\partial}{\partial x}\left(\circ\ u\right) dx\, dy\, dz - \frac{\partial}{\partial y}\left(\circ \text{ v}\right) dx\, dy\, dz - \frac{\partial}{\partial z}\left(\circ \text{ w}\right) dx\, dy\, dz$$

From law of conservation of Mass, the net gain in mass rate of flow should be zero and hence

$$\left[\frac{\partial}{\partial x}\left(\circ\ u\right) + \frac{\partial}{\partial y}\left(\circ \text{ v}\right) + \frac{\partial}{\partial z}\left(\circ \text{ w}\right)\right] dx\, dy\, dz = 0$$

$$\left[\frac{\partial}{\partial x}\left(\circ\ u\right) + \frac{\partial}{\partial y}\left(\circ \text{ v}\right) + \frac{\partial}{\partial z}\left(\circ \text{ w}\right)\right] = 0$$

This expression is known as the general Equation of Continuity in three dimensional form or differential form.

Q.3If for a two dimensional potential flow, the velocity potential is given by ¤ = x (2y-1). Determine the velocity at the point P (4, 5). Determine also the value of stream function · at the point 'P'. (July2014, July 2013)

Ans:

(i) The velocity at the point P (4, 5), x =4, y = 5

ϕ = x (2y-1).

$$\frac{\partial ¤}{\partial x} = -u = (2y-1), \quad u = (1-2y)$$

$$\frac{\partial ¤}{\partial y} = -v = x \times 2, \qquad v = -2x$$

u at 'P'(4,5) = -9 Units/s

v(4,5) at 'P' = -8 Units/s

Velocity at P = -9i-8j, Velocity $\sqrt{(-9)^2+(-8)^2} = 12.04 \quad Units$

(ii) Stream function $\psi_{P\,(4,\,5)}$

Given ϕ = x (2y-1)

$$\frac{\partial \phi}{\partial x} = -u = (2y-1) = \frac{\partial \psi}{\partial y}$$
$$\frac{\partial \phi}{\partial y} = -v = x \times 2 = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial \psi}{\partial y} = -u = (2y-1) \cdots Eq.1$$
$$\frac{\partial \psi}{\partial x} = u = -2x \cdots Eq.2$$

Integrating Eq.1 with respect ‘y’ we get

$$\int d\psi = \psi = \frac{2 \times y^2}{2} - y + C(f(x)) \cdots Eq.3$$

Differentiating Eq.3 with respect to ’x’

$$\frac{\partial \psi}{\partial x} = \frac{\partial C}{\partial x} \qquad from \quad Eq.2 \quad \frac{\partial \psi}{\partial x} = -2x$$
$$\frac{\partial C}{\partial x} = -2x \quad Integrating \rightarrow C = -x^2$$

Substituting value of C in Eq.3

$$\psi = \left(y^2 - y - x^2\right)$$

Q.4 Distinguish between: (i) Steady and unsteady flow (ii) Uniform and non-uniform flow (iii) Compressible and incompressible flow (Dec 2013, Jan 2015)

Ans: (i) Steady and unsteady flow: The flow parameters does not vary with respect to time at a given location is called steady flow while as if the flow parameters (like depth, velocity, acceleration) vary with respect to time is called Unsteady flow

(ii) Uniform and non-uniform flow: The flow parameters does not vary with respect to distance at a time is called Uniform flow while as if the flow parameters (like velocity, acceleration, depth vary along the length of the channel or river is called Non-uniform flow

(iii) Compressible and incompressible flow: If the fluid density vary with respect to applied pressure at a given location and time is called compressible flow (gases are compressible) while as if fluid density does not vary with respect to applied pressure at a given location and time is called incompressible flow (Liquids are normally considered incompressible)

Q. 5 Define terms velocity potential function, stream function and establish relation between them (Dec 2013, Jan 2014, july 2015)

Ans: Velocity Potential ¤is a scalar function of space and timesuch that its negative derivative with respect to any direction gives the velocity component in that direction

Thus ¤ = ¤ *(x,y,z,t)* and flow is steady then,

***u* = -(ô¤ /ô *x*); v = -(ô¤ /ô *y*) ; w = -(ô¤ /ô *z*)**

Stream Function (·)

Stream Function · is a scalar function of space and timesuch that its partial derivative with respect to any direction gives the velocity component at right angles to that direction.

Thus · = · ***(x,y,z,t)*** and flow is steady then,

***u* = -(ô · /ô *y*); v = (ô · /ô *x*)**

Relation between (¤ and ·):

$$u = -\frac{\partial ¤}{\partial x} = -\frac{\partial \cdot}{\partial y}$$

$$v = -\frac{\partial ¤}{\partial y} = \frac{\partial \cdot}{\partial x}$$

Q.6 A stream function is given by ψ = $2x^2$-$2y^2$. Determine the velocity and velocity potential function at (1, 2) (Dec 2013, jan 2015)

Ans: Given: $\psi = 2x^2 - 2y^2$

$$\frac{\partial \psi}{\partial x} = 4x = -v;\; v = -4x \Rightarrow \textit{Velocity at } (1,2), \quad v = -4 \quad \textit{Units}$$

$$\frac{\partial \psi}{\partial y} = -4y = u;\; u = -4y \Rightarrow \textit{Velocity at } (1,2), \quad u = -8 \quad \textit{Units}$$

Resultant velocity $V_{(1,2)}$ = $\sqrt{(-4)^2 + (-8)^2} = 8.94$ ***Units***

$$\frac{\partial \phi}{\partial x} = -u \quad \Rightarrow \frac{\partial \phi}{\partial x} = -(-4y) = 4y \Rightarrow \phi = 4 \times x \times y + C(f(y) only) \cdots eq1$$

$$\frac{\partial \phi}{\partial y} = -v \quad \Rightarrow \frac{\partial \phi}{\partial x} = -(-4x) = 4x \Rightarrow \phi = 4 \times x \times y + C(f(x) only) \cdots eq2$$

From Eq.1

$$\frac{\partial \phi}{\partial y} = (4x + \frac{\partial C}{\partial y}) \Rightarrow \frac{\partial C}{\partial y} = 4x - \frac{\partial \phi}{\partial y} \Rightarrow \frac{\partial C}{\partial y} = 4x - \left(\frac{\partial \psi}{\partial x}\right) \Rightarrow \frac{\partial C}{\partial y} = 4x - 4x = 0$$

$$\frac{\partial C}{\partial y} = 0 \;\; \textit{Integrating} \quad C = 0$$

$$\therefore \phi = 4 \times x \times y \qquad \Rightarrow \phi = 4 \times 1 \times 2 = 8 \; \textit{Units}$$

Q. 7 In a flow the velocity vector is given by V = 3xi + 4yj -7zk. Determine the equation of the stream line passing through a point M (1, 4, 5). (July2013, july 2015)

Ans: Given the Velocity vector V = 3xi+4yj -7zk

⇒ u = 3x ; v = 4y; w = -7z

To determine the equation of the stream line passing through a point M (1, 4, 5)

The 3-D equation of streamline is given by,

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\frac{dx}{3x} = \frac{dy}{4y} = \frac{dz}{-7z} \cdots Eq.1$$

The streamline equation at point **M (1, 4, 5),** x =1, y = 4, z = 5

Substituting the values of x, y, and z in Eq.1

$$\frac{dx}{3} = \frac{dy}{16} = \frac{dz}{-35}$$

The equation of a streamline ds = 3i +16k – 35k

Q 8The velocity potential φ for a two dimensional flow is given by (x^2 – y^2) + 3xy. Calculate: i) the stream function ψ and ii) the flow rate passing between the stream lines through (1,1) and (1, 2). (July 2013, 2014)

Ans: Given $\phi = (x^2 - y^2) + 3xy$
(i) To determine the ψ function

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy \cdots Eq.(1)$$

$$d\psi = -v\,dx + u\,dy \cdots Eq.(2)$$

As per definition of velocity potential (φ) and stream function (ψ);

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = u \text{ and } \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} = v$$

$$u = \frac{\partial \phi}{\partial x} = (2x + 3y) = \frac{\partial \psi}{\partial y} \text{ and } \frac{\partial \phi}{\partial y} = (-2y + 3x) = \left(-\frac{\partial \psi}{\partial x}\right) = v$$

Substituting the value of u and v in terms of x and y in equation 2, we obtain

$$d\psi = -v\,dx + u\,dy = -(-2y + 3x)dx + (2x + 3y)dy$$

$$d\psi = (2y + 3x)dx + (2x + 3y)dy \cdots Eq.3$$

Integrating the equation-3 (partially w.r.t ‘x’ the ‘dx-term’ and w.r.t ‘y’ the ‘dy-term’)

$$\psi = \left(2xy + \frac{3}{2}x^2\right) + \left(2xy + \frac{3}{2}y^2\right) = 4xy + \frac{3}{2}\left(x^2 + y^2\right)$$

$$\boxed{\psi = 4xy + \frac{3}{2}\left(x^2 + y^2\right)}$$

(ii) The flow rate passing between the stream lines through (1, 1) and (1, 2).

The equation of stream function is given by $\boxed{\psi = 4xy + \frac{3}{2}\left(x^2 + y^2\right)}$

The value of Point streamline at (1, 1) is obtained by substituting x = 1, y = 1

$$\psi_{(1,1)} = 4xy + \frac{3}{2}\left(x^2 + y^2\right) = 4 \times 1 \times 1 + \frac{3}{2}(1^2 + 1^2) = 7\,Units$$

The value of Point streamline at (1, 2) is obtained by substituting x = 1, y = 2

$$\psi_{(1,2)} = 4xy + \frac{3}{2}\left(x^2 + y^2\right) = 4 \times 1 \times 2 + \frac{3}{2}(1^2 + 2^2) = 15.5\,Units$$

The flow rate passing between the stream lines through (1, 1) and (1, 2)
$\psi_{(1,2)} - \psi_{(1,1)}$ = (15.5-7)

q = 8.5 m^2/s/unit width

Q.9 What are the practical uses of streamlines and velocity potential lines? July 2013

Ans: The practical use of streamlines and velocity potential lines are:

(i) Quantity of seepage

(ii) Upward lift pressure below the hydraulic structures (dam, gate, locks etc.)

(iii) Velocity and pressure distribution, for given boundaries of flow

(iv) To design streamlined structure flow pattern near well

Q.10List the method of construction of flow net for a given boundary configurations. What are thelimitations of flow net? (July 2013,Jan 2015)

Ans: Methods of Drawing flow net

- Analytical Method
- Graphical Method
- Electrical Analogy Method
- Hydraulic Models
- Relaxation Method
- Hele-Shaw or Viscous Analogy Method

Limitations of flow net:

- The flow should be two dimensional
- The flow should be steady
- The flow should be Irrotational
- The flow is not governed by gravity force

Q**. 11 The velocity components in a 2-dimensional incompressible flow field are expressed as**

$$u = \left(\frac{y^3}{3} + 2x - x^2 \times y \right), \quad v = \left(x \times y^2 - 2y - \frac{x^3}{3} \right)$$

Dec 2013

Is the flow irrotational? If so determine the corresponding stream function.

Ans: Given the components of velocity

$$u = \left(\frac{y^3}{3} + 2x - x^2 \times y \right), \quad v = \left(x \times y^2 - 2y - \frac{x^3}{3} \right)$$

The condition for Irrorational flow

$$\left(\frac{\partial v}{\partial x} \right) = \left(\frac{\partial u}{\partial y} \right)$$

$$LHS \quad \frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(x \times y^2 - 2y - \frac{x^3}{3} \right) \text{and } RHS \quad \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y^3}{3} + 2x - x^2 \times y \right)$$

i.e. LHS = $(y^2 - x^2)$ and RHS = $(y^2 - x^2)$

Hence the flow is Irrorational

The corresponding stream function ‘ψ’can be obtained by using following relationship

$$\frac{\partial \cdot}{\partial x} = v = \left(x \times y^2 - 2y - \frac{x^3}{3} \right) \cdots Eq.1$$

$$\frac{\partial \cdot}{\partial y} = -u = -\left(\frac{y^3}{3} + 2x - x^2 \times y \right) \cdots Eq.2$$

Integrating Eq.1 with respect to ‘x’

$$\cdot = \frac{x^2 \times y^2}{2} - 2 \times x \times y - \frac{x^4}{12} + C_1\left(f(y) \right) \cdots Eq.3$$

Differentiating Eq.3 with respect to ‘y’

$$\frac{\partial \cdot}{\partial \boldsymbol{y}} = \boldsymbol{x}^2 \times \boldsymbol{y} - 2\boldsymbol{x} + \frac{\partial \boldsymbol{C}_1}{\partial \boldsymbol{y}}$$

$$\frac{\partial \boldsymbol{C}_1}{\partial \boldsymbol{y}} = -\frac{\boldsymbol{y}^3}{3}$$

$$\textbf{\textit{Integratin g}}, \boldsymbol{C}_1 = -\frac{\boldsymbol{y}^4}{12} + \boldsymbol{C}; \quad (\textbf{\textit{assu}} \min \boldsymbol{g} \ \boldsymbol{C} = 0)$$

$$\boldsymbol{C}_1 = -\frac{\boldsymbol{y}^4}{12}$$

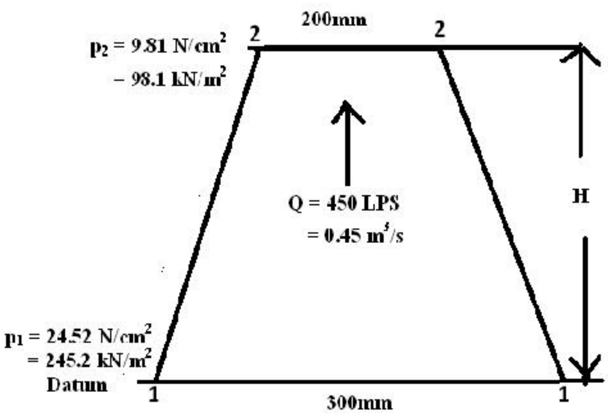
$$\cdot \;= \frac{x^2 \times y^2}{2} - 2 \times x \times y - \frac{x^4}{12} + - \frac{y^4}{12}$$

PART-B

UNIT-5: DYNAMICS OF FLUID FLOW

Q1.Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.52 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the difference in datum head if the flow through pipe is 40 LPS. (July 2014, jan 2015)

Ans:



Let 'H' difference in datum head (Z_2-Z_1)

From Continuity equation

$$Q = \frac{\text{®}}{4} \times (0.3)^2 \times V_1 = \frac{\text{®}}{4} \times (0.2)^2 \times V_2 = 0.45$$

V_1 = 6.37 m/s, V_2 = 14.324 m/s

The velocity head at 1 and 2 is given by

<u>At Section 1-1</u>

$$\frac{V_1^2}{2 \times g} = \frac{(6.37)^2}{2 \times 9.81} = 2.068m$$

<u>At Section 2-2</u>

$$\frac{V_2^2}{2 \times g} = \frac{(14.324)^2}{2 \times 9.81} = 10.457m$$

Applying Bernoulli's equation between the section 1-1 and 2-2 assuming no-loss

$$\frac{p_1}{\text{¥}_w} + Z_1 + \frac{V_1^2}{2 \times g} = \frac{p_2}{\text{¥}_w} + Z_2 + \frac{V_2^2}{2 \times g} + \cdots\cdots Eq.1$$

25 + 0 + 2.068 = 10 + H + 10.457

H = 6.611m

Q. 2Namethe different forces present in a fluid flow. What are the forces considered for the Euler's equation of motion? (Dec 2013, Dec 2014)

Ans: (i) Gravity force (ii) Pressure force (iii) Viscous force (iv) Force due to turbulence (v) Force due to compressibility

Forces considered in Euler's equation: Gravity, pressure forces

Q.3 State and prove Bernoulli's theorem (Dec 2013, July 2014, Jan 2015)

Ans: The Bernoulli's theorem states that for a "for a steady, streamline flow of an ideal, incompressible fluid, the sum of kinetic, potential and pressure energy is constant"

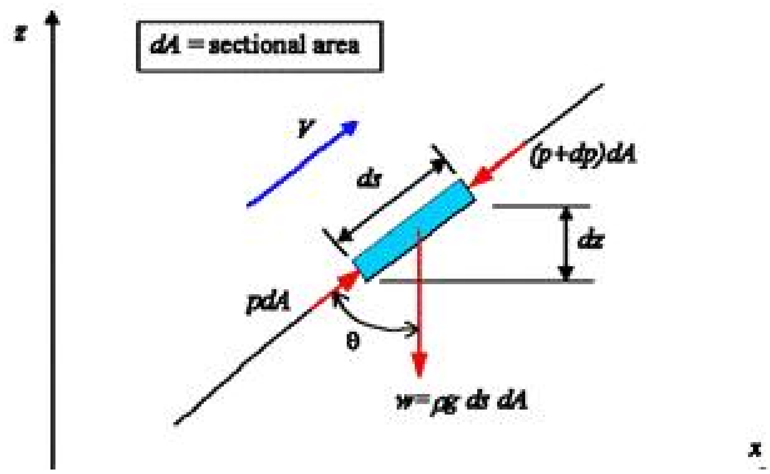
Assumptions:

- Only Gravitational and Pressure forces are considered
- Fluid motion along a stream line is considered
- Flow is steady & Incompressible
- Flow is irrotational
- Flow is in viscid (Zero Viscosity)

Consider a stream line along direction x as shown in Fig. Consider a cylindrical fluid element of cross-sectional area 'dA' and length ds along the stream line direction.

