

LECTURE NOTES

SUB:- MACHINE DESIGN-I (BME 310)

MODULE-II (DESIGN OF JOINTS)

Riveted Joint

Developed by Dr. J. R. Mohanty

Often small machine components are joined together to form a larger machine part. Design of joints is as important as that of machine components because a weak joint may spoil the utility of a carefully designed machine part. Mechanical joints are broadly classified into two classes viz., non-permanent joints and permanent joints. Non-permanent joints can be assembled and disassembled without damaging the components. Examples of such joints are threaded fasteners (like screw-joints), keys and couplings etc.

Permanent joints cannot be disassembled without damaging the components. These joints can be of two kinds depending upon the nature of force that holds the two parts. The force can be of mechanical origin, for example, riveted joints, joints formed by press or interference fit etc, where two components are joined by applying mechanical force. The components can also be joined by molecular force, for example, welded joints, brazed joints, joints with adhesives etc. Not until long ago riveted joints were very often used to join structural members permanently. However, significant improvement in welding and bolted joints has curtailed the use of these joints. Even then, rivets are used in structures, ship body, bridge, tanks and shells, where high joint strength is required.

Rivets and riveting

A Rivet is a short cylindrical rod having a head and a tapered tail. The main body of the rivet is called shank (see figure 2.1).

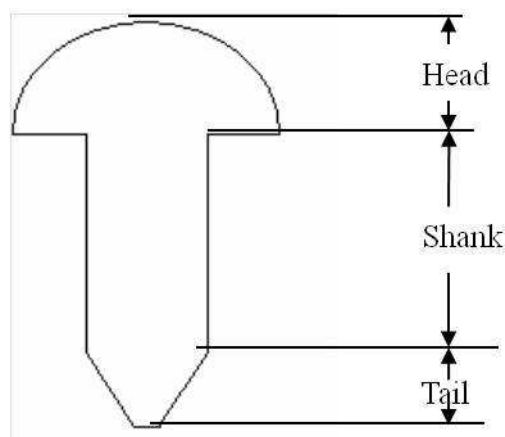


Fig. 2.1 Rivets and its parts

According to Indian standard specifications rivet heads are of various types. Rivets heads for general purposes are specified by Indian standards IS: 2155-1982 (below 12 mm diameter) and IS: 1929-1982 (from 12 mm to 48 mm diameter). Rivet heads used for boiler works are specified by IS: 1928-1978. To get dimensions of the heads see any machine design handbook.

Riveting is an operation whereby two plates are joined with the help of a rivet. Adequate mechanical force is applied to make the joint strong and leak proof. Smooth holes are drilled (or punched and reamed) in two plates to be joined and the rivet is inserted. Holding, then, the head by means of a backing up bar as shown in figure 2.2, necessary force is applied at the tail end with a die until the tail deforms plastically to the required shape. Depending upon whether the rivet is initially heated or not, the riveting operation can be of two types: (a) cold riveting is done at ambient temperature and (b) hot riveting rivets are initially heated before applying force. After riveting is done, the joint is heat-treated by quenching and tempering. In order to ensure leak-proofness of the joints, when it is required, additional operation like caulking is done.

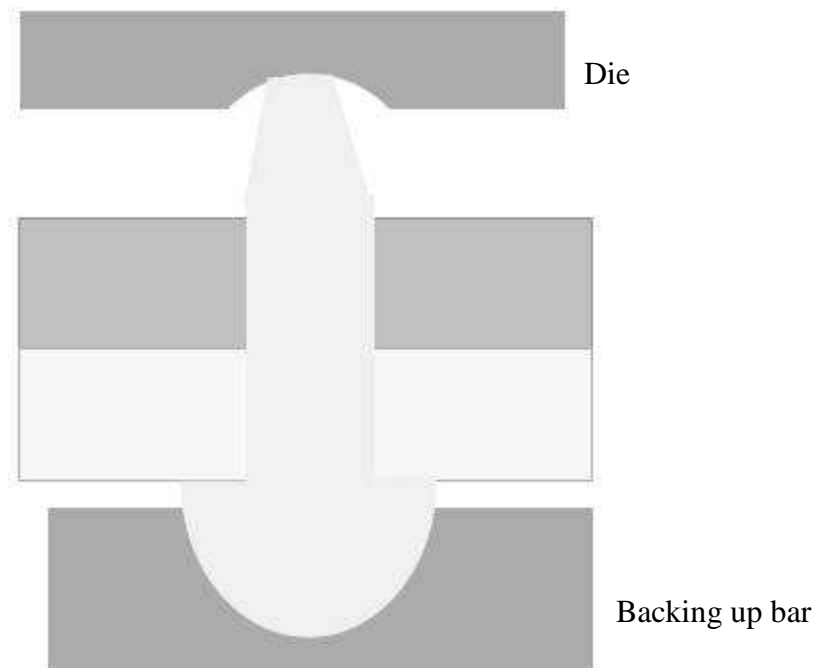


Fig. 2.2 Riveting operation

Types of rivet joints

Riveted joints are mainly of two types

1. Lap joints
2. Butt joints

Lap joints

The plates that are to be joined are brought face to face such that an overlap exists, as shown in figure 10.1.3. Rivets are inserted on the overlapping portion. Single or multiple rows of rivets are used to give strength to the joint. Depending upon the number of rows the riveted joints may be classified as single riveted lap joint, double or triple riveted lap joint etc. When multiple joints are used, the arrangement of rivets between two neighbouring rows may be of two kinds. In chain riveting the adjacent rows have rivets in the same transverse line. In zig-zag riveting, on the other hand, the adjacent rows of rivets are staggered.

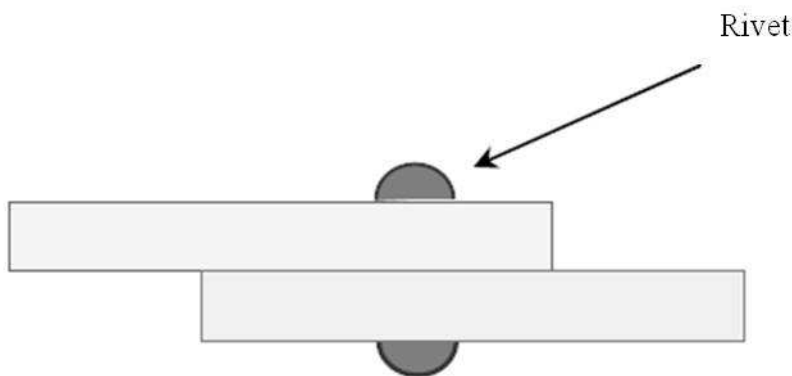


Fig. 2.3 Lap joint

But joints

In this type of joint, the plates are brought to each other without forming any overlap. Riveted joints are formed between each of the plates and one or two cover plates. Depending upon the number of cover plates the butt joints may be single strap or double strap butt joints. A single strap butt joint is shown in figure 2.4. Like lap joints, the arrangement of the rivets may be of various kinds, namely, single row, double or triple chain or zigzag.

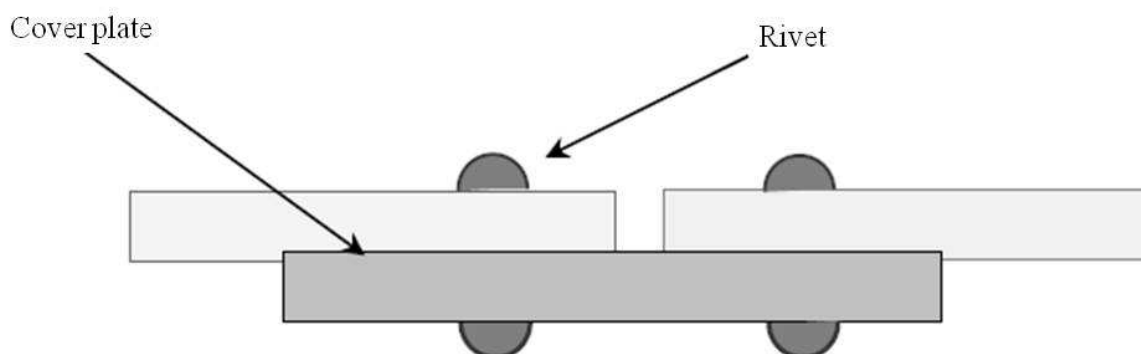


Fig. 2.4 Butt joint

Important terms used in rivet joints

Few parameters, which are required to specify arrangement of rivets in a riveted joint are as follows:

- a) *Pitch*: This is the distance between two centers of the consecutive rivets in a single row. (usual symbol p)
- b) *Back Pitch*: This is the shortest distance between two successive rows in a multiple riveted joint. (usual symbol b_p)
- c) *Diagonal pitch*: This is the distance between the centers of rivets in adjacent rows of zigzag riveted joint. (usual symbol p_d)
- d) *Margin or marginal pitch*: This is the distance between the centre of the rivet hole to the nearest edge of the plate. (usual symbol m)

These parameters are shown in figure 2.5.

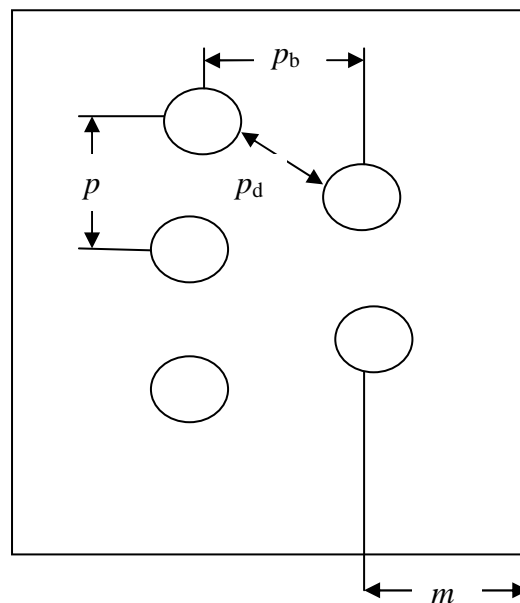


Fig. 2.5 Important design parameters of riveted joints

Modes of failure of rivet joints

- (1) *Tearing of the plate at the edge*: Figure 2.6 shows the nature of failure due to tearing of the plate at the edge.

Such a failure occurs due to insufficient margin. This type of failure can be avoided by keeping margin, $m = 1.5d$, where d is the diameter of the rivet.

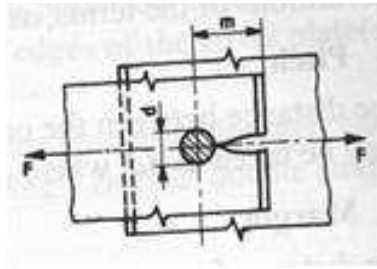


Fig. 2.6 Tearing of the plate at the edge

(2) *Tearing of the plate across a row of rivets:* In this, the main plate or cover plates may tear-off across a row of rivets, as shown in Fig. 2.7. Considering one pitch length,

Tearing strength per pitch length, $F_t = \sigma_t (p - d)t$ (2.1)

Where, σ_t = permissible tensile stress for the plate material; p = pitch; d = diameter of the rivet; t = thickness of the plate.

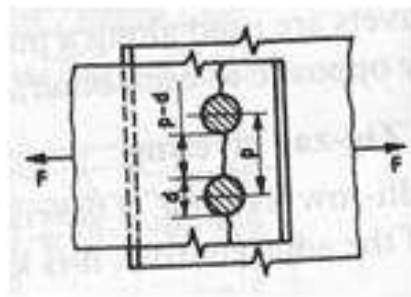
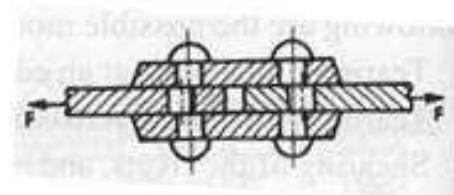
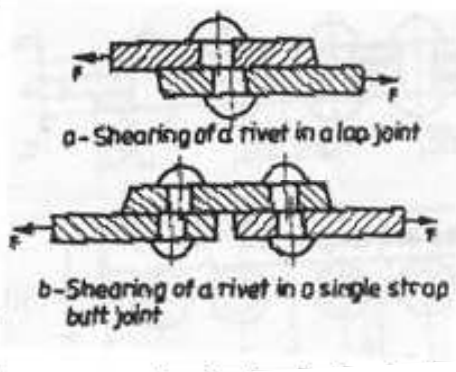


Fig. 2.7 Tearing of the plate across a row of rivets

(3) *Shearing of rivets:* Rivets are in single shear (Fig. 2.8a) in lap joints and in double shear in double strap butt joints (Fig. 2.8b). Considering one pitch length,



(b)

(a)

Fig. 2.8 Shearing of rivets

$$\text{Shearing resistance per pitch length, } F_s = \frac{\pi d^2}{4} n \tau \text{ in single shear} \quad (2.2)$$

$$= 2 \times \frac{\pi d^2}{4} \times n \tau \text{ in double shear} \quad (2.3)$$

Where, n = number of rivets per pitch length

(4) *Crushing of rivets (plates)*: When the joint is loaded, compressive stress is induced over the contact area between rivet and the plate (Fig. 2.9).

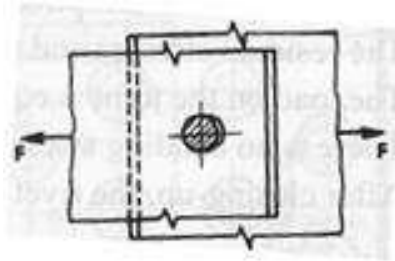


Fig. 2.9 Crushing of a rivets

The contact area is given by the projected area of the contact. Considering one pitch length,

$$\text{Crushing resistance per pitch length, } F_c = n d t \sigma_c \quad (2.4)$$

Where, n = number of rivets per pitch length; σ_c = permissible compressive stress.