The forces acting on the fluid element are:

- The pressure force pdA along the flow direction s
- The pressure force $[p+\partial p]dA$ against the flow direction s
- Weight of the fluid element = $\rho gdA\,ds$ acting vertically downwards at an angle 'θ' with the vertical. Let 'θ' is the angle between the direction of flow and the line of action of the weight of element.
- The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the directions.



$$p\,dA-(p+dp)\,dA-\rho\,g\,ds\,dA\cos\theta=\rho\,ds\,dA\times a_s$$

Where ‘as’is the acceleration in the direction of s. as= dV/dt, ‘V’isa function of ‘s’and t.

If the flow is steady, $\frac{dv}{dt}=0$, $\cos\theta=\frac{dz}{ds}$ and $a_s=V\frac{dV}{ds}$

$$-dp\;dA-\rho\;g\;ds\;dA\cos\theta=\rho\;ds\;dA\times V\frac{dV}{ds}$$

.On substituting and dividing the equation by ρgdA, we can obtain Euler’s equation:

$$\frac{dp}{\rho}+gdz+vdv=0-Eq.(1)$$

The above equation is known as Euler’s equation of motion.

Bernoulli’s Equation: Integrating the Eq(1) the ‘total energy head’ H of the fluid is found by adding the three types of mechanical energy possessed by the fluid at that point.

$$H=\frac{p}{\rho g}+\frac{v^2}{2g}+z$$

As water flows between two points, or sections of a pipeline or channel, no energy can be created or destroyed (fundamental law of physics concerning conservation of energy). If any mechanical energy is converted into say heat energy through friction then it is lost to the mechanical system and the 'total energy head' H is reduced.

The energy heads involved are measured in meters and can be represented as vertical distances (heights) on an energy diagram.

Considering flow between 2-points labelled 1 and 2:

Bernoulli's Equation for Ideal fluid flow: $\frac{p_1}{{}^\circ g}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{{}^\circ g}+\frac{v_2^2}{2g}+z_2$

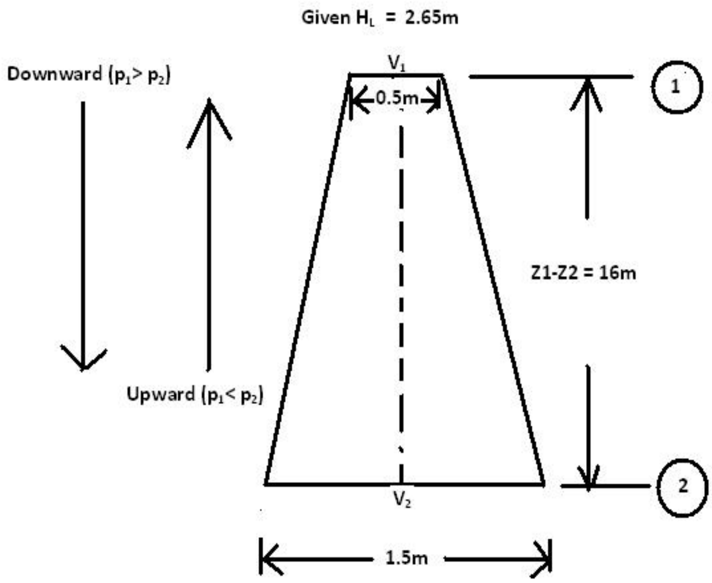
Bernoulli's Equation for Real fluid flow:

$$\boldsymbol{H}_1 = \boldsymbol{H}_2 + \boldsymbol{h}_f + \boldsymbol{h}_L$$

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2+\sum h_f+\sum h_L$$

Q. 4 A tapered section is 2.65m in either direction when the velocity at the smaller section is 9m/s. If the smaller section is at the top and pressure head at this section is 2.15m of water. Find the pressure head at the lower end when the flow is (i) Downwards (ii) Upward (Dec 2013, Jan 2015)

Ans: Based on continuity equation



Given D1 = 0.5m, D2 = 1.5m

V1 =3m/s , V2 = ?

Q = A1V1 = A2V2

$$\boldsymbol{Q} = \frac{®}{4}\times(0.5)^2\times 9 = \frac{®}{4}\times(1.5)^2\times \boldsymbol{V}_2$$

V_2 = 1m/s

Case-1: When flow the is downward

i.e. $p_1 > p_2$

Applying Bernoulli's Equation
between '1' and '2' assuming section-2 as datum

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2 \times g} = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_1^2}{2 \times g} + H_L$$

$$16 + 2.15 + \frac{9^2}{19.62} = 0 + \frac{p_2}{\gamma_w} + \frac{1^2}{19.62} + 2.65$$

$$\frac{p_2}{\gamma_w} = 19.577 \, m \text{ of water column}$$

Case-2: When flow the is Upward: i.e. $p_1 < p_2$

Applying Bernoulli's Equation between '1' and '2' assuming section-2 as datum

$$Z_1 + \frac{p_1}{\gamma_w} + \frac{V_1^2}{2 \times g} + H_L = Z_2 + \frac{p_2}{\gamma_w} + \frac{V_1^2}{2 \times g}$$

$$16 + 2.15 + \frac{9^2}{19.62} + 2.65 = 0 + \frac{p_2}{\gamma_w} + \frac{1^2}{19.62}$$

$$\frac{p_2}{\gamma_w} = 24.877 \, m \text{ of water column}$$

Q.5. Figure shows nozzle at the end of a pipe line discharging oil from a tank to atmosphere. Estimate the discharge from the nozzle when the head 'H' in the tank is 4m. The loss in the pipe can be taken as 20 $V^2/2g$, where 'V' is the velocity in the pipe. The loss of energy in the nozzle can be assumed to be zero. Also, determine the pressure at the base of the nozzle. (July 2013,2014 july 2015)

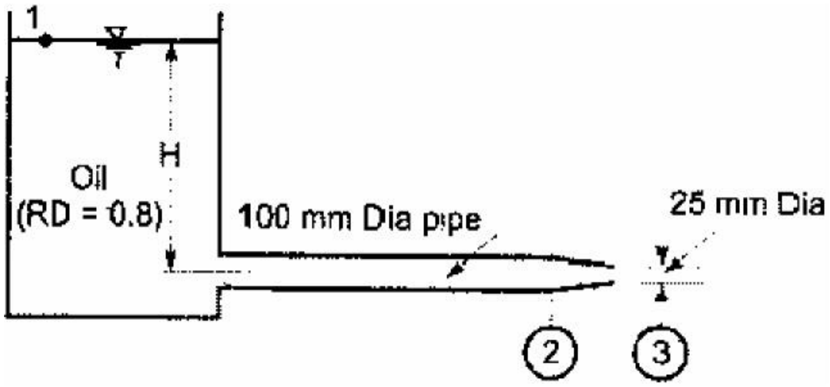


Fig.Q5.(a)

Ans: Let 'V' be the velocity in 100mm diameter pipe and 'V_N' be the velocity of flow at the exit of the nozzle (25mm dia.)

The relationship between the V and V_N can be derived from the continuity equation

$$Q = \frac{®}{4} \times (0.2)^2 \times V^2 = \frac{®}{4} \times (0.025)^2 \times V_N^2$$

$$V_N = 16\,V \cdots\cdots Eq.(1)$$

Applying Bernoulli's Equation between 1-1 and 3-3 [Nozzle free exit, $p_3 = 0$ (atm)]

$$0 + 0 + 4 = 0 + 0 + \frac{V_N^2}{2 \times 9.81} + (H_{loss})_{1\text{-}3}$$

$$4 = \frac{V_N^2}{2 \times 9.81} + 20 \times \frac{V^2}{2 \times 9.81}$$

Substituting for Eq.1 $V_N = 16\ V$

V = 0.533 m/s

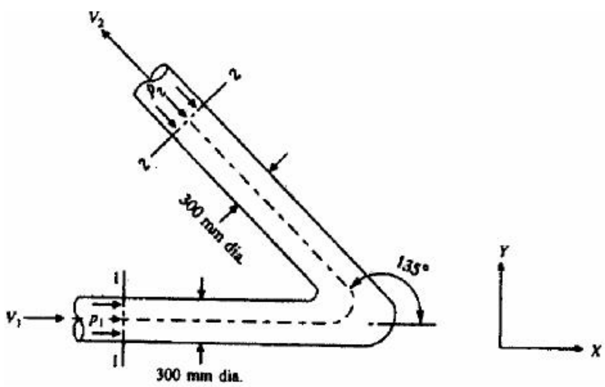
Applying Bernoulli's Equation between 1-1 and 2-2

$$0 + 0 + 4 = 0 + \frac{p_2}{°_{oil} \times 9.81} + \frac{V}{2 \times 9.81} + 20 \times \frac{V^2}{2 \times 9.81}$$

Substituting V = 0.533 m/s and solving for pressure at the base of the nozzle

P_2 = 29037.6 Pa = 29.037 kPa

Q.6 250LPS of water is flowing in a pipe having a diameter of 300mm. If the pipe is bent by 135î, find the magnitude and direction of the resultant force on the bend. The pressure of the water flowing is 400 kN/m^2. Take specific weight of water as 9.81 kN/m^3. (Figure) (July 2013)



Figure

Ans: The pipe is of uniform cross-sectional area. Therefore the velocities at section 1-1 and section 2-2 are same.

$$V_1 = V_2 = \frac{Q}{A} = \frac{0.25}{\left(\frac{®}{4} \times 0.3^2\right)} = 3.537\ m/s$$

The pressure intensity is also same $p_1 = p_2 = 400$ kPa = 400 ×1000 N/m^2

Force along x-axis ‘Fx’: (Dynamic + Static) Force

$$F_x = \frac{¥_W Q}{g} \times \left(V_1 - V_2 \times Cos135^{ˉ}\right) + p_1 A_1 - p_2 A_2 \cos 135^{ˉ}$$

$$F_x = \frac{9810 \times 0.25}{9.81} \times \left(3.54 - 3.54 \times (-0.707)\right) + 400 \times 1000 \times \frac{®}{4} \times 0.3^2 - 400 \times 1000 \times \frac{®}{4} \times 0.3^2 \times (-0.707)$$

F_x = 49750.51 N (= ì)

Force along Y-axis ‘Fx’: (Dynamic + Static) Force

$$F_x = \frac{¥_W Q}{g} \times \left(0 - V_2 \times Sin135^{ˉ}\right) - p_2 A_2\ Sin135^{ˉ}$$

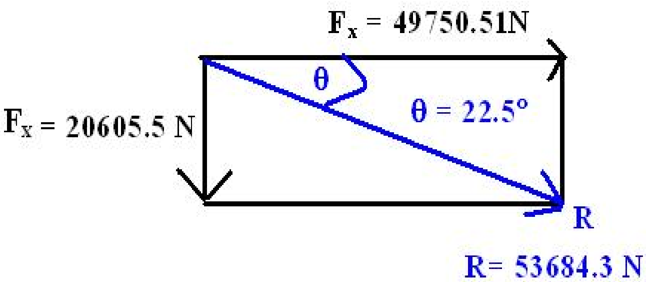
$$F_x = \frac{9810 \times 0.25}{9.81} \times \left(0 - 3.54 \times (0.707)\right) - 400 \times 1000 \times \frac{®}{4} \times 0.3^2 \times (0.707)$$

F_y = -20605.51 N (í)

$$R = \sqrt{F_x^2 + F_y^2}$$

$$R = \sqrt{(49750.5)^2 + (-20605.5)^2}$$

R = 53848.9 N



$$\tan ¯ = -\frac{20605.5}{49750.5}$$

$$¯ = \tan^{-1}\left(-22.5^{ˉ}\right)$$

Q7 For the Venturimeter of 150mm x75mm, determine the reading of the mercury manometer, if the pipe carries a discharge of 35.32 litres/sec. of oil of relative density 0.80. TakeC_d = 0.97. (July 2013)

Ans: Given D_1 = 150mm = 0.15m, D_2 = 75mm = 0.075m,

Q = 35.32 Litres/s = 0.03532 m^3/s, Sp. Gr Oil = 0.8, C_d = 0.97

$$\mathbf{A_1} = \frac{\pi}{4}\times(0.15)^2 \; ; \; \mathbf{A_2} = \frac{\pi}{4}\times(0.075)^2$$

$$\mathbf{h} = \mathbf{x}\times\left(\frac{\mathbf{S_{mercury}}}{\mathbf{S_{oil}}}-1\right) = \mathbf{x}\times\left(\frac{13.6}{0.8}-1\right)\cdots\mathbf{Eq.}1$$

$$\mathbf{Q} = \mathbf{C_d}\times\frac{\mathbf{A}_1}{\sqrt{\left(\frac{\mathbf{A}_1}{\mathbf{A}_2}\right)^2-1}}\times\sqrt{2\times\mathbf{g}\times\mathbf{h}} \qquad \mathbf{Eq.2}$$

$$\frac{35.32}{1000} = 0.03532 = \mathbf{0.97}\times\frac{\left(\frac{\pi}{4}\right)\times(0.15)^2}{\sqrt{\left(\frac{150}{75}\right)^4-1}}\times\sqrt{2\times9.81\times\mathbf{h}} \qquad \mathbf{Eq.2}$$

On solving h = 0.65m

Substituting the value of 'h' in Eq.1

$$\mathbf{0.6} = \mathbf{x}\times\left(\frac{\mathbf{S_{mercury}}}{\mathbf{S_{oil}}}-1\right) = \mathbf{x}\times\left(\frac{13.6}{0.8}-1\right)\cdots\mathbf{Eq.}1$$

x = 0.0406m = 40.6mm

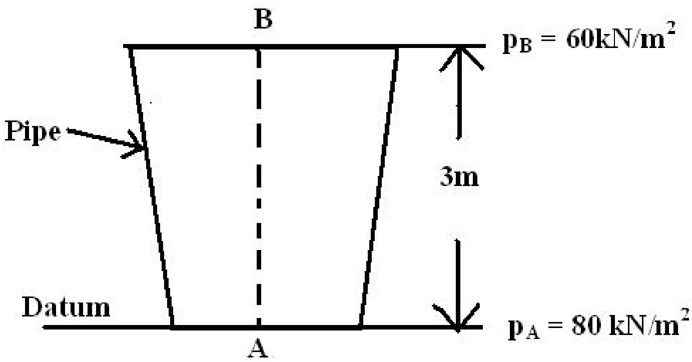
Q.8 A vertical pipeline carrying water changes in diameter from 200mm at a position 'A' to 500mm at another position 'B' which is 3m at a higher level. If the pressure at 'A' and 'B' are 80kN/m2 and 60kN/m^2 respectively and discharge is 200 litres/sec, determine the loss of head and direction of flow (Dec2013, July 2014)

Ans: Given Q = 200 lit/sec = 0.2m^3/s,

D_A = 200mm = 0.2m, D_B = 500mm = 0.5m

$(Z_B – Z_A)$= 3m, p_A = 80kN/m^2, p_B = 60kN/m^2,

The velocity at 'A' and 'B' can be obtained by continuity equation



$$V_A = \frac{Q}{A_A} = \frac{0.2}{\left(\frac{\pi}{4} \times 0.2^2\right)} = 6.366\ m/s$$

$$V_B = \frac{Q}{A_B} = \frac{0.2}{\left(\frac{\pi}{4} \times 0.5^2\right)} = 1.018\ m/s$$

Total Energy head at 'A'

$$E_A = Z_A + \frac{p_A}{\gamma_w} + \frac{V_A^2}{2 \times g}$$

$$E_A = 0 + \frac{80}{9.81} + \frac{(6.366)^2}{2 \times g} = 10.21m$$

Total Energy head at 'B'

$$E_B = Z_B + \frac{p_B}{\gamma_w} + \frac{V_B^2}{2 \times g}$$

$$E_B = 3 + \frac{60}{9.81} + \frac{(1.018)^2}{2 \times g} = 9.169m$$

Since $E_A > E_B$, The direction of flow will be from "A" to "B" i.e. Upward ↑

The loss of head = $(E_A – E_B)$ = (10.21-9.169) = 1.041m

UNIT – VI PIPE FLOW

Q.1 Define (i) Hydraulic gradient (ii) Energy gradient (July 2013,July 2014,Jan 2015)

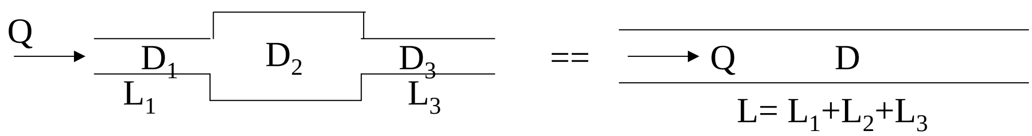
Ans:(i) Hydraulic gradient: A Line joining the peizometric heads at various points in a flow is known as Hydraulic Grade Line (HGL)

(ii) Energy gradient:
It is a line joining the elevation of total energy of a flow measured above a datum, i.e. EGL Line lies above HGL by an amount $V^2/2g$.

Q. 2Distinguish between compound pipe and equivalent pipe (July 2014, Jan 2015)

Ans:

Compound pipe **Equivalent pipe**



lent pipe is such that the entire system is replaced by a single pipe of uniform diameter D, but of the same length $L=L_1+L_2+L_3$ such that the head loss due to friction for both the pipes, viz equivalent pipe & the compound pipe are the same. For a compound pipe or pipes in series.

$$h_f = hf_1 + hf_2 + hf_3$$

$$h_f = \frac{8fL_1Q^2}{g\text{®}^2D_1^5} + \frac{8fL_2Q^2}{g\text{®}^2D_2^5} + \frac{8fL_3Q^2}{g\text{®}^2D_3^5} --- (1)$$

For an equivalent pipe

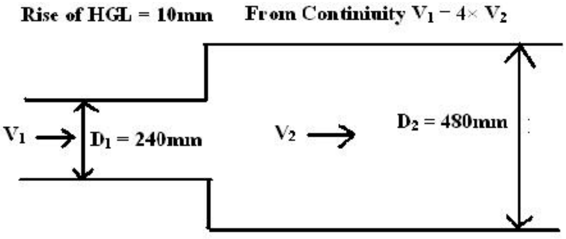
$$h_f = \frac{8fLQ^2}{g\text{®}^2D_1^5} --- (2)$$

Equating (1) & (2) and simplifying

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

$$D = \left\{ \frac{L}{\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}} \right\}^{\frac{1}{5}}$$

Q. 3 At a sudden enlargement of water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow. (July 2014, july 2015)
Ans: Rise of hydraulic gradient line = 10mm



Sudden Enlargement

From Continuity equation

$$Q = \frac{\pi}{4} \times (0.24)^2 \times V_1 = \frac{\pi}{4} \times (0.48)^2 \times V_2$$

$V_1 = 4\times V_2$

Loss of head in sudden expansion

$$H_{expansion} = \left(\frac{(V_1 - V_2)^2}{2\times g} \right)$$

Applying Bernoulli's equation between upstream and downstream of sudden expansion

$$\frac{p_1}{\gamma_w} + Z_1 + \frac{V_1^2}{2\times g} = \frac{p_2}{\gamma_w} + Z_2 + \frac{V_2^2}{2\times g} + H_{expansion} \cdots\cdots Eq.1$$

Given rise of hydraulic gradient line = 10mm = 0.01m = $\frac{(p_2 - p_1)}{\gamma_w}$

The elevation is same for horizontal enlargement $Z_1 = Z_2$ and also $V_1 = 4\times V_2$

Substituting in eq.1

$$\frac{p_1}{\gamma_w} + Z_1 + \frac{(4V_2)^2}{2\times g} = \frac{p_2}{\gamma_w} + Z_1 + \frac{V_2^2}{2\times g} + \left(\frac{(V_1 - V_2)^2}{2\times g} \right)$$

$$\frac{(p_2 - p_1)}{\gamma_w} = 0.01 = \frac{15\times V_2^2}{2\times g} - \frac{9\times V_2^2}{2\times g}$$

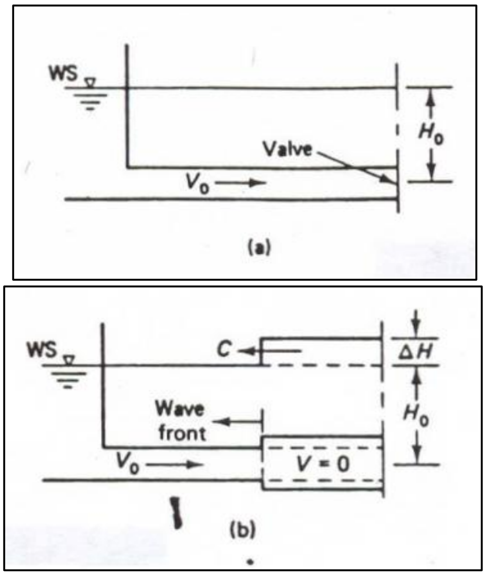
$$V_2 = 0.181 m/s$$

$$Q = \frac{\pi}{4} \times (0.48)^2 \times 0.181 = 0.03275 m^3/s$$

The rate of flow Q = 0.03275 m^3/s

Q.4 Explain the phenomenon of water hammer. List the four factors affecting water hammer (July 2013 ,Dec 2013, Jan 2015)

Ans: Water Hammer Phenomenon in pipelines: A sudden change of flow rate in a large pipeline (due to valve closure, pump turnoff, etc.) involve a great mass of water moving inside the pipe. The force resulting from changing the speed of the water mass may cause a pressure rise/ pressure drop in the pipe with a magnitude several times greater/less than the normal static pressure in the pipe. This may set up a noise known as knocking. This phenomenon is commonly known as the water hammer phenomenon



(a) Steady state prior to valve closure

(b) Rapid valve closure – pressure increase, pipe walls expand, liquid compression; transient conditions propagate upstream

Factors affecting water hammer phenomenon:

(i) Length of Pipeline (ii) Diameter of the pipeline (iii) Material of the pipeline (iv) Discharge (v) Thickness of pipeline (vi) Time of valve closure

Q. 5Derive an expression for head loss due to sudden enlargement in a pipe flow (Dec 2013, July 2015)

Ans: Equation for head loss due to Sudden Enlargement or Expansion in Pipe:

Consider the sudden expansion of flow between the two section (1)- (1) & (2)- (2) as shown. p_1&p_2 are the pressure acting at (1) - (1) and (2) - (2), while V_1 and V_2 are the velocities. From experiments, it is proved that pressure P1 acts on the area (a_2 – a_1) i.e. at the point of sudden expansion. From II Law of Newton Force = Mass x Acceleration.

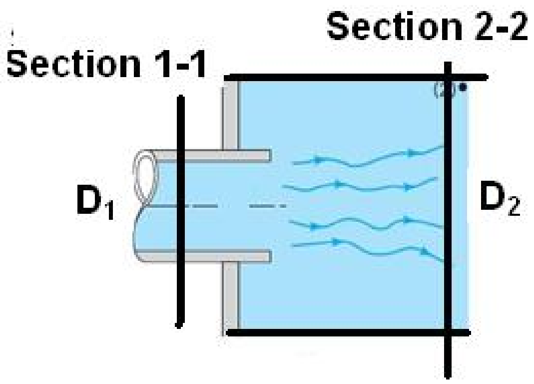


Fig. Head Loss due to Sudden Expansion

Consider LHS of eq(1)

$$or, \sum forces = a_2(p_1 - p_2) --- (ii)$$

Consider RHS of eq(1)

Mass x acceleration = × Vol × change in velocity /time

$$° \times \boldsymbol{Q} \times (\boldsymbol{V}_1 - \boldsymbol{V}_2) - - - (\boldsymbol{iii})$$

ρ =volume/time × change in velocity

$$a_2(p_1 - p_2) = pQ(V_1 - V_2)$$

Substitution (ii) & (iii) in eq(i)

$$(p_1 - p_2) = ° V_2 (V_1 - V_2)$$

Both sides by (Specific weight 'γ')

$$\therefore \left(\frac{p_1 - p_2}{¥} \right) = \frac{V_2(V_1 - V_2)}{g} - - - (iv)$$

Applying Bernoulli's equation between (1) and (2) with the centre line of the pipe as

$$\therefore \left(\frac{p_1 - p_2}{¥} \right) = \frac{V_1^2 - V_2^2}{2g} = h_L$$

$$h_L = \frac{2V_2(V_1 - V_2) + (V_1^2 - V_2^2)}{2g}$$

datum and

Considering head loss due to sudden expansion $h_L$only.

$$\boldsymbol{Z}_1 = \boldsymbol{Z}_2 \therefore \textbf{\textit{pipe is horizontal}}$$

$$h_L = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g} \qquad h_L = \frac{2V_1V_2 - 2V_2^2 + V_1^2 - V_2^2}{2g}$$

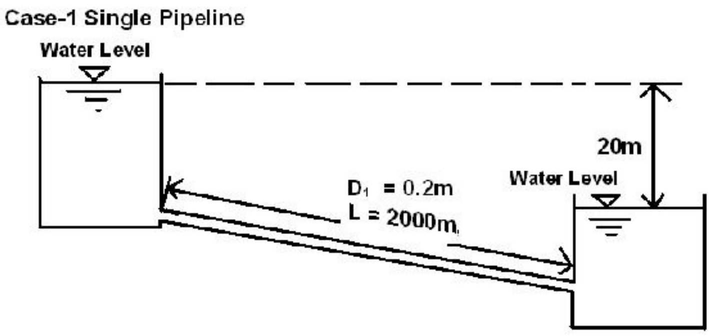
$$Z_1 + \frac{p_1}{¥} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{¥} + \frac{V_2^2}{2g}$$

Q.6 A pipe of 200mm diameter and length 2000m connects two reservoir, having

$$h_L = \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g} \qquad \boldsymbol{h_L} = \frac{(\boldsymbol{V}_1 - \boldsymbol{V}_2)^2}{2\boldsymbol{g}}$$

difference of water level as 20m. Determine the discharge through the pipe. If an additional pipe of diameter 200mm and length 1200m is attached to the last 1200m of the existing pipe, find the increase in discharge. Take f = 0.015 and neglect minor losses. (Dec2013,July 2014)

Ans; Case-**1: When single pipe connects two reservoirs**



H = 20m, f = 0.015, L_1 = 2000m

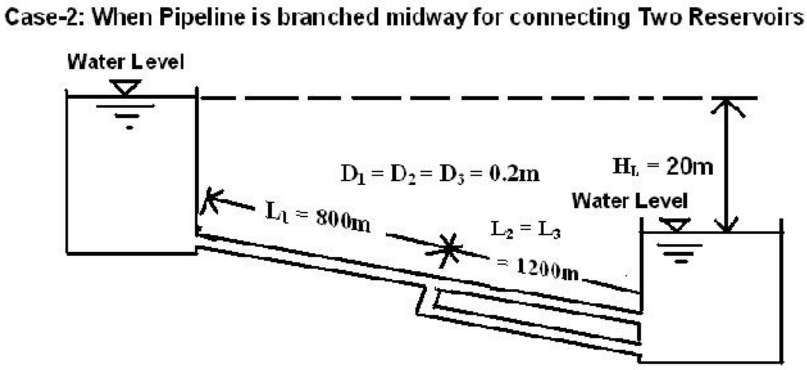
D_1 = 0.2m $V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi}{4} \times D_1^2\right)}$

$$H = \frac{4fL_1V_1^2}{2gD_1} = \frac{4fL_1Q^2}{2gD_1\left(\frac{\pi}{4} \times D_1^2\right)} = \frac{32fL_1Q^2}{\pi^2 gD_1^5}$$

$$H = 20 = \frac{32 \times 0.015 \times 2000 \times Q^2}{\pi^2 \times g \times (0.2)^5}$$

$$Q = 0.0254\ m^3/\sec$$

Case-2: When Pipeline is branched midway for connecting two reservoirs



H = 20m, f = 0.015,

L_1 = 800m, D_1 = 0.2m

L_2 = 1200m, D_2 = 0.2m

L_3 = 1200m, D_3= 0.2m

$Q_1 = Q_2 + Q_3 = 2Q_2$

$Q_2 = \left(\frac{Q_1}{2}\right)$

$$H = \frac{4fL_1V_1^2}{2gD_1} + \frac{4fL_2V_2^2}{2gD_2}$$

$$20 = \frac{32fL_1Q_1^2}{\pi^2 gD_1^5} + \frac{32fL_2Q_2^2}{\pi^2 gD_2^5} = \frac{32 \times 0.015 \times 800 \times Q_1^2}{\pi^2 \times g \times (0.2)^5} + \frac{32 \times 0.015 \times 1200 \times (0.5Q_1)^2}{\pi^2 \times g \times (0.2)^5}$$

$$Q_1 = 0.0342\ m^3/\sec$$

Increase in Discharge = (Q_1 – Q) = (0.0342-0.0254) = 0.088m^3/sec

Q 7For the distribution main of a City water supply a 0.3m diameter pipe is required. But the main is replaced by laying two parallel pipes of same diameter. Find the diameter of parallel pipe (July 2013)