Note: Number of rivets under crushing is equal to the number of rivets under shear.

Efficiency of a riveted joint

The efficiency of a riveted joint is defined as the ratio of the strength of the joint (least of calculated resistances) to the strength of the solid plate.

$$\text{Efficiency of a riveted joint, } \eta = \frac{F_t, F_s, \text{ or } F_c \text{ (least)}}{p t \sigma_t} \quad (2.5)$$

Where, ' $p t \sigma_t$ ' is the strength of the solid plate per pitch length.

Design of boiler joints

In general, for longitudinal joint, butt joint is adopted, while for circumferential joint, lap joint is preferred.

Design of longitudinal butt joint

1. *Thickness of the plate*: The thickness of the boiler shell is determined, by using thin cylinder formula, i.e.

$$t = \frac{P_i d_i}{2 \sigma_t \eta} + 1 \text{ mm} \quad (2.6)$$

Where, 1 mm is the allowance for corrosion; P_i = internal steam pressure; d_i = internal diameter of the boiler shell; σ_t = permissible stress of the shell material.

2. *Diameter of rivets*: The diameter of the rivets may be determined from the empirical relation, $d = 6\sqrt{t}$ for $t \geq 8\text{mm}$

Note: (1) The diameter of rivet should not be less than the plate thickness.

(2) If the plate thickness is less than 8 mm, the diameter of the rivet is determined by equating the shearing resistance of the rivet to its crushing resistance.

3. *Pitch of the rivets*: The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. However, it should be noted that,

- (i) the pitch of the rivets should not be less than $2d$.
- (ii) the maximum value of the pitch, for a longitudinal joint is given by,

$$P_{\max} = ct + 41.28\text{mm} \text{ where 'c' is a constant.}$$

Note: If the pitch of the rivets obtained by equating the tearing resistance to the shearing resistance is more than P_{\max} , then the value of P_{\max} can be adopted.

4. *Row (transverse) pitch*:

- (i) For equal number of rivets in more than one row for lap joint or butt joint, the row pitch should not be less than, $0.33p + 0.67d$ for zig-zag riveting and $2d$, for chain riveting.
- (ii) For joints in which the number of rivets in the rows is half the number of rivets in the inner rows, and if the inner rows are chain riveted, the distance between the outer row and the next row, should not be less than, $0.33p + 0.67d$ or $2d$, whichever is greater. The distance between the rows in which there are full number of rivets, should not be less than $2d$.
- (iii) For joints in which the number of rivets in outer row is half the number of rivets in inner rows, and if the inner rows are zig-zag riveted, the distance between the outer row and the next row, should not be less than, $0.2p + 1.15d$. The distance between the rows in which there are full number of rivets (zig-zag), should not be less than, $0.165p + 0.67d$.

Note: p is the pitch of the rivets in the outer row.

5. *Thickness of butt straps*: The thickness of butt strap(s) is given by, (in no case it should not be less than 10 mm).

$$t_1 = 1.125t, \text{ for ordinary single butt strap (chain riveting)}$$

$$= 1.125t \left(\frac{p-d}{p-2d} \right), \text{ for a single butt strap, with alternate rivets in the outer rows}$$

omitted

$$= 0.625t, \text{ for ordinary double straps of equal width (chain riveting)}$$

$$= 0.625t \left(\frac{p-d}{p-2d} \right), \text{ for double straps of equal width, with alternate rivets in the}$$

outer rows omitted

When two unequal widths of butt straps are employed, the thickness of butt straps are given by, $t_1 = 0.75t$, for wide strap on the inside and $t_2 = 0.625t$, for narrow strap on the outside.

Note: The thickness of butt strap, in no case, shall be less than 10 mm.

6. *Margin*: The margin 'm' is generally followed as $1.5d$.

Design of circumferential lap joint

1. *Diameter of rivets*: It is usual practice to adopt the rivet diameter and plate thickness, same as those used for longitudinal joint.
2. *Number of rivets*: The rivets are in single shear, since lap joint is used for circumferential joint.

Total number of rivets to be used for the joint,

$$N = \frac{\text{steam load}}{\text{Shear strength of one}} = \frac{(\frac{\pi d_i^2}{4} \times p_i) / (\frac{\pi d^2}{4} \tau)}{1} = \left(\frac{d_i}{d} \right)^2 \times \frac{p_i}{\tau}$$

Where, d_i = inner diameter of boiler; d = rivet diameter; τ = allowable shear strength of rivet material

3. *Pitch of rivets*: In general, the efficiency of the circumferential joint may be taken as 50% of the tearing efficiency of the longitudinal joint. If intermediate circumferential joints are used, the strength of the seam should not be less than 62% of the strength of the undrilled plate. Knowing the (tearing) efficiency of the circumferential joint, the pitch of the rivets can be obtained from,

$$\text{Efficiency, } \eta = \frac{(p-d)t\sigma_t}{pt\sigma_t} = \frac{p-d}{p}$$

4. *Number of rows*: Number of rivets per row, $n = \frac{\pi(d_i + t)}{d}$

$$\text{Number of rows, } Z = \frac{\text{Total number of rivets}}{\text{No. of rivets per row}}$$

5. *Selection of the type of joint:* After determining the number of rows, the type of joint (single riveted, double riveted etc.) may be decided.
6. *Row (back) pitch and margin:* The proportions suggested for longitudinal joint, may be followed for the circumferential joint as well.

Example problem – 1: Design a triple riveted lap joint, to join two plates of 6 mm thick. The allowable stresses are: $\sigma_t = 80$ MPa, $\sigma_c = 100$ MPa, and $\tau = 60$ MPa. Calculate the rivet diameter, rivet pitch, and distance between the rows of rivets. Use zig-zag riveting. State how the joint will fail.

Solution: As the thickness of the plate is less than 8 mm, the diameter of the rivet may be determined by equating the shearing resistance to the crushing resistance. Further, as the joint is triple riveted zig-zag lap joint, there will be three rivets per pitch length (Fig. 2.10) and are under single shear, and same number of rivets under crushing.