Ans:

Case- 1 Single Pipe f Length L Discharge Q D = 0.3m

The head loss through single pipe is given by

$$\mathbf{h_f} = \frac{\mathbf{f \times L \times V^2}}{2 \times \mathbf{g} \times \mathbf{D}} = \frac{\mathbf{f \times L}}{2 \times \mathbf{g} \times \mathbf{D}} \times \left(\frac{\mathbf{Q}}{\mathbf{A}}\right)^2$$

$$\mathbf{h_f} = \frac{\mathbf{f \times L}}{2 \times 9.81 \times \mathbf{D}} \times \left(\frac{\mathbf{Q}}{\frac{\pi}{4} \times (0.3)^2}\right)^2 = 84.61 \times \mathbf{f} \times \mathbf{L} \times \mathbf{Q}^2 \cdots\cdots \mathbf{Eq.}1$$

For single pipe and Parallel Pipes $\boxed{h_f = h_{f1} = h_{f2}}$

Case- 1 Single Pipe f Length L Discharge Q D = 0.3m

Case- 2 Parallel Pipe f Length L Discharge Q_1 $D_1 = D_2 = d$

$Q = Q_1 + Q_2 = 2Q_1$ (as both the diameter and length are same)

Case- 2 Parallel Pipe f Length L Discharge Q_1 $D_1 = D_2 = d$

$Q = Q_1 + Q_2 = 2Q_1$ (as both the diameter and length are same)

$$\mathbf{h_{f1}} = \mathbf{h_{f2}} = \frac{\mathbf{f} \times \mathbf{L}}{2 \times \mathbf{g} \times \mathbf{D}} \times \left(\frac{\mathbf{Q_1}}{\mathbf{A}}\right)^2 \quad \because \mathbf{Q_1} = \frac{\mathbf{Q}}{2} = 0.5 \times \mathbf{Q}$$

$$\mathbf{h_{f1}} = \mathbf{h_{f2}} = \frac{\mathbf{f} \times \mathbf{L}}{2 \times 9.81 \times \mathbf{d}} \times \left(\frac{0.5 \times \mathbf{Q}}{\frac{\pi}{4} \times \mathbf{d}^2}\right)^2 = \frac{0.0207 \times \mathbf{f} \times \mathbf{L} \times \mathbf{Q}^2}{\mathbf{d}^5} \cdots\cdots \mathbf{Eq.}2$$

Equating the values of h_f and h_{f1} from Eq.1 and Eq.2

$$84.61 \times \mathbf{f} \times \mathbf{L} \times \mathbf{Q}^2 = \frac{0.0207 \times \mathbf{f} \times \mathbf{L} \times \mathbf{Q}^2}{\mathbf{d}^5}$$

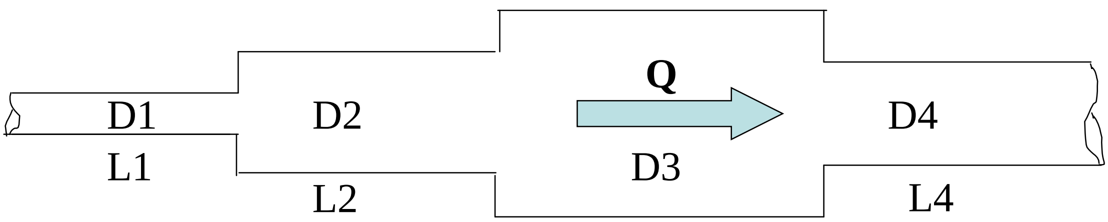
$$\mathbf{d}^5 = \frac{0.0207}{84.61}$$

$$\mathbf{d} = 0.19\mathbf{m}$$

Q.8 Define Pipes in Series (Dec 2013, Jan2015)

Ans: <u>Pipes in Series or Compound Pipe</u>

D_1, D_2, D_3, D_4 are diameters.
L_1, L_2, L_3, L_4 are lengths of a number of Pipes connected in series
$(hf)_1$, $(hf)_2$, $(hf)_3$ & $(hf)_4$ are the head loss due to friction for each pipe.



The total head loss due to friction h_f for the entire pipe system is given by

$$h_f = hf_1 + hf_2 + hf_3 + hf_4$$

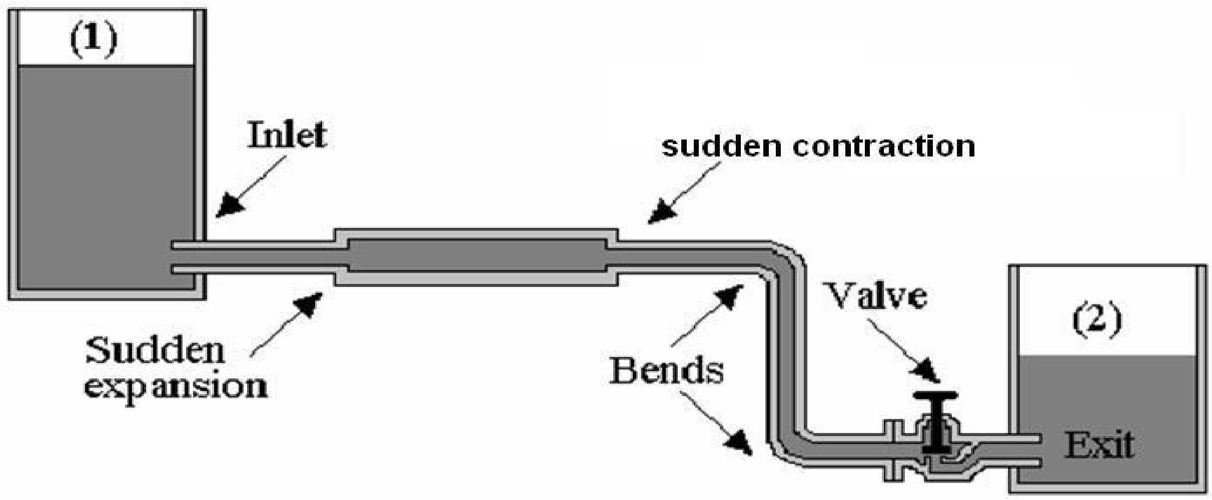
$$h_f = \frac{8 fL_1Q^2}{g\text{®}^2 D_1^5} + \frac{8 fL_2Q^2}{g\text{®}^2 D_2^5} + \frac{8 fL_3Q^2}{g\text{®}^2 D_3^5} + \frac{8 fL_4Q^2}{g\text{®}^2 D_4^5}$$

Q. 9Explain minor and major losses in pipeline (Dec 2014, july 2015)

Ans: Minor and Major Losses in Pipes: Minor losses in a pipe flow can be either due to change in magnitude or direction of flow. They can be due to one or more of the following reasons.

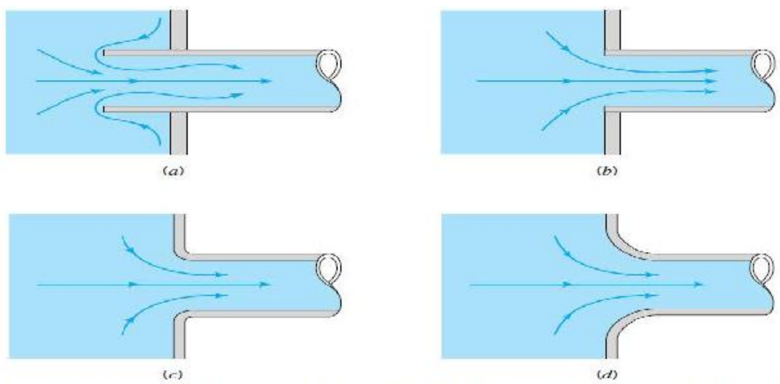
I. Entry loss
II. Exit loss
III. Sudden expansion loss
IV. Sudden contraction loss
V. Losses due to pipe bends and fittings
VI. Losses due to obstruction in pipe.

Major Loss: Friction loss



Loss due to Entrance

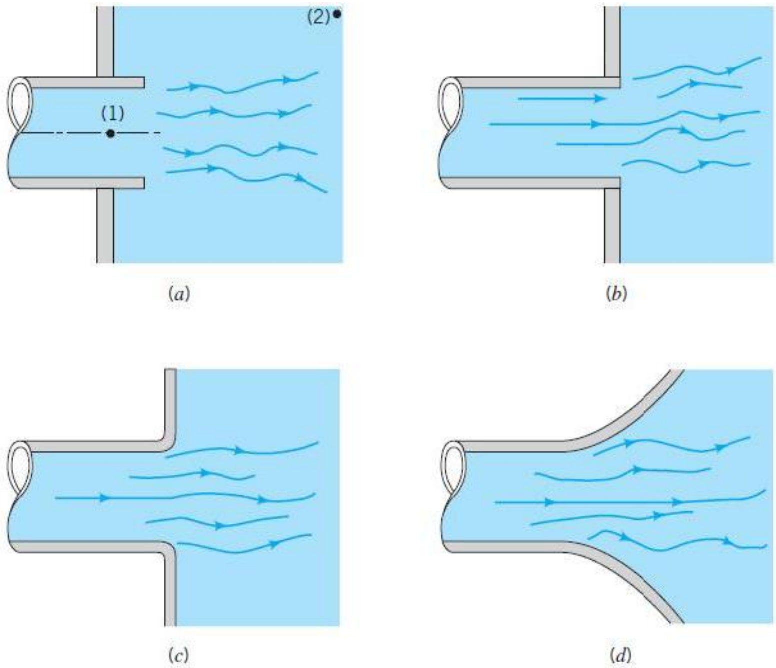
$$h_{L\,entry} = \frac{0.5V^2}{2g}$$



Entrance flow conditions and loss coefficient (Refs. 28, 29). (a) Reentrant, $K_L = 0.8$, (b) sharp-edged, $K_L = 0.5$, (c) slightly rounded, $K_L = 0.2$ (see Fig. 8.24), (d) well-rounded, $K_L = 0.04$ (see Fig. 8.24).

Loss due to Exit:

$$h_{L\,exit} = \frac{V^2}{2g}$$



Exit flow conditions and loss coefficient. (*a*) Reentrant, $K_L = 1.0$, (*b*) sharp-edged, $K_L = 1.0$, (*c*) slightly rounded, $K_L = 1.0$, (*d*) well-rounded, $K_L = 1.0$.

Equation for head loss due to Sudden Enlargement or Expansion in Pipe:

Consider the sudden expansion of flow between the two section (1) (1)& (2) (2) as shown. P_1& P_2 are the pressure acting at (1) (1) and (2) (2), while V_1 and V_2 are the velocities. From experiments, it is proved that pressure P_1 acts on the area (a_2 – a_1) i.e. at the point of sudden expansion. From II Law of Newton Force = Mass x Acceleration.Consider LHS

of

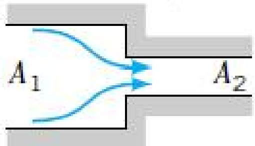
$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

Equations for other minor losses

Loss due to bends & fittings

$$h_L = \frac{KV^2}{2g}$$ K=coefficient of bend

Equation for head loss due to Sudden Contraction in pipe:



$$h_L = \frac{V^2}{2g}\left(\frac{1}{Cc} - 1\right)^2$$

Where Cc – Coefficient of contraction = 0.63 – 0.67

Q10 A piping system consists of three pipes arranged in series

Pipe	Length	Diameter
AB	1800m	50 cm
BC	1200m	40 cm
CD	600m	30 cm

Transform the system to
(i) An equivalent length of 40cm pipe
(ii) An equivalent diameter for the pipe of 3600m length (Dec 2014)

Ans: The Dupits equation for equivalent pipe length for pipes in series

$$\frac{L_{eq}}{D^5{}_{eq}} = \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}\right]$$

(i) Given D_{eq}= 40 cm = 0.4m

$$\frac{L_{eq}}{(0.4)^5} = \left[\frac{1800}{(0.5)^5} + \frac{1200}{(0.4)^5} + \frac{600}{(0.3)^5}\right]$$

L_{eq}= 4318.2 m

(ii) Given L_{eq}= 3600 m

$$\frac{3600}{D_{eq}^5} = \left[\frac{1800}{(0.5)^5} + \frac{1200}{(0.4)^5} + \frac{600}{(0.3)^5}\right]$$

D_{eq}= 0.39 m

UNIT VI- DEPTH AND VELOCITY MEASUREMENTS

Q.1Compare manual and self-recording depth gauges. (July 2013)

Ans: Manual Depth gauges: Measurement of water depth

The river stage has been defined as the height of the water surface in the river at a given section above any arbitrary datum. It is usually expressed in meters. In many cases, the datum is taken as the mean sea level. Sometimes the datum may be selected at or slightly below the lowest point on the river bed.Stage can be very easily measured by installing Non-recording (manual) or Recording (automatic) stream gauge stations. The various methods adopted can be listed as;

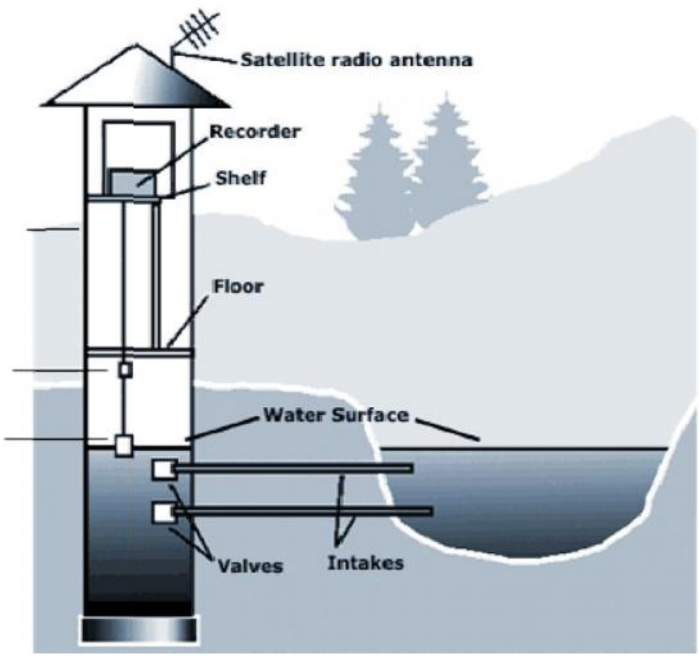
Non recording and recording type stream gauge are listed below:

1. Point and Hook Gauge
2. Staff
3. Weight gauge
4. Float gauge
5. Recordinggauge

Self-recording depth gauges: manual gauges described above are simple and inexpensive, they must be read frequently to get a continuous curve of the stream flow, especially when, the stage is, changing rapidly. Also, it is likely that the peak stage may be missed when it occurs in between the observations. *Recording type gauges* may be installed to overcome these difficulties.

Recording type gauge used to measure the stage continuously with time is also known as an *automatic stage recorder.* It usually consists of a float tied to one end of a cable running over pulley. To the other end of the cable a counterweight is attached. The float would be resting on the water surface and the counterweight always keeps the cable in tension. Any change in water surface makes the float either to raise or lower and this in turn makes the pulley rotate. The movement of the pulley would actuate a pen arm which rests on a clock-driven drum wrapped with a chart. The circumference of the drum represents the time axis while the height of the drum represents the stage. So, either sufficient height of the drum or some scaling mechanism is provided to cover the expected range of the stage. The clock and the drum may be so designed that the chart runs for a specified period of time (like a day or a week or a month) unattended.

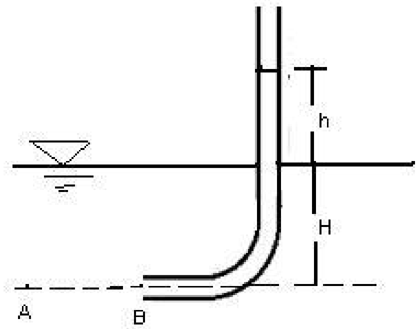
A float type automatic stage recorder requires a shelter in the form of a stilling well as shown is Figure. This stilling well gives protection to the float and counterweight from floating debris and with proper design of intake pipes it suppresses the fluctuations resulting from surface waves in the river, Generally two or more intake pipes are placed to allow the water from the river into the well so that at least one will admit water at all the times. As the stilling well is likely to get filled with sediment, it is necessary to make provision for the removal of silt from time to time. It is customary to install staff gauges inside and



Outside the well. These staff gauges serve to check the performance of the recorder and these are read each time the station is visited.

Q. 2With a neat sketch, describe the construction and working of a Pitot tube.
(July 2013 , Jan 2014, Jan 2015)

Ans: Pitot tube: It is one of the means of measuring of the local velocity in a flowing fluid. Pitot tube named after Henri Pitot who used a bent glass tube in 1730 to measure velocity in the river Seine. Used for measurement of velocity using either an inclined manometer or other type of manometer.



In a simple form, it is made up of a glass tube in which the lower end bent at right angles

Arrangement shown is for measuring velocity in free flow- Open Channel flows

The Liquid level in the tube (h) depends on velocity of stream. The term, H, is depth of tube from the free surfaceThe points A and B are at the same level as that of 'H'. The point A is just u/s of Pitot tube entry point and point B is at tube inlet point. The inlet of the Pitot tube acts as an obstruction to flow, where in KE of flowing fluid converts to PE. Hence at the point of inlet, i.e., at B, the velocity of flow become zero. This point is called as stagnation point.

Let v be the velocity at A. The Pressure head, H, at A is given by,

$$\frac{P_a}{w} = H$$

Where, w is the specific weight of the liquid.

Pressure at B: There is no velocity at B. It is a stagnation point, which means that the KE flowing fluid converts in to potential energy. i.e., pressure head, h above liquid surface. Pressure head at B is expressed as,

$$\frac{P_b}{w} = H + h$$

,
Now applying Bernoulli's Equation between A and B,

$$\frac{P_a}{w} + \frac{v^2}{2g} = \frac{P_b}{w}$$

$$H + \frac{v^2}{2g} = H + h$$

the converted energy head,h, can be represented by,

$$h = \frac{v^2}{2g}$$

From the above equation the theoretical velocity ,'v', can be calculated as,

$$v = \sqrt{2gh}$$

The above expression would give the theoretical velocity because in the above analysis the energy losses occurring in the system is not considered. The actual velocity can be determined by introducing a coefficient, C_V, which is the ratio of Actual Velocity to Theoretical Velocity. Hence the actual velocity is given by,

$$v = \boldsymbol{C_v}\sqrt{2gh}$$

Where C_V is the coefficient of Pitottube near to Unity

Q. 3 Apitot-tube is mounted on an airplane to indicate the speed of the plane relative to the prevailing wind. What differential pressure intensity in kPa will the instrument register when the plane is traveling at a speed of 200km/hr in a wind of 60 km/hr blowing against the direction of the plane? ÷°$_{air}$= 1.2 kg/m^3. (July 2013,Jan 2014)

Ans:Given :

$$\vec{V} = 200\ Km/hr; \qquad \overleftarrow{V}_{Air} = 75\ Km/hr$$

V_{net}= (200-60) = 140 km/hr = 38.89 m/s

$$V_{net} = C_V \sqrt{2 \times g \times h}$$

$$33.89 = 1.0\sqrt{2 \times 9.81 \times h}$$

h = 77.086m of Air

$$(\Delta p)_{Air} = \circ_{Air} \times g \times h$$
$$(\Delta p)_{Air} = 1.2 \times 9.81 \times 77.086$$

$(\Delta p)_{Air}$ = 907.456 Pa = 0.907456 kPa

Q.4 Define Hydraulic coefficient and determine the hydraulic coefficients experimentally (July 2014, July 2015)

Ans: **Hydraulic Coefficients of an orifice**

(i) **Coefficient of discharge** (C_d): It is defined as the ratio of actual discharge (Q_{act}) to the theoretical discharge (Q_{th})

$$\therefore C_d = \left(\frac{Q_{act}}{Q_{th}}\right)$$

Value of C_d varies in the range of 0.61 to 0.65

(ii) **Coefficient of Velocity** (C_V): It is defined as the ratio of actual velocity (V_{act}) to the theoretical velocity (V_{th}).

$$\therefore C_V = \left(\frac{V_{act}}{V_{th}}\right)$$

Value of C_V varies in the range of 0.95 to 0.99

(iii) Coefficient of Contraction (C_C): It is defined as the ratio of the area of cross section of the jet at Vena of cross section of the jet at Vena Contracta (a_C) to the area of the orifice (a).

$$\therefore C_C = \left(\frac{a_c}{a}\right)$$

$$But\ V = C_V\sqrt{2gH}$$

Value of C_C will be generally more than 0.62

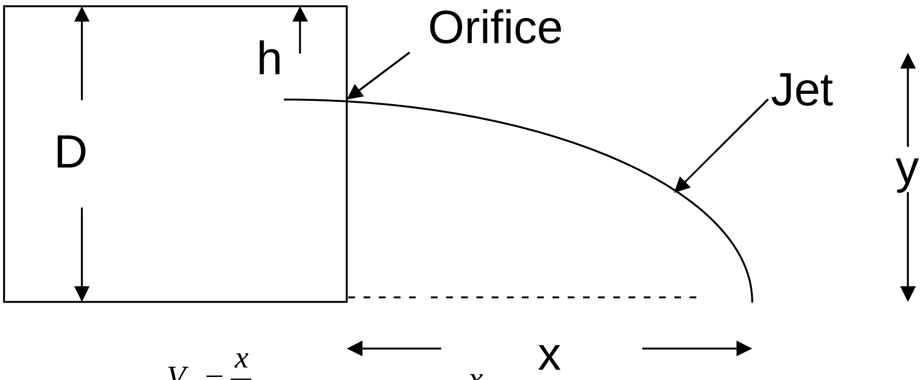
Torricelli's equation:

$$h_L = (H - HxC_V{}^2)$$

$$h_L = H(1 - C_V{}^2)$$

Equation for Coefficient of Velocity (C_V) (Trajectory method)

Consider a point P on the centre line of the jet, such that its horizontal and vertical coordinates are x and y respectively. By definition, velocity



$$V_a = \frac{x}{t} \qquad t = \frac{x}{V_a}$$

Since, the jet falls through a vertical distance y under the action of gravity during this time (t)

$$y = \frac{gt^2}{2} \quad t = \left(\frac{2y}{g}\right)^{\frac{1}{2}} ---(2)$$

But,

$$V_a = C_V\sqrt{2gH} \qquad \frac{x}{C_V\sqrt{2gH}} = \left(\frac{2y}{g}\right)^{\frac{1}{2}}$$

$$C_V = \frac{x}{2^{\frac{1}{2}} g^{\frac{1}{2}} H^{\frac{1}{2}}}\, x\, \frac{g^{\frac{1}{2}}}{2^{\frac{1}{2}} y^{\frac{1}{2}}}$$

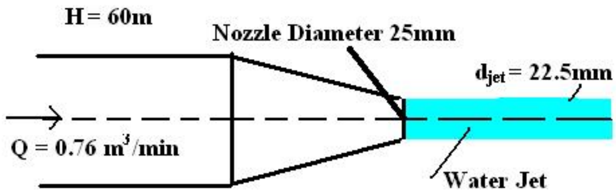
$$C_V = \frac{x}{2\sqrt{Hy}}$$

$$C_V = \left[\sqrt{\frac{x^2}{4yH}}\right]$$

Q. 5 A 25mm diameter nozzle discharges 0.76m^3/minute, when the head is 60m. The diameter of the jet is 22.5mm. Determine the values of C_c, C_v, C_d and loss of head due to fluid resistance (July 2014)

Ans: Given Nozzle Diameter D= 25mm, Q = 0.76 m^3/min, H = 60m,

d_{jet}= 22.5mm



(i) Values of coefficients

$$C_C = \frac{Area\ of\ \ jet}{Area\ of\ \ Nozzle} = \frac{\frac{\pi}{4}\times d_j^2}{\frac{\pi}{4}\times D^2} = \frac{d_j^2}{D^2} = \frac{(22.5)^2}{(25)^2} = 0.81$$

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01267}{A\times\sqrt{2\times g\times H}} = \frac{0.01267}{\frac{\pi}{4}\times(0.025)^2\times\sqrt{2\times 9.81\times 60}} = 0.7522$$

From the relationship $C_d = C_C \times C_V$

$$Cv = \frac{C_d}{C_c} = \frac{0.7523}{0.81} = 0.928$$

Loss of head (H_L) due to fluid resistance can be obtained by applying Bernoull's Equation between the outlet of the nozzle and jet of water

$$\frac{p_1}{\gamma_w} + Z_1 + \frac{V_1^2}{2\times g} = \frac{p_2}{\gamma_w} + Z_2 + \frac{V_2^2}{2\times g} + H_L \cdots\cdots Eq.1$$

For Outlet of Nozzle, $\frac{p_1}{\gamma_w} = Atmospheric \quad Pressure = 0$, $Z_1 = Z$

For Water jet, $\frac{p_2}{\gamma_w} = Atmospheric \ Pressure = 0$, $Z_2 = Z$

Substituting in Eq.1

$$\frac{V_1^2}{2 \times g} = \frac{V_2^2}{2 \times g} + H_L \cdots\cdots Eq.2$$

$$\frac{\left(\sqrt{2 \times 9.81 \times 60}\right)^2}{2 \times 9.81} = \frac{(Cv)^2\left(\sqrt{2 \times 9.81 \times 60}\right)^2}{2 \times 9.81} + H_L$$

$$H_L = 60\left(1 - C_V^2\right) = 60\left(1 - (0.928)^2\right) = 8.329m$$

Q. 6 With the help of a neat sketch, explain working of a current meter (Dec 2013, July 2014, Jan 2015)

Ans: Current meter: A current meter (Figure 1) consists of a rotating element which when placed in flowing water at a point where velocity (v) is to be measured. The current meter at a speed (N) related to the velocity of water (V) as shown in the calibration graph Fig. 2

V = a + Nb Eq….1

Where ‘V’ velocity in m/s ,’N’ rpm, ‘a’ and ‘b’ are constants

For the measurement of velocity the current meters are most commonly used. IS: 3910 - 1966 gives specifications for cup type current meter and IS: 3918 - 1966 gives the code of practice for use of this type of current meter. To obtain a mean velocity in a vertical, velocity distribution observations can be made at a number of points along the vertical. This is done when results are required to be accurate, or for purpose of calibration.

In two-point method the velocity observations are made at 0.2 and 0.8 depth below the surface while in one point method observations is made at 0.6 depth below the surface.

Fig 1 Propeller and Cup Cone type Current meter

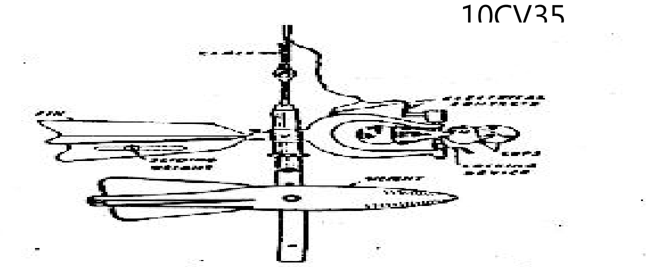
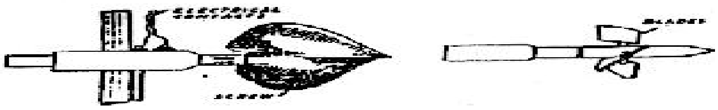


Fig.1 Price current meter



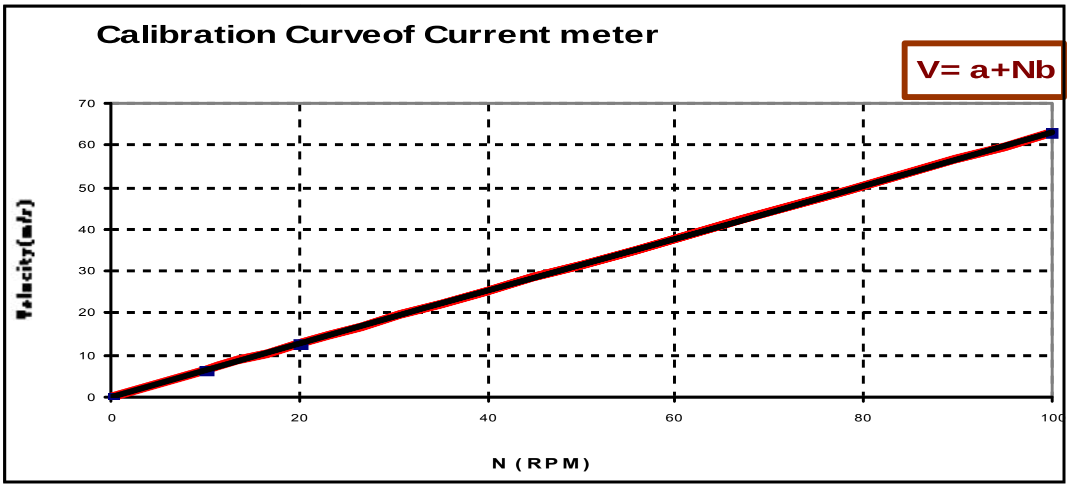


Fig 2 Typical Calibration Curve of a Current meter

Both the two-point and one point methods are in common use in India, though sub-surface method comprising making velocity observations just below the surface is also used during floods when other methods are not feasible.

Q.7 Briefly explain (i) Staff gauge (ii) Weight gauge (iii) float gauge (Dec 2013, July 2014)

Ans: Staff Gauge: The Staff Gage has a long history of visually providing a direct indicator for determining water level. The gages are designed with heavy metal grommets with a 0.2 inch opening for easy mounting to a wall or pier. The gage consists of a metal plate with accurately positioned markings. The metal plates are heavy 16 gage

(0.075 in / 1.9 mm) enameled iron or steel, which is completely covered with a baked-on porcelain enamel finish to resist rust or discoloration. Different colors of enamel are used to provide the markings; typically black numbers on a white background.