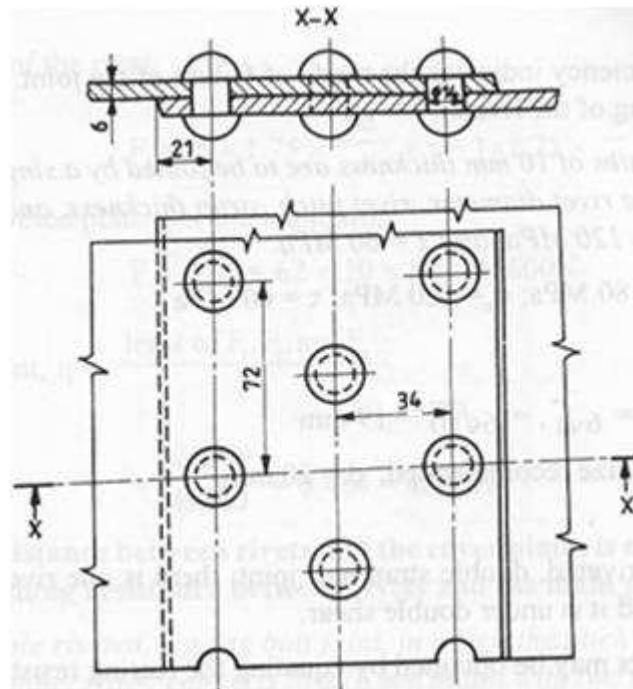


Fig. 2.10

$$\text{Shearing resistance, } F_s = \frac{\pi d^2}{4} n \tau = 3 \times \frac{\pi d^2}{4} \times 60 = 141.37 d^2 \quad (\text{i})$$

$$\text{Crushing resistance, } F_c = n d t \sigma_c = 3 \times d \times 6 \times 100 = 1800 d \quad (\text{ii})$$

Reference

Design
Data Book
by K.
Mahadevan
& K. B.
Reddy

Table-5.3b;
Page-68A;

Reference

<p>Equating equations (i) and (ii) we have, $141.37d^2 = 1800d$</p> $d = 12.73 \text{ mm}$ <p>The nearest standard diameter of the rivet recommended, $d = 14 \text{ mm}$</p> <p><i>Pitch of the rivets:</i> The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.</p> <p>Tearing resistance, $F_t = \sigma_t(p - d)t = (p - 14) \times 6 \times 80 \text{ N}$ (iii)</p> <p>Shearing resistance, $F_s = n \times \frac{\pi d^2}{4} \times \tau = 3 \times \frac{\pi 14^2}{4} \times 60 = 27708.8 \text{ N}$ (iv)</p> <p>Equating equations (i) and (ii) we have, $(p - 14) \times 480 = 27708.8$</p> $p = 71.73 \text{ mm, say } 72 \text{ mm.}$ <p>Distance between the rows of rivets, p_b (or p_t) = $0.33p + 0.67d = 33.14 \text{ mm}$, say 34 mm.</p> <p><i>Mode of failure of the joint:</i></p> <p>Tearing efficiency = $\frac{p - d}{p} = \frac{72 - 14}{72} = 0.801 = 80.1\%$</p> <p>Crushing efficiency = $\frac{ndt\sigma_c}{pt\sigma_t} = \frac{3 \times 14 \times 6 \times 100}{72 \times 6 \times 80} = 0.802 = 80.2\%$</p> <p>The lowest efficiency indicates the mode of failure of the joint. In the present case, the joint will fail by crushing of the rivets.</p>	<p>Page-66, Eq. 5.31b</p>
<p>Example problem – 2: A double riveted, zig-zag butt joint, in which the pitch of the rivets in the outer row is twice that in the inner rows; connects two 16 mm plates with two cover plates, each 12 mm thick. Determine the diameter of the rivets and pitch of the rivets, if the working stresses are: $\sigma_t = 100 \text{ MPa}$, $\sigma_c = 150 \text{ MPa}$, and $\tau = 75 \text{ MPa}$.</p> <p>Solution:</p> <p><i>Diameter of the rivet:</i></p> <p>Diameter of the rivet, $d = 6\sqrt{t} = 24 \text{ mm}$</p> <p><i>Pitch of the rivets:</i></p> <p>Let p_o = pitch of the rivets in the outer row</p> <p>P_i = pitch of the rivets in the inner row</p>	<p>Reference</p> <p>Design Data Book by K. Mahadevan & K. B. Reddy</p>

The pitch of the rivets may be obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

Referring Fig. 2.11, since the pitch in the outer row is twice the pitch of inner row; for one pitch length in the outer row, there are three rivets, which are under double shear.

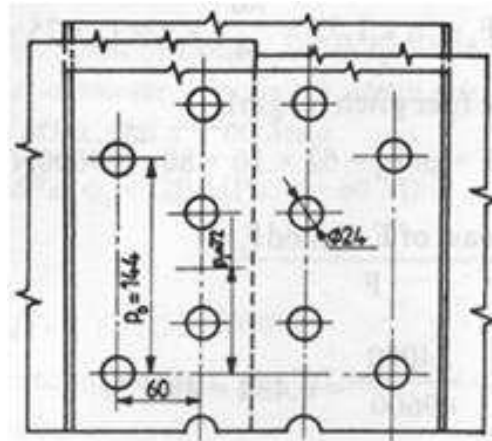


Fig. 2.11- Double riveted, double strap, zig-zag butt joint

Tearing resistance, $F_t = (p - d)t\sigma_t = (p_o - 14) \times 16 \times 100 = (p_o - 24) \times 1600$ N
(i)

Shearing resistance, $F_s = n \times 1.875 \times \frac{\pi d^2}{4} \tau$, assuming that the rivets under double shear are 1.875 times as strong as those under single shear =
 $3 \times 1.875 \times \frac{\pi}{4} \times 24^2 \times 75 = 190851.8$ N
(ii)

Equating equations (i) and (ii) we have, $(p_o - 24) \times 1600 = 190851.8$

$$p_o = 143.3, \text{ say } 144 \text{ mm}$$

Pitch of rivets in the inner row, $p_i = \frac{p_o}{2} = \frac{144}{2} = 72$ mm

Distance between the rows of rivets:

For zig-zag riveting, the row (back) pitch, $p_b \geq 0.2p_o + 1.15d \geq 0.2 \times 144 + 1.15 \times 24 \geq 56.4$ mm.

A back/row pitch of 60 mm may be recommended.

Page-66,
Eq. 5.33a

Welded joints

Welded joints and their advantages:

Welding is a very commonly used permanent joining process. Thanks to great advancement in welding technology, it has secured a prominent place in manufacturing machine components. A welded joint has following advantages:

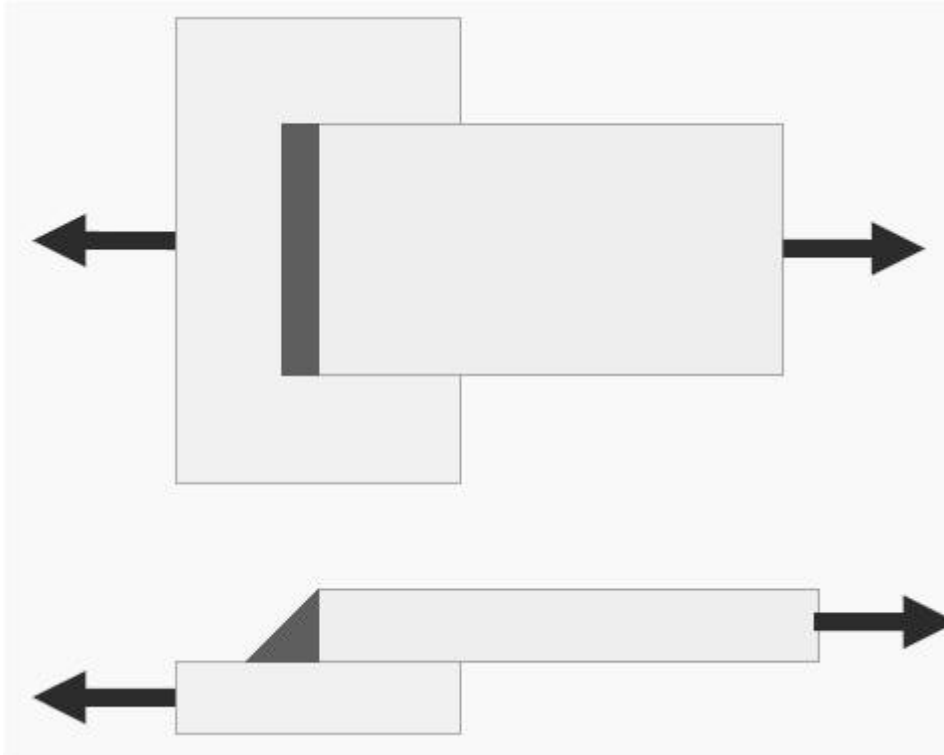
- (i) Compared to other type of joints, the welded joint has higher efficiency. An efficiency $> 95\%$ is easily possible.
- (ii) Since the added material is minimum, the joint has lighter weight.
- (iii) Welded joints have smooth appearances.
- (iv) Due to flexibility in the welding procedure, alteration and addition are possible.
- (v) It is less expensive.
- (vi) Forming a joint in difficult locations is possible through welding.

The advantages have made welding suitable for joining components in various machines and structures.

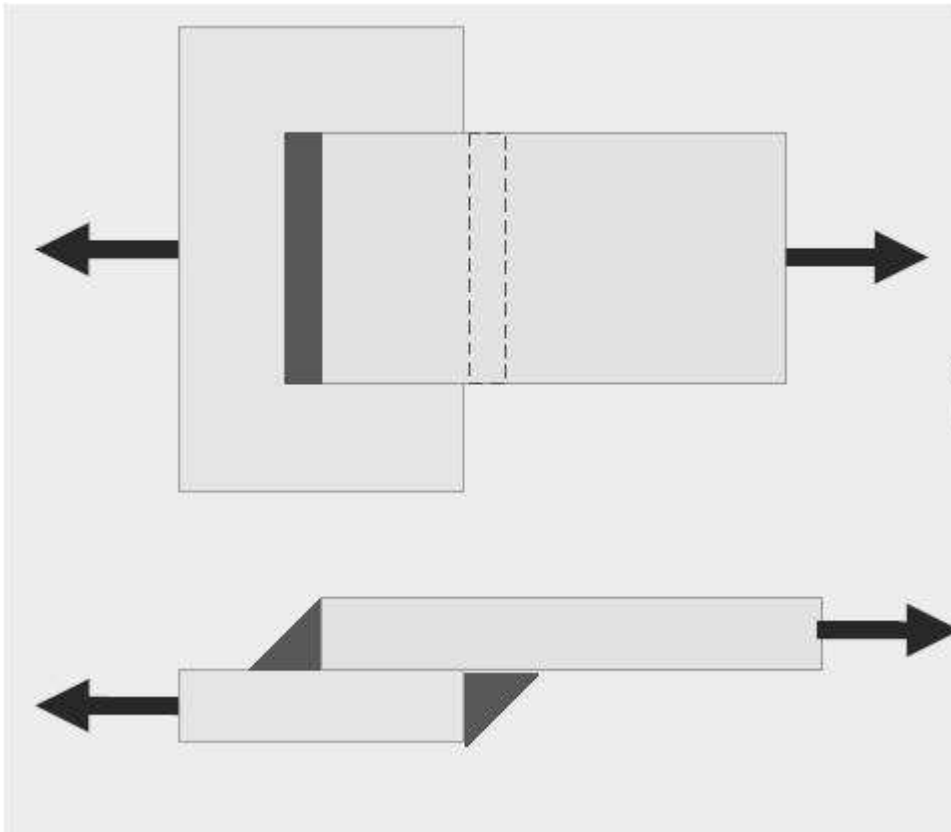
Types of welded joints

Welded joints are primarily of two kinds

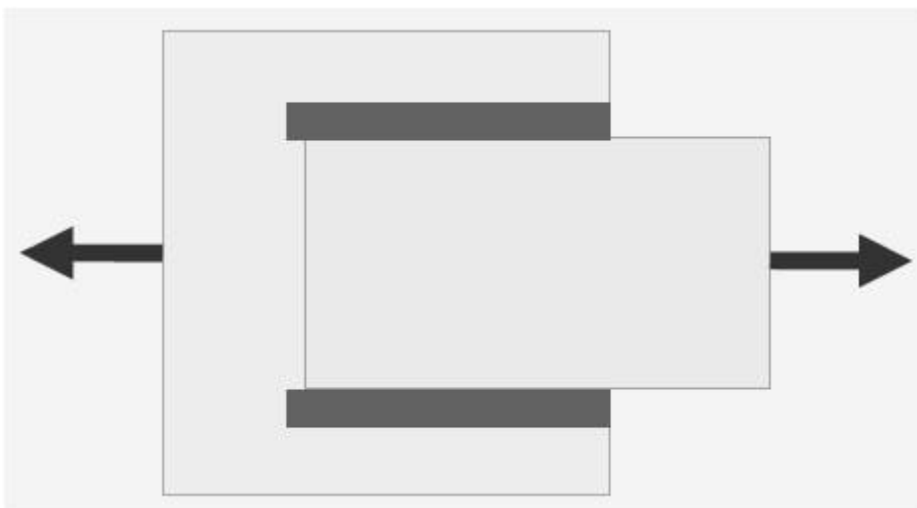
a) *Lap or fillet joint*: obtained by overlapping the plates and welding their edges. The fillet joints may be single transverse fillet, double transverse fillet or parallel fillet joints (see figure 2.12).



Single transverse lap joint



Double transverse lap joint



Parallel lap joint

Fig. 2.12 – Different types of lap joint

b) *Butt joints*: formed by placing the plates edge to edge and welding them. Grooves are sometimes cut (for thick plates) on the edges before welding. According to the shape of the grooves, the butt joints may be of different types, e.g.,

- Square butt joint
- Single V-butt joint, double V-butt joint
- Single U-butt joint, double U-butt joint
- Single J-butt joint, double J-butt joint
- Single bevel-butt joint, double bevel butt joint

These are schematically shown in figure 2.13.