(ii) Weight Gauge: Wire-weight gages house a weight attached by wire cable to a graduated reel (gradations are tenths and hundredths of a foot) with a counter at one end. The weight should be lowered to touch the surface of the water (causing a slight ripple). At that position, the counter value should be recorded to the nearest whole number and the point indicated by the stylus on the graduated reel to the nearest hundredth of a foot. The wire-weight gage could be a movable type to accommodate braided streams. If the gage needs to be moved, use the correction value on the bridge near the repositioned gage location.

Q.8 A Pitot tube inserted in a pipe of 300mm diameter. The static pressure in pipe is 100mm of mercury (vacuum). The stagnation pressure at the centre of the pipe recorded by Pitot tube is 9.81KPa. Calculate the rate of flow of water through pipe. Take mean velocity as 0.85 times central velocityand C_v = 0.98.(Dec 2013,Jan 2015)

Ans: Given:

(i) D = 300mm = 0.3m $\Rightarrow$ Area of flow $A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07068\, m^2$

(ii) Static pressure head = 100mm of Vacuum = $-\frac{100}{1000} \times 13.6 = -1.36 m\ of\ water$

(ii) Stagnation Pressure Head = 9.81 kPa , $H_{stagnation} = \frac{p}{\gamma} = \frac{9.81}{9.81} = 1m\ \ of\ \ water$

H = (1.0-(-1.36)) = 2.36m

Velocity at the centre of pipe $V_c = C_V \sqrt{2 \times g \times H} = 0.98\sqrt{2 \times 9.81 \times 2.36} = 6.668\, m/s$

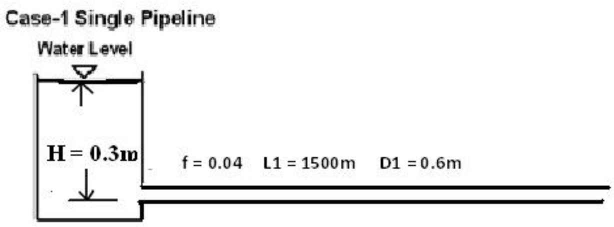
Mean Velocity V = 0.85×6.668 = 5.668 m/s

Rate of flow Q = A × V = 0.07068 × 5.668 = 0.4 m^3/s

Case**-1: When single pipe connects two reservoirs**

H = 0.3m, f = 0.04, L_1 = 1500m

D_1 = 0.6m $V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi}{4} \times D_1^2\right)} = \frac{Q}{\left(\frac{\pi}{4} \times 0.6^2\right)}$



$$H = \frac{4fL_1V_1^2}{2gD_1} = \frac{4fL_1Q^2}{2gD_1\left(\frac{\circledR}{4}\times D_1^2\right)} = \frac{32fL_1Q^2}{\circledR^2 gD_1^5}$$

$$H = 0.3 = \frac{32\times 0.04\times 1500\times Q^2}{\circledR^2\times g\times(0.2)^5}$$

$$Q = 0.0343\, m^3/\sec$$

Case-2: When Pipeline is branched midway for connecting two reservoirs

H = 20m, f = 0.015,

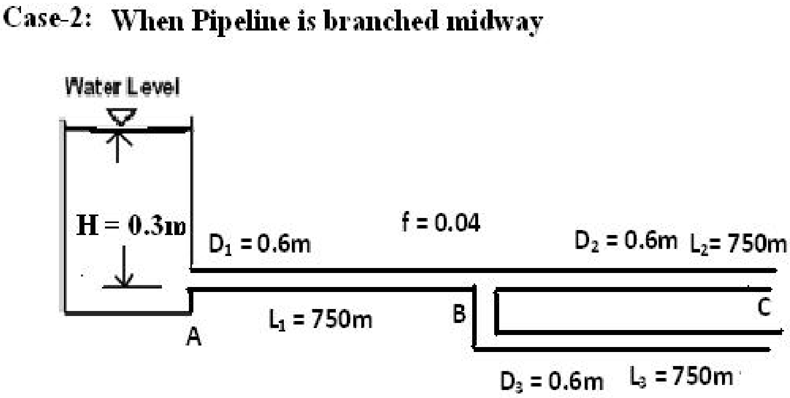
L_1 = 800m, D_1 = 0.2m

L_2 = 1200m, D_2 = 0.2m

L_3 = 1200m, D_3= 0.2m

$Q_1 = Q_2 + Q_3 = 2Q_2$

$Q_2 = \left(\frac{Q_1}{2}\right)$



$$H = \frac{4fL_1V_1^2}{2gD_1} + \frac{4fL_2V_2^2}{2gD_2}$$

$$20 = \frac{32fL_1Q_1^2}{\circledR^2 gD_1^5} + \frac{32fL_2Q_2^2}{\circledR^2 gD_2^5} = \frac{32\times 0.04\times 750\times Q_1^2}{\circledR^2\times g\times(0.6)^5} + \frac{32\times 0.04\times 750\times(0.5Q_1)^2}{\circledR^2\times g\times(0.6)^5}$$

$$Q_1 = 0.04338\, m^3/\sec$$

Increase in Discharge = (Q_1 – Q) = (0.04338-0.0343) = 0.00908m^3/sec

Q.9 The velocity of water in a 60cm diameter and 15mm thick cast iron pipe (E=1.04x10^{11} Pa) is changed from 3 m/s to zero in 1.25 s by closure of a valve i) if the pipe length is 800m what will be the water hammer pressure at the valve? What will be the corresponding pressure rise if the closure takes place in; ii) 2s and iii) 0.8s respectively? Bulk module of elasticity of water is 2.11 x 10^9 N/m^2. (July2013, July 2015)

Ans: Given: D = 60cm = 0.6m, L = 800m, t = 15mm = 0.015m

E = 1.04 × 10^{11} Pa, K = 2.11 × 10^9 N/m^2 , V = 3m/s

(i) Case-1 Time of Closure T_c = 1.25 sec

The celerity of wave $C = \sqrt{\frac{K}{\circ}} = \sqrt{\frac{2.11 \times 10^9}{1000}} = 1452.6 \quad m/s$

The ratio $\left(\frac{2 \times L}{C} = \frac{2 \times 800}{1452.6} = 1.1 \quad \text{sec}\right)$

The value of Time of Closure **T_c = 1.25 sec > 1.1 sec**

Hence GRADUAL ClOSURE OF VALVE

The value ofpressure risein pipeline due to gradual closure is given by,

$$p = \left(\frac{\circ \times L \times V}{T_c}\right) = \left(\frac{1000 \times 800 \times 3}{1.25}\right) = 1920 \quad kPa$$

(ii) Case-2 Time of Closure T_c = 2 sec

The value of Time of Closure **T_c = 2 sec > 1.1 sec**

Hence GRADUAL COSURE OF VALVE

The value ofpressure risein pipeline due to gradual closure is given by,

$$p = \left(\frac{\circ \times L \times V}{T_c}\right) = \left(\frac{1000 \times 800 \times 3}{2}\right) = 1200 \quad kPa$$

(iii) Case-3 Time of Closure T_c = 0.8 sec

The value of Time of Closure **T_c = 0.8 sec < 1.1 sec**

Hence INSTANTANEOUS COSURE OF VALVE

The value ofpressure risein pipeline due toinstantaneous closureis given by,

$$p = V\sqrt{\frac{\circ}{\left(\frac{1}{k} + \frac{D}{t \times E}\right)}} = 3\sqrt{\frac{1000}{\left(\frac{1}{2.11 \times 10^9} + \frac{0.6}{0.015 \times 1.04 \times 10^{11}}\right)}} = 3238\, kPa$$

Q. 10 Distinguish between small and large orifice. (July 2013)

Ans:

Based on size Small orifice (when the head over the orifice is more than five times its size i.e. H>5d, Large orifice

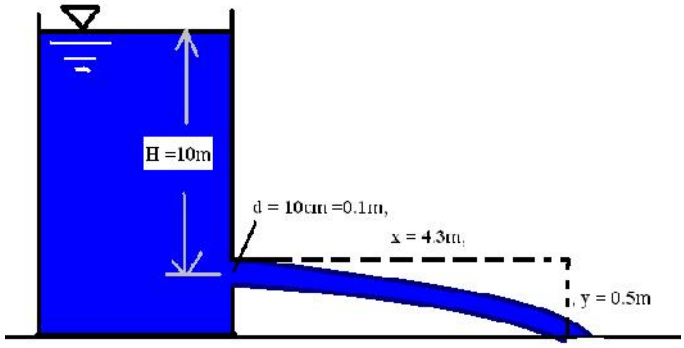
Q. 11 The head of water over an orifice of diameter 10cm is 10m. The water coming out fromorifice is collected in a circular tank of diameter 1.5m. The rise of water level in this tank is1.0m in 25 secs. Also the coordinate of a point on the jet measured from Vena Contracts is 4.3m horizontal and 0.5m vertical. Find the hydraulic co-efficient.

(July 2013, Jan 2015)

Ans: Given: d = 10cm =0.1m, x = 4.3m, H =10m, y = 0.5m

$Q_{act} = \frac{\left(\frac{\pi}{4} \times 1.5^2 \times 1.0\right)}{25} = 0.07069 \, \mathbf{m^3/sec}$,

$\mathbf{a_{jet}} = \frac{\pi}{4} \times (0.1)^2 = 0.007854\mathbf{m}^2$



$Q_{ideal} = 0.007854 \times \sqrt{2 \times 9.81 \times 10} = 0.11 \, \mathbf{m^3/s}$

$$\mathbf{C_d} = \frac{\mathbf{Q_{act}}}{\mathbf{Q_{ideal}}} = \frac{0.07069}{0.11} = 0.6426$$

$$\mathbf{C_V} = \frac{\mathbf{x}}{\sqrt{4 \times \mathbf{y} \times \mathbf{H}}} = \frac{4.3}{\sqrt{4 \times 0.5 \times 10}} = 0.9615$$

$$\mathbf{C_C} = \frac{\mathbf{C_d}}{\mathbf{C_V}} = \frac{0.6426}{0.9615} = 0.67$$

Q. 12 Distinguish between the following: (Dec 2013)

(i) Notch and Weir (ii) Coefficient of Velocity and Coefficient of contraction (iii) Float gauge and weight gauge (iv) Orifice and Venturimeter

Ans:

(i) Notch and Weir: Notch is sharp and carries less discharge as compared to weir

(ii) Coefficient of Velocity and Coefficient of contraction: Cv is near to 1.0 while typical Cc value is near 0.67

(iii) Float gauge and weight gauge: Float gauge records the depth based on floatation while the weight gauge fully submerge and gives the depth reading at a given location.

(iv) Orifice and Venturimeter: Orifice meter is a small insertion while venturimeter is long insertion. The typical Cd values for orifice meter is 0.68 while for venturimeter it is around 0.9. Orifice meter is cheaper as compared to venturimeter.

Q.13 A Pitot tube was used to measure the velocity of water at the centre of a 25cm diameter pipe. The stagnation and static pressure heads are indicated as 6m and 5m of water head. Given the coefficient of velocity C_v= 0.98. Determine the velocity at the centre of the pipe (Dec 2013, July 2014)

Ans: Given $C_v = 0.98$, $h_{stagnation} = 6m$, $h_{static} = 5m$

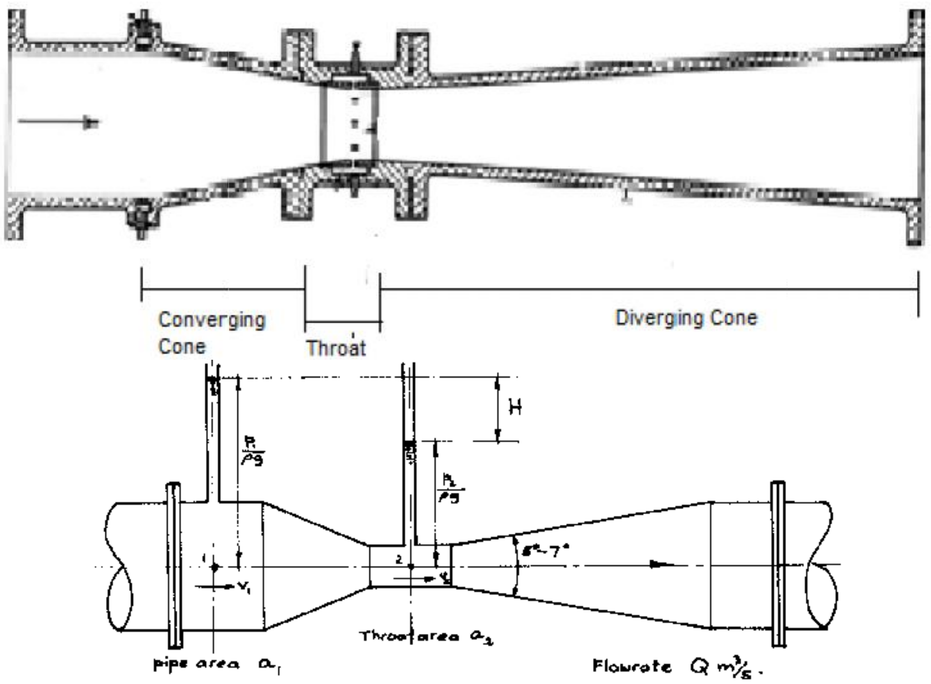
h= (6-5) = 1m

The velocity through Pitot tube $V = C_V \times \sqrt{2 \times g \times h}$

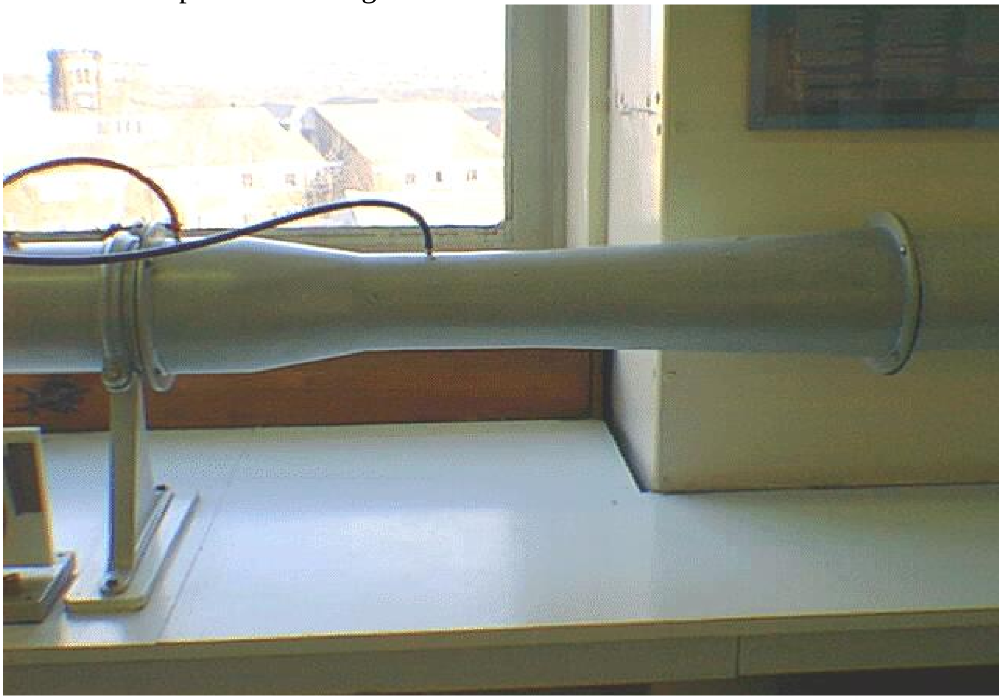
$$V = 0.98 \times \sqrt{2 \times 9.81 \times 1} = 4.34 m/s$$

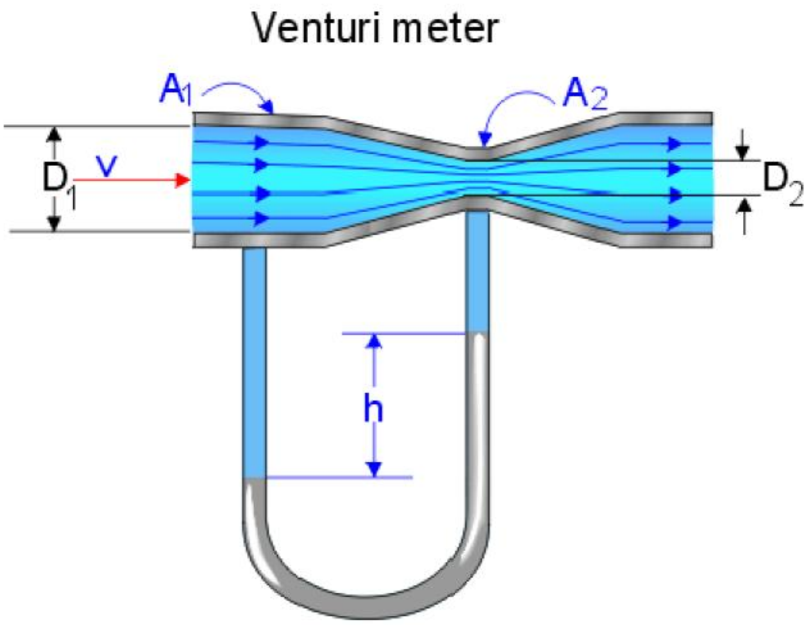
Q.14 Derive the equation for the discharge through Venturimeter (July 2014, July 2015)

Ans:Venturimeter: The venturimeter consists essentially of a convergence in a pipeline followed by a short parallel sided 'throat' and then a divergence.



In the venturi meter, the fluid is accelerated through a converging cone of angle 15-20° and the pressure difference between the upstream side of the cone and the throat is measured and provides the signal for the rate of flow.





Applying the Bernoulli equation along the streamline from point 1 to point 2 in the narrow *throat* of the Venturi meter, we have

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

By using the continuity equation we can eliminate the velocity V_2, $Q = A_1V_1 = A_2V_2$ or $V_2 = A_1V_1 / A_2$.

Substituting this into and rearranging the Bernoulli equation we get

$$V_1 = \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{(A_1/A_2)^2 - 1}}$$

To get the theoretical discharge this is multiplied by the area. To get the actual discharge taking in to account the losses due to friction, we include a coefficient of discharge

$$Q_{\text{ideal}} = A_1V_1; \qquad Q_{\text{actual}} = C_dQ_{\text{ideal}} = C_dA_1V_1 = C_dA_1\sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{(A_1/A_2)^2 - 1}}$$

Suppose a differential manometer is connected between (1) and (2). Then the terms inside the square brackets can be related to the manometer reading h as given by

$$p_1 + \rho g z_1 = p_2 + \rho_{man}gh + \rho g(z_2 - h) \;\Rightarrow\; \frac{p_1 - p_2}{\rho g} + z_1 - z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

Thus the discharge can be expressed in terms of the manometer reading:

$$Q_{\text{actual}} = C_dA_1\sqrt{\frac{2gh\left[\frac{\rho_{man}}{\rho} - 1\right]}{(A_1/A_2)^2 - 1}}$$

Notice how this expression does not include any terms for the elevation or orientation (z_1 or z_2) of the Venturi meter. This means that the meter can be at any convenient angle to function.

Q.15.The following data were collected for a stream at a gauging station. Compute the discharge through the stream. Rating equation of current meter V = 0.3N + 0.05, N = revolutions per second, V = velocity in m/s. (July 2013)

Distance from one end of water surface (m)	Water depth 'd' (m)	Current meter immersion at					
		0.6 d		0.2 d		0.8 d	
		Rev.	S	Rev.	S	Rev.	S
3	1.4	12	50	-	-	-	-
6	3.3	-	-	38	52	23	55
9	5.0	-	-	40	58	30	54
12	9.0	-	-	48	60	34	58
15	5.4	-	-	34	52	30	50
18	3.8	-	-	35	52	30	54
21	1.8	18	50	-	-	-	-

Ans: The average velocity is calculated based on Velocity at 0.6d i.e. $V_{0.6d} = V_{av}$

Or

The average velocity is calculated based on $V_{av} = \left(\dfrac{V_{0.2d} + V_{0.8d}}{2}\right)$

Distance (m)	Water Depth (d) in m	Area **(m²)**	Current meter Velocity V = 0.3N + 0.05 in m/s			V_{av} (m/s0	Discharge (Q) **(m³/s)**
			0.6d	0.2d	0.8d		
3	1.4	4.2	V = 0.3(12/50)+0.05 = **0.122**			0.122	0.5124
6	3.3	9.9		0.2692	0.1745	0.2219	2.1968
9	5.0	15.0		0.2900	0.2167	0.2534	3.8010
12	9.0	27.0		0.2900	0.2000	0.2450	6.6150
15	5.4	16.2		0.2462	0.2300	0.2381	3.8572
18	3.8	11.4		0.2519	0.2167	0.2343	2.6710
21	1.8	5.4	V = 0.3(18/50)+0.05 = **0.158**			0.1580	0.8532
The Total Discharge through the stream (m³/s)							**20.507**

The Total Discharge through the stream (Q) = 20.507 m^3/s

UNIT VIII: DISCHARGE MEASUREMENTS

Q. 1 Distinguish between:

- **i) Sharp crested and broad crested weirs**
- **ii) Orifice and mouth piece**
- **iii) Broad crested weir and submerged weir**

(July 2014, Jan 2015)

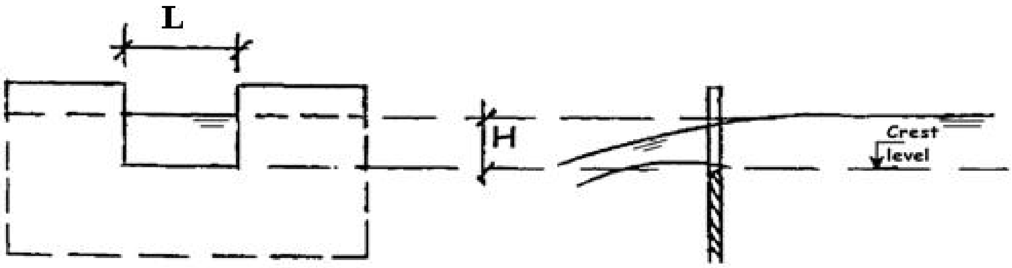
Ans:

Sharp crested Weir	Broad crested weirs
Crest is sharp, it carries relatively less discharge, affected by silt	Crest is broad, it carries relatively largeflow, affected by silt, and debris
Orifice	Mouth piece
A hole is provided at the side of a tank and vena contracta is formed outside the orifice	A pipe piece is provided projecting either outside or inside of a tank and efficient in handling flows
Broad crested weir	Submerged weir
Crest is broad, it carries relatively largeflow, affected by silt, and debris. Used for river flow measurements	Weir is fully submerged and discharge equation need to modify based on experimental results. Error in measurement increases as submergence level increases above weir top level

Q.2Derive an expression for discharge over a rectangular notch

(July 2013 ,July 2015)

Ans: Flow over a Rectangular Notch



Rectangular Notch/Weir

L=length of the notch, H=head over the notch,

Consider a small strip of thickness ‘dh’ at a depth h below the liquid surface

Discharge through the strip $dq = Area \times Velocity$

$$dq = Lxdhx\sqrt{2gh}$$

Total discharge $\int_0^Q dq = L\sqrt{2g}\int_0^H h^{\frac{1}{2}}dh$

$$Q = L\sqrt{2g}\,\frac{2}{3}h^{\frac{3}{2}}\int_0^H$$

$$\therefore Q = \frac{2}{3}L\sqrt{2g}H^{\frac{3}{2}}$$

$$Q_{act} = \frac{2}{3}C_d\sqrt{2g}LH^{\frac{3}{2}} ---(1)$$

C_d=Coefficient of discharge, its average value is about 0.62.

End Contraction

When the length of the weir (L) is less than the width of the channel (B), the nappe contracts at the sides, and this is knows as end contractions.(fig34)

According to Francis, the effective length of flow over the notch is given by

Substituting this value in EQ(1) and simplifying

$$Q = \frac{2}{3}C_d\sqrt{2g}\left(L - 0.1nH\right)H\frac{3}{2} ---(2)$$

A notch without end contraction is known as a suppressed notch.

Velocity of approach (V_a)

The total head over the weir will be the sum of static head (H) and velocity head (h_a), velocity head $h_a = \frac{V_a^2}{2a}$ is due to the Velocity of the liquid approaching the notch.

On similar lines, considering a strip of uniform thickness dh at a depth h below the liquid surface.

Discharge through the strip dq=area x velocity. $dQ = Lxdhx\sqrt{2g\left(H+h_a\right)}$

Therefore Total discharge is given by

$$\int_0^Q dq = L\sqrt{2g}\int_0^H \left(H+h_a\right)^{\frac{1}{2}} dh$$

$$= L\sqrt{2g}\,\frac{2}{3}\left\{\left(H+ha\right)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right\}$$

$$Q = \frac{2}{3}C_d\sqrt{2g}L\left\{\left(H+h_a\right)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right\}$$

Empirical Formula

(i)Francis Formula $Q = 1.84\left(L-0.1nH\right)\left[\left(H+ha\right)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right]$

Q3.Water flows over a rectangular weir 1m wide at a depth of 15cm and afterwards passes through a right angled weir. Taking C_d for rectangular weir 0.62 and for triangular 0.59 calculate the depth over the triangular weir (July2014,July 2015)

Ans: Given

- For Rectangular Weir Cd = 0.62, H = 15cm, L = 1m

- For Triangular Weir Cd = 0.59, θ = 90°

Let the depth over triangular weir be 'H_1'.

As the same discharge passes through both the weirs equating the discharge for Rectangular and triangular weir

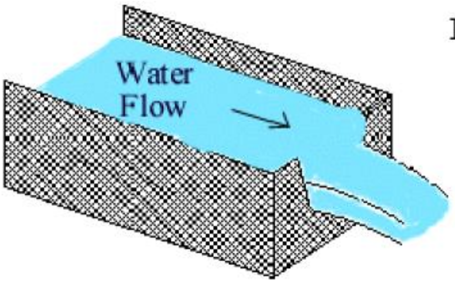
$$Q = \frac{2}{3}\times\left(C_d\right)_{Rectangular}\times L\times\sqrt{2\times g}\times H^{3/2} = \frac{8}{15}\times\left(C_d\right)_{Triangular}\times\sqrt{2\times g}\times\tan\left(\frac{-}{2}\right)\times\left(H_1\right)^{5/2}$$

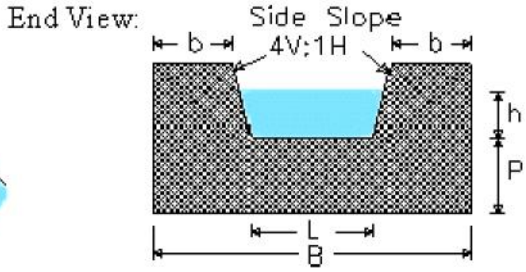
$$Q = \frac{2}{3}\times 0.62\times 1\times\sqrt{2\times 9.81}\times\left(0.15\right)^{3/2} = \frac{8}{15}\times 0.59\times\sqrt{2\times 9.81}\times\tan\left(\frac{90^{\circ}}{2}\right)\times\left(H_1\right)^{5/2}$$

$$H_1 = 0.3573m$$

Q. 4 With the help of neat sketches explain (i) Cipolletti Notch and (ii) Ogee Weir (Dec 2013, July 2015)

Ans:(i) Cipolleti Weirs: The Cipolletti or Trapezoidal Sharp-edge Weir is similar to a rectangular weir with end contractions except that the sides incline outwardly at a slope of 1 horizontal to 4 vertical. This slope causes the discharge to occur essentially as though it were without end contraction. The advantage of this weir is that no correction for end contraction is required. A disadvantage is that measurement accuracy is inherently less than that obtainable with a rectangular suppressed or V-notch weir. The Cipolletti Weir is commonly used in irrigation systems.





The discharge formula for this type of weir was given by Cipoletti as:

$$Q = 1.86\ L\ H^{3/2}$$

Where 'Q' is the discharge in m^3/s; 'L' is the length of the crest in meters; and 'H' is the head in meters. The discharge measurements using the above formula for the trapezoidal weir are not as accurate as those obtained from rectangular weirs using the Francis formula.

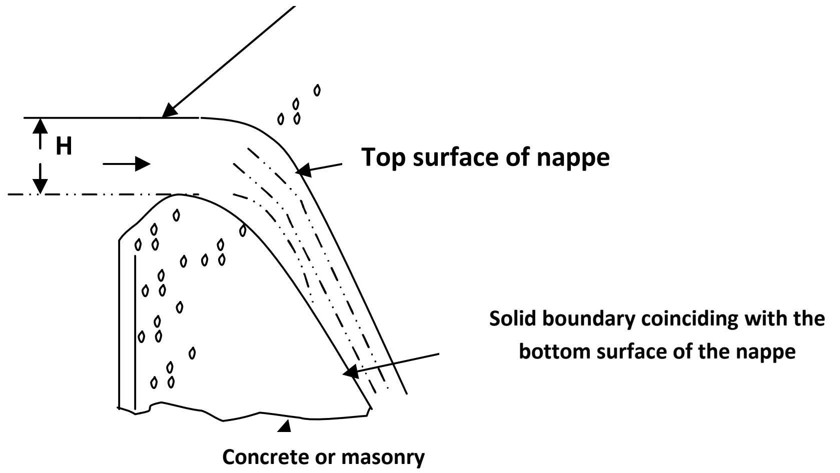
OGEE WEIR

When the weir is suppressed and its height is large, the nappe emerging out may be subjected to the problems of ventilation. Hence, in such cases the weir profile downstream is constructed conforming to the shape of the lower side of the nappe. Such a weir is known as a spillway or ogee weir.