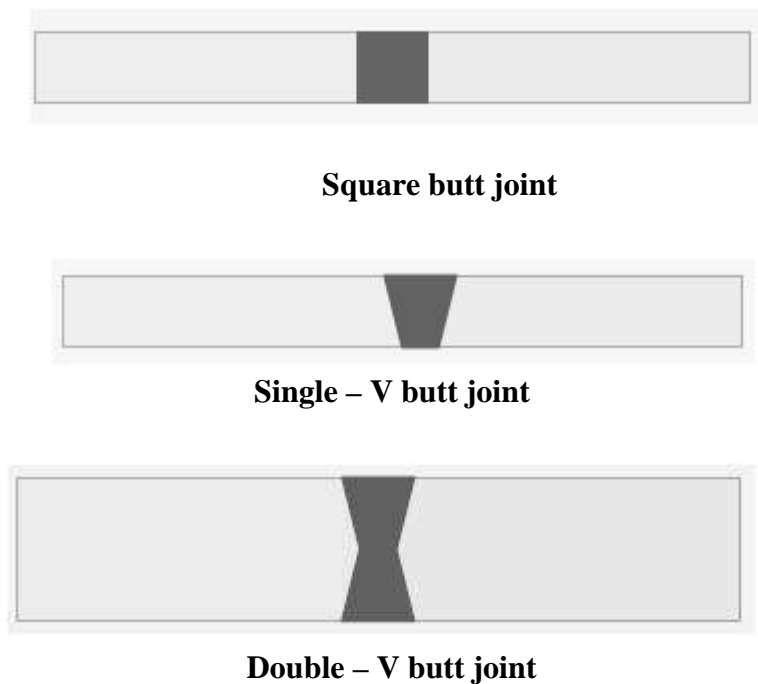


Fig. 2.13 – Different types of butt joints

Strength of welds: in-plane loading

There are different forms of welded joints, subjected to in-plane loading under tension.

1. Transverse fillet weld

Figure 2.14a shows a double transverse fillet weld under tension. It is assumed that the section of the weld is an isosceles right angled triangle, ABC, i.e. 45° fillet weld (Fig. 2.14b).

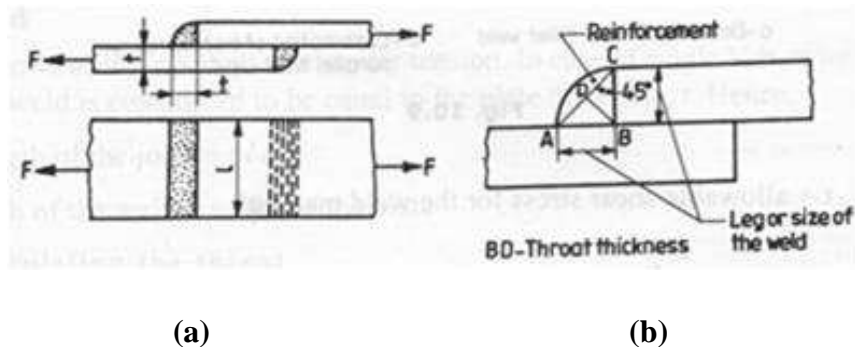


Fig. 2.14 – Double transverse fillet weld

The length of each side ($AB=BC$) is known as leg length or size of the weld. The minimum cross-sectional dimension, BD (at 45° from the plate surface or edge) is termed as throat thickness. Transverse fillet welds are assumed to fail in tension across the throat.

Let t = thickness of the plate or size of the weld
 l = length of the weld
 σ_t = allowable tensile stress for the weld material

From the geometry of Fig. 2.14b,

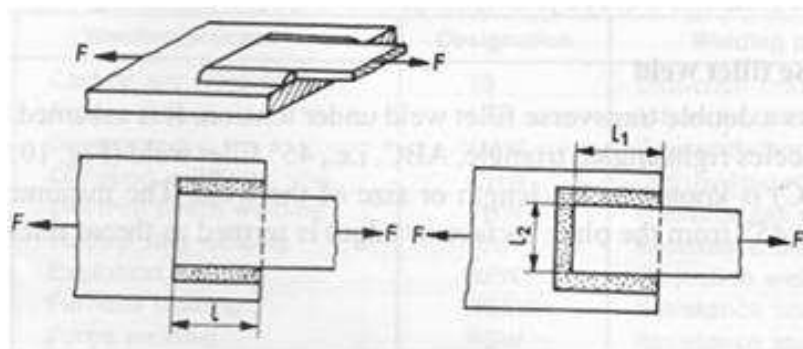
$$\text{Throat thickness, } BD(=h) = t \sin 45^\circ = \frac{t}{\sqrt{2}}$$

$$\text{Resisting throat area} = hl = \frac{tl}{\sqrt{2}}$$

$$\begin{aligned} \text{Tensile strength of the joint} &= \frac{tl}{\sqrt{2}} \sigma_t, \text{ for single fillet} \\ &= \frac{2tl}{\sqrt{2}} \sigma_t = \sqrt{2}tl \sigma_t, \text{ for double fillet} \end{aligned}$$

2. Parallel fillet weld

Figure 2.15a shows a double parallel fillet weld under tension. Parallel fillet welds are assumed to fail in shear across the throat.



(a) Double parallel fillet weld (b) Combination of transverse and parallel fillet weld

Fig. 2.15

Let τ = allowable shear stress for the weld material

$$\text{Resisting throat area} = \frac{tl}{\sqrt{2}}$$

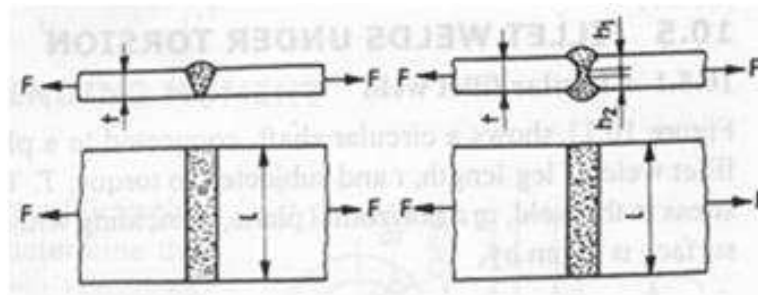
$$\text{Shear strength of the joint} = \frac{tl}{\sqrt{2}} \tau, \text{ for single fillet}$$

(Tensile strength)

$$= \frac{2tl}{\sqrt{2}} \tau = \sqrt{2}tl \tau, \text{ for double fillet}$$

3. Butt weld

Fig. 2.15a shows a single V-butt joint under tension.



(a) Single-V butt joint (b) Double-V butt joint

Fig. 2.15 – Butt joints under tension

In case of single V-butt weld, the throat thickness of the weld is considered to be equal to the plate thickness, t . Hence, tensile strength of the joint = $tl\sigma_t$

Where, l = length of the weld = width of the plate.

Figure 2.15b shows a double V-butt joint under tension.