The cross section of an ogee weir is shown. The coordinates of the spillway profile can be worked out for the head H using the equation. $x^{1.85} = 2H^{0.85}y$

The u/s face of the spillway is generally kept vertical. The discharge equation for an ogee weir will be $Q = \frac{2}{3}C_d\sqrt{2g}LH^{\frac{3}{2}}$

Same as that for a suppressed rectangular notch



Q. 5 Derive an expression for discharge through a triangular notch. (Dec 2013, July 2014, Jan 2015)

Ans: Flow through a Triangular Notch:

A sharp edged triangular notch with an included angle of θ is shown in Figure 4

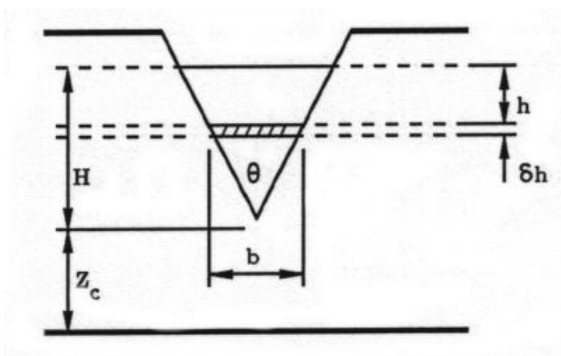


Figure 4 Triangular or V Notch

Again consider an element of height δh at a depth h

Breadth of element $b = 2(H-h)\tan \theta/2$

Hence area of element $a = 2(H-h)\tan \theta/2 \ \delta h$

Velocity through element $V = \sqrt{2gh}$

Discharge through element $\delta \dot{Q} = 2(H-h)\tan\frac{\theta}{2}\sqrt{2gh}\,\delta h$

Integrating to obtain the total discharge between h = o and h = H

$$\dot{Q} = 2\tan{\theta}/{2}\sqrt{2g}\int_0^H \left[H h^{1/2} - h^{3/2}\right]dh$$

$$= 2\tan\frac{\theta}{2}\sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$\dot{Q} = \frac{8}{15}\tan\frac{\theta}{2}\sqrt{2g}\,H^{5/2}$$

Again, a coefficient of discharge C_d has to be introduced.

Actual discharge $\dot{Q} = C_d\frac{8}{15}\tan\frac{\theta}{2}.\sqrt{2g}.H^{5/2}$

The triangular notch has advantages over the rectangular notch since the shape of the nappe does not change with head so that the coefficient of discharge does not vary so much. A triangular notch can also accommodate a wide range of flow rates.

Q. 6A rectangular notch of crest width 400mm is used to measure flow of water in a rectangular channel 600mm wide and 450mm deep. If the water level in the channel is 225mm above the weir crest, find the discharge in the channel. For the notch assume Cd = 0.63 and take velocity approach into account (Dec 2014)

Ans: Initially neglecting velocity of approach

$$Q = \frac{2}{3}\times C_d\times\sqrt{2g}\left[L - 0.1\times n\times H\right]H^{3/2}$$

$$Q = \frac{2}{3}\times 0.63\times\sqrt{2\times 9.81}\left[0.4 - 0.1\times 2\times 0.225\right](0.225)^{3/2}$$

Q = 0.0705 m^3/s

$V_a = \frac{Q}{A} = \frac{0.0705}{0.6\times 0.45} = 0.261\, m/s$

$$h_a = \frac{V_a^2}{2\times g} = \frac{0.261^2}{19.62} = 0.0035m$$

$$Q = \frac{2}{3}\times C_d \times \sqrt{2g}\left[L - 0.1\times n\times\left(H + h_a\right)\right]\left(H + h_a\right)^{3/2}$$

$$Q = \frac{2}{3}\times 0.63\times\sqrt{19.62}\left[0.4 - 0.1\times 2\times\left(0.225 + 0.0035\right)\right]\left(0.225 + 0.0035\right)^{3/2}$$

Q = 0.07186 m^3/s

New $V_a = \frac{Q}{A} = \frac{0.07186}{0.6\times 0.45} = 0.266\ m/s$

$$h_a = \frac{V_a^2}{2\times g} = \frac{0.266^2}{19.62} = 0.0036m$$

New Q = 0.0719 m^3/s

Q.7It is required to establish the throat diameter of a venturimeter in an installation of 100mm diameter pipe conveying water. The maximum range available in mercury-water differential manometer gauge is 50cm of mercury deflection. Find the maximum throat diameter which will indicate the fill gauge deflection when the flow rate is 20 LPS assuming coefficient of venturimeter as 0.984. (July 2014)

Ans: Given D_1 = 100mm = 0.1m, Q = 20 LPS = 0.02 m^3/s, C_d = 0.984

Deflection of mercury manometer deflection (x) = 50cm = 0.5m

The discharge through Orifice meter is given by,

$$\boldsymbol{Q} = \boldsymbol{C_d}\times\left(\frac{\boldsymbol{A}_1}{\sqrt{\left(\frac{\boldsymbol{A}_1}{\boldsymbol{A}_2}\right)^2 - 1}}\right)\times\sqrt{2\times\boldsymbol{g}\times\boldsymbol{h}} = 0.984\times\left(\frac{0.031416}{\sqrt{\left(\frac{\boldsymbol{D}_1}{\boldsymbol{D}_2}\right)^4 - 1}}\right)\times\sqrt{2\times 9.81\times\boldsymbol{h}}$$

Where, D_1 = 0.2m, $\boldsymbol{A}_1 = \frac{®}{4}\times\left(0.2\right)^2 = 0.031416\boldsymbol{m}^2$

$$\boldsymbol{h} = \boldsymbol{x}\left(\frac{\boldsymbol{S_m}}{\boldsymbol{S_f}} - 1\right) = 0.5\left(\frac{13.6}{1.0} - 1\right) = 6.3\boldsymbol{m}$$, Q = 0.02 m^3/s

$$0.02 = 0.984 \times \left(\frac{0.031416}{\sqrt{\left(\frac{0.2}{D_2}\right)^4 - 1}} \right) \times \sqrt{2 \times 9.81 \times 6.3}$$

D_2 = 0.04864m = 48.64 mm

Q.8 A discharge of 0.06 m^3/s was measured over a right angled notch. While measuring the head over the notch an error of 1.5mm was made. Determine the percentage error in discharge if the coefficient of discharge for the notch is 0.6 (July 2014)

Ans: Solution: Given Q = 0.06 m^3/s, θ = 90˚, C_d = 0.6
Error in Head measurement (dH) = 1.5mm = 0.0015m

Let 'H' be the height of water, above the apex of the notch

$$Q = \frac{8}{15} \times C_d \times \sqrt{2 \times 9.81} \times \tan\left(\frac{90°}{2}\right) \times H^{5/2}$$

$$0.06 = \frac{8}{15} \times 0.6 \times \sqrt{2 \times 9.81} \times \tan\left(\frac{90°}{2}\right) \times H^{5/2}$$

H = 0.282m

Using the relation,

$$\frac{dQ}{Q} = \frac{5}{2} \times \left(\frac{dH}{H}\right)$$

$$\frac{dQ}{Q} = \frac{5}{2} \times \left(\frac{0.0015}{0.282}\right) = 0.0133$$

Percentage error in discharge $\frac{dQ}{Q} = 0.0133 \times 100 = 1.33\%$

Q.9 What are the advantages of triangular notch over rectangular notch? (July 2013 , Jan 2014)

Ans:

ADVANTAGES OF TRIANGULAR NOTCH OR WEIR OVER RECTANGULAR NOTCH OR WEIR

A triangular notch or weir is preferred to a rectangular weir or notch due to following reasons

1. The expression for discharge for a right-angled V-notch or weir is very simple.
2. For measuring low discharge, a triangular notch gives more accurate results than rectangular weir or notch.
3. In case of triangular notch, only one reading, i.e., (H) is required for the computation of discharge.
4. Ventilation of a triangular notch is not necessary.

Q10. Define velocity approach. How does the velocity of approach affect the discharge over aweir? (July 2014 , Jan 2015)

Ans: It is the velocity of water which approaches just upstream of weir or notch. It is affected by the nappe drawdown and correction 'h_a'need to be applied based on velocity of head calculated with velocity of approach 'Va'

Velocity of approach (V_a)

The total head over the weir will be the sum of static head (H) and velocity head (h_a), velocity head $h_a = \frac{V_a^2}{2a}$ is due to the Velocity of the liquid approaching the notch.

On similar lines, considering a strip of uniform thickness 'dh' at a depth h below the liquid surface.

Discharge through the strip d_q=area x velocity. $dQ = Lxdhx\sqrt{2g\left(H+h_a\right)}$

Therefore Total discharge is given by

$$\int_0^Q dq = L\sqrt{2g}\int_0^H \left(H+h_a\right)^{\frac{1}{2}} dh$$

$$= L\sqrt{2g}\,\frac{2}{3}\left\{\left(H+ha\right)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right\}$$

$$Q = \frac{2}{3}C_d\sqrt{2g}L\left\{\left(H+h_a\right)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right\}$$

Empirical Formula

(i)Francis Formula $Q = 1.84\left(L-0.1nH\right)\left[\left(H+ha\right)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right]$

(ii) Bazin's formula $Q = \left(0.405 + \frac{0.003}{H}\right) L\sqrt{2g} H^{\frac{3}{2}}$

(iii)) Rehbock formula $Q = \left[0.403 + \frac{0.053(H + 0.011)}{Z}\right] L\sqrt{2g}\,(H + 0.0011)^{\frac{3}{2}}$

Considering velocity of approach and End contraction, we have

$$Q = \frac{2}{3} C_d \sqrt{2g}\,(L - 0.1nH)\left\{(H + h_a)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right\}$$

Considering velocity of approach and End contraction, we have

$$Q = \frac{2}{3} C_d \sqrt{2g}\,(L - 0.1nH)\left\{(H + h_a)^{\frac{3}{2}} - ha^{\frac{3}{2}}\right\}$$

Q 11A rectangular channel 2m wide has a discharge of 250 LPS which is measured by a right-angled V-notch. Find the position of the notch from the bed of the channel if maximum depthof water is not to exceed 1.3m.Take C_d = 0.62. (July 2013 , july 2015)

Ans: Given: Q = 250 LPS = 0.25m^3/s, C_d = 0.62, θ = 90°,

Maximum depth in the channel not to exceed 1.3m

The discharge through a right angled triangular notch is given by,

$$\mathbf{Q} = \frac{8}{15} \times \mathbf{C_d} \times \sqrt{2 \times \mathbf{g}} \times \mathbf{tan}\left(\frac{\theta}{2}\right) \times \mathbf{H}^{5/2}$$

$$0.25 = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \times \mathbf{tan}\left(\frac{90^{\circ}}{2}\right) \times \mathbf{H}^{5/2}$$

$$\mathbf{H} = 0.5\mathbf{m}$$

The position of apex of the from the bed of channel = Maximum Depth of water in channel – Height of water over V-notch notch

The position of apex of the from the bed of channel = (1.3 – 0.5) **= 0.8m**

Q.12 Explain Cipolleti notch with a neat sketch and mention its advantages(Dec 2014)

Ans:

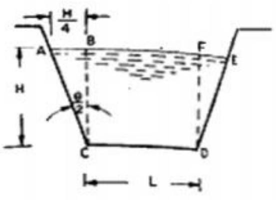
CIPOLLETTI WEIR

Cipolletti weir is a trapezoidal weir, which has side slopes of 1 horizontal to 4 vertical as shown in Fig.

Thus in triangle ABC

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{H/4}{H} = \frac{1}{4}$$

$$\therefore \quad \frac{\theta}{2} = \tan\frac{1}{4} = 14^\circ\, 2'.$$



By giving this slopes to the sides, an increase in discharge through the triangular portions *ABC* and *DEF* of the weir is obtained.

If this slope is not provided the weir would be a rectangular one, and due to end contraction, the discharge would decrease. Thus in case of cipolletti weir, the factor of end contraction is not required which is shown below.

The discharge through a rectangular weir with two end contractions is

$$Q = \frac{2}{3} \times C_d \times (L - 0.2\,H)\sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g}\, H^{3/2} - \frac{2}{15} C_d \times \sqrt{2g} \times H^{5/2}$$

Thus due to end contraction, the discharge decreases by

$$\frac{2}{15} C_d \times \sqrt{2g} \times H^{5/2}$$

This decrease in discharge can be compensated by giving such a slope to the sides that the discharge through two triangular portions is equal to

$$\frac{2}{15} C_d \times \sqrt{2g} \times H^{5/2}$$

Let the slope is given by θ/2. The discharge through a V-notch of angle θ is given by

$$= \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} H^{5/2}$$

Thus $$\frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} H^{5/2} = \frac{2}{15} \times C_d \times \sqrt{2g} \times H^{5/2}$$

$$\therefore \quad \tan\frac{\theta}{2} = \frac{2}{15} \times \frac{15}{8} = \frac{1}{4} \quad \text{or} \quad \theta/2 = \tan^{-1}\frac{1}{4} = 14^\circ\, 2'.$$

Thus discharge through cipolletti weir is

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\, H^{3/2}$$

If velocity of approach, V_a is to be taken into consideration,

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\, [(H + h_a)^{3/2} - h_a^{3/2}]$$

Q13Distinguish between Cipolleti notch and Ogee weir (Dec 2014,jan 2013)

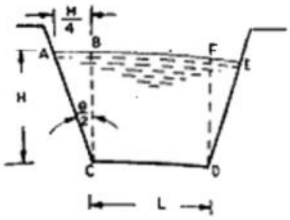
CIPOLLETTI WEIR

Cipolletti weir is a trapezoidal weir, which has side slopes of 1 horizontal to 4 vertical as shown in Fig.

Thus in triangle ABC

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{H/4}{H} = \frac{1}{4}$$

$$\therefore \quad \frac{\theta}{2} = \tan\frac{1}{4} = 14° 2'.$$



By giving this slopes to the sides, an increase in discharge through the triangular portions *ABC* and *DEF* of the weir is obtained.

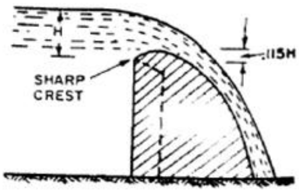
If this slope is not provided the weir would be a rectangular one, and due to end contraction, the discharge would decrease. Thus in case of cipolletti weir, the factor of end contraction is not required which is shown below.

The discharge through a rectangular weir with two end contractions is

$$Q = \frac{2}{3} \times C_d \times (L - 0.2\,H)\sqrt{2g} \times H^{3/2}$$
$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g}\, H^{3/2} - \frac{2}{15} C_d \times \sqrt{2g} \times H^{5/2}$$

DISCHARGE OVER AN OGEE WEIR

Below fig shows an Ogee weir, in which the crest of the weir rises up to maximum height of 0.115 x *H* (where *H* is the height of water above inlet of the weir) and then falls as shown in Fig.



The discharge **for** an Ogee weir is the same as that of a rectangular weir, and it is given by

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2}$$

Q. 14 A rectangular notch of crest width of 10m X 2.5m depth is used to measure flow of water in a rectangular channel. If the water level in the channel is 2m above the weir crest, find the discharge in the channel. For the notch assume C_d = 0.6 and take velocity of approach into account (Dec 2013, jan 2015)

Ans: Given - H = 2m, L = 10m, C_d = 0.6

Without considering velocity of approach

$$Q = C_d \times \frac{2}{3} \times L \times \sqrt{2 \times g} \times H^{3/2}$$ (Without end contraction)

$$Q = 0.6 \times \frac{2}{3} \times 10 \times \sqrt{2 \times 9.81} \times 2^{3/2} = 50.11\, m^3 / s$$

Velocity of approach $V_a = \frac{Q}{A} = \frac{50.11}{(20 \times 2.5)}$ =1 m/s

Approach Head $H_a = \frac{V_a^2}{2 \times g} = \frac{1^2}{(2 \times 9.81)} = 0.051m$

Discharge equation considering end contraction

$$Q = C_d \times \frac{2}{3} \times L \times \sqrt{2 \times g} \times \left([(H + H_a]^{3/2} - H_a^{3/2} \right)$$

$$Q = 0.6 \times \frac{2}{3} \times 10 \times \sqrt{2 \times 9.81} \times \left([(2 + 0.051]^{3/2} - (0.051)^{3/2} \right) = 51.81 m^3/s$$