Let h_1 = throat thickness at the top

h_2 = throat thickness at the bottom

Then tensile strength of the joint = $(h_1 + h_2)l\sigma_t$

4. Fillet welds under torsion

Circular fillet weld: Figure 2.16 shows a circular shaft, connected to a plate, by a fillet weld of leg length, t and subjected to torque, T . The shear stress in the weld, in a horizontal plane, coinciding with the plate surface

is given by,

$$\tau = \frac{T \times d / 2}{J}$$

Where,

$$J = \pi d \left(\frac{d}{2} \right)^2$$

$$\tau = \frac{T \times d / 2}{\pi d \left(\frac{d}{2} \right)^2} = \frac{2T}{\pi d^2}$$

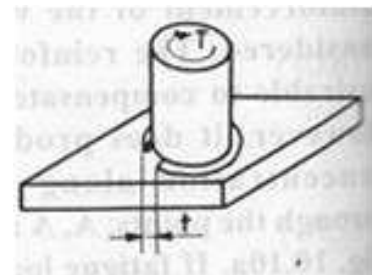


Fig. 2.16

The maximum value of the shear stress occurs in the weld throat, the length of which is $\frac{t}{\sqrt{2}}$

Therefore,

$$\tau_{\max} = \frac{2T\sqrt{2}}{\pi d^2} = \frac{2.83T}{\pi d^2}$$

Long adjacent fillet welds: Fig 2.17 shows a vertical plate, connected to a horizontal plate by two identical fillet welds, and subjected to torque, T about the vertical axis of the joint.

Let $l =$ length of the joint

$T =$ leg length of the weld

The effect of the applied torque is to produce shear stress, varying from zero at the axis and maximum at the plate ends (This is similar to the variation of normal stress over the depth of a beam, subjected to bending).

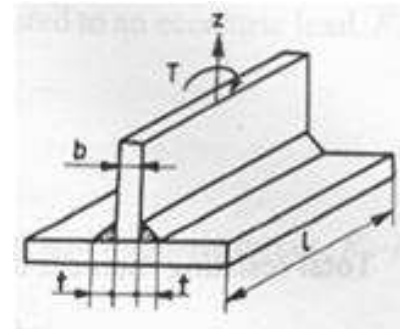


Fig. 2.17

The torsional shear stress, induced at the plate ends, and in a horizontal plane, coinciding with the top surface of the horizontal plane, is given by,

$$\tau_{\max} = \frac{3T\sqrt{2}}{tl^2} = \frac{4.2T}{tl^2}$$

5. Fillet welds under bending moment

Annular fillet weld: Figure 2.18 shows one example of an annular fillet weld, subjected to bending moment, M . To determine the maximum bending stress induced in the joint; let us consider a small element of the weld, at an angle, θ , subtending an angle, $d\theta$ at the centre of the shaft.

Area of the element $= r \cdot d\theta \cdot t$

Where, $t =$ size of the weld

Normal force acting on the weld element,

$$dF = r \times d\theta \times t \times \sigma_t$$

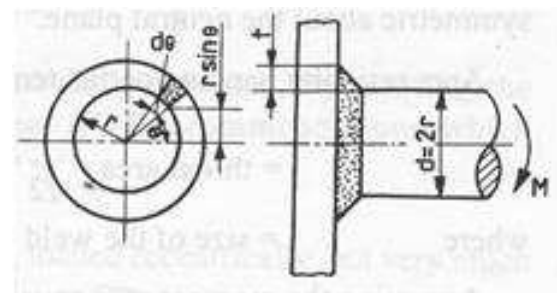


Fig. 2.18

Since the normal stress in the element is proportional to the distance from the neutral plane,

$$\frac{\sigma_{t\max}}{r} = \frac{\sigma_t}{r \sin \theta}$$

Where, $\sigma =$ normal (bending) stress induced in the weld element

$\sigma_{t\max} =$ maximum bending stress

$$\sigma_t = \sigma_{t\max} \sin \theta$$

Moment due to the force, dF about the neutral plane,

$$= dF \times r \sin \theta$$

$$= r \cdot d\theta \cdot t \cdot \sigma_t \cdot r \sin \theta$$

$$= r \cdot d\theta \cdot t \cdot \sigma_{t\max} \cdot \sin \theta \cdot r \sin \theta$$

$$= r^2 t \sigma_{t \max} \cdot \sin^2 \theta \cdot d\theta$$

$$\begin{aligned} \text{Total resisting moment offered by weld} &= \int_0^{2\pi} r^2 t \sigma_{t \max} \cdot \sin^2 \theta \cdot d\theta \\ &= r^2 t \sigma_{t \max} \pi = \text{external moment, } M \end{aligned}$$

Therefore, $M = r^2 t \sigma_{t \max} \pi$

or $\sigma_{t \max} = \frac{M}{r^2 t \pi}$

Considering the throat area, for evaluation of the stress,

$$\sigma_{t \max} = \frac{M}{\left(\frac{d}{2}\right)^2 \times \frac{t}{\sqrt{2}} \times \pi} = \frac{5.66M}{d^2 \pi}$$

Parallel fillet weld:

Figure 2.19 shows a double parallel fillet weld, subjected to bending moment, M . The joint is symmetric about the neutral plane.

Area resisting bending on the tensile (compressive) side,

$$= \text{throat area} = \frac{t}{\sqrt{2}} \times l$$

Where, t = size of the weld

Assuming the moment arm equal to $(b+t)$,

$$\text{Resisting moment} = \frac{t}{\sqrt{2}} \times l(b+t) \sigma_t$$

Where, b = thickness of the plate

$$\text{Therefore, } M = \frac{t}{\sqrt{2}} \times l(b+t) \sigma_t$$

$$\text{And } \sigma_t = \frac{\sqrt{2}M}{tl(b+t)}$$

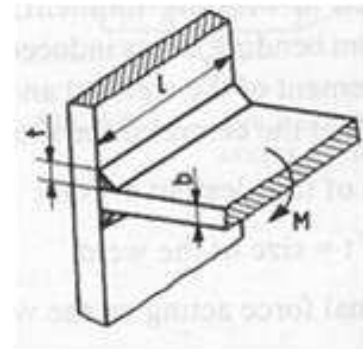


Fig. 2.19

6. Welded joints under eccentric loading

Case – I

Figure 2.20 shows a T-joint, with double parallel fillet weld, subjected to an eccentric load, F at a distance, e

Let t = size of the weld

l = length of the weld and b = thickness of the plate

To analyse the effect of the eccentric load, F , introduce two equal and opposite forces, $F_1 - F_2$ such that $F_1 = F_2 = F$, as shown in Fig. 2.20.

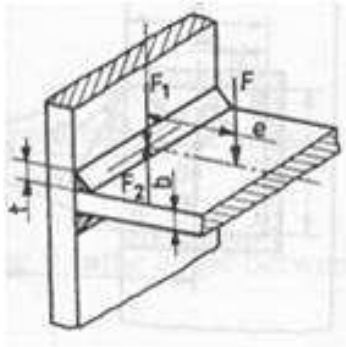


Fig. 2.20

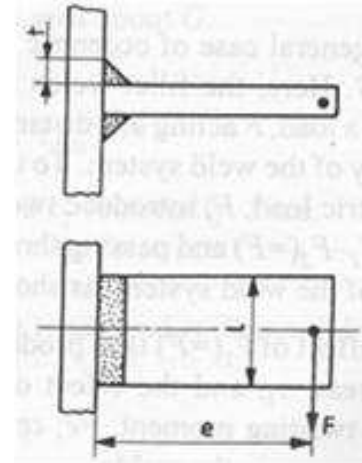


Fig. 2.21

The effect of $F_1 (=F)$ is to produce transverse shear stress, given by,

$$\tau = \frac{F}{2 \times \frac{t}{\sqrt{2}} \times l} = \frac{F}{\sqrt{2}tl}$$

Where, $\frac{t}{\sqrt{2}}$ = throat thickness (h)

The effect of $F - F_2 (F - F)$ is to produce bending moment, M , given by, Fe .

Bending stress induced due to M is,

$$\sigma_b = \frac{\sqrt{2}M}{tl(b+t)} = \frac{\sqrt{2}Fe}{tl(b+t)}$$

The resultant (maximum) normal stress is given by,

$$\sigma_{\max} = \sqrt{\sigma_b^2 + \tau^2} = \frac{F}{tl(b+t)} \times \sqrt{2e^2 + \frac{(b+t)^2}{2}}$$

Case - II

Figure 2.21 shows a T-joint with double parallel fillet weld, loaded eccentrically, but very much different from that of the joint as shown in Fig. 2.20.

Let F = load; e = eccentricity; t = leg length; l = length of the weld

Similar to previous case, to analyse the effect of the eccentric load, F , introduce two equal and opposite forces, $F_1 - F_2$ such that $F_1 = F_2 = F$, as shown in Fig. 2.21.

The effect of $F_1 = F$ is to produce transverse shear stress, given by,

$$\tau = \frac{F}{2 \times \frac{t}{\sqrt{2}} \times l} = \frac{F}{\sqrt{2}tl}$$

Where, $\frac{t}{\sqrt{2}}$ = throat thickness

The effect of $F - F_2$ ($F - F$) is to produce bending moment, M , given by, Fe .

Bending stress induced due to M is,

$$\sigma_b (= \sigma_t = \sigma_c) = \frac{M}{Z}$$

Where,

$$Z = 2 \times \frac{1}{6} \left(\frac{t}{\sqrt{2}} \right) l^2 = \frac{\sqrt{2}}{6} tl^2 = \frac{tl^2}{4.242}$$

$$\sigma_b = \frac{4.242Fe}{tl^2}$$

The resultant (maximum) normal stress is given by,

$$\sigma_{\max} = \sqrt{\sigma_b^2 + \tau^2} = \frac{0.707F}{tl} \times \sqrt{1 + \left(\frac{6e}{l} \right)^2}$$

Case – III

A more general case of eccentric loading is shown in Fig. 2.22. Here, the fillet welds are subjected to the action of a load, F acting at a distance, e from the centre of gravity of the weld system. To understand the effect of eccentric load, F ; introduce two equal and opposite forces, $F_1 - F_2$ ($=F$) and passing through G , the centre of gravity of the weld system, as shown.

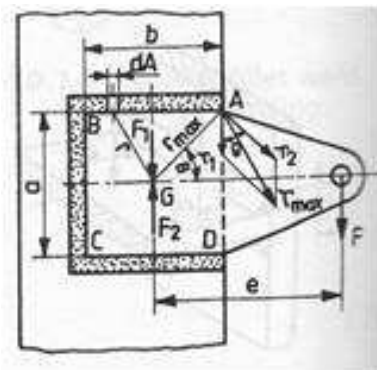


Fig. 2.22

The effect of $F_1(=F)$ is to produce direct or primary shear stress, τ_1 , and the effect of $F - F_2$ ($=F-F$) is to produce twisting moment, Fe ; resulting in secondary shear stress, τ_2 in the welds.

$$\text{Primary shear stress, } \tau_1 = \frac{\sqrt{2}F}{tl}$$

Where, t = size of the weld

$$l = \text{total length of the weld} \approx a + 2b$$

Considering bending action, the shear stress induced is proportional to the distance of the weld section from G . Obviously, it is maximum at the corners of the weld.

Let τ_2 = maximum secondary shear stress at, say corner, A . Then from Fig. 2.22,

$$\frac{\tau}{r} = \frac{\tau_2}{GA} = \frac{\tau_2}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$$

$$\tau = 2\tau_2 \times \frac{r}{\sqrt{a^2 + b^2}}$$

Where, τ is the secondary shear stress at distance, r from G .

The moment of the shear force on the weld element of area, dA and at distance, r from G is,

$$dM = \tau \cdot dA \cdot r = \frac{2\tau_2 r^2 \cdot dA}{\sqrt{a^2 + b^2}}$$

Total resisting moment due to the welds AB, BC, CD shall be equal to the external (applied) twisting moment, Fe .

$$Fe = \frac{2\tau_2}{\sqrt{a^2 + b^2}} \left[\int_A^B r^2 \cdot dA + \int_B^C r^2 \cdot dA + \int_C^D r^2 \cdot dA \right] = \frac{2\tau_2}{\sqrt{a^2 + b^2}} \cdot J$$

Where $J = \sum r^2 \cdot dA$ = polar moment of inertia of the throat area about G .

$$\tau_2 = \frac{Fe \times \sqrt{a^2 + b^2}}{2I_G} = \frac{Fe}{J} \times r_{\max}$$

The resultant stress, τ_{\max} is obtained by adding τ_1 and τ_2 vectorially. Thus,

$$\tau_{\max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos \theta}$$

Where θ is the angle between primary and secondary shear loads, and is obtained from,

$$\cos \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

Example Problem-1: Figure 2.23 shows a cylindrical rod of 50 mm diameter, welded to a flat plate. The cylindrical fillet weld is loaded eccentrically, by a force of 10 kN acting at 200 mm from the welded end. If the size of the weld is 20 mm, determine the maximum normal stress in the weld.

Solution: Let h = throat thickness = $\frac{t}{\sqrt{2}}$

Referring Fig. 2.23, let us introduce two equal and opposite forces, $F_1 - F_2$ and

Reference

Design
Data Book
by K.
Mahadevan

parallel to F , and passing through the centre of the rod at the fixed end such that $F_1 = F_2 = F$. Effect of $F_1 (=F)$ is to produce transverse shear stress, τ .

$$\text{Throat area, } A = \pi dh = \pi \times 50 \times \frac{t}{\sqrt{2}} = \pi \times 50 \times \frac{20}{\sqrt{2}} = 2221.8 \text{ mm}^2$$

Transverse shear stress,

$$\tau = \frac{F}{A} = \frac{10 \times 1000}{2221.8} = 4.5 \text{ N/mm}^2$$

Effect of $F_1 - F_2 (=F - F)$ is to produce bending moment, M , given by,

$$M = FL = 10 \times 1000 \times 200 = 20 \times 10^5 \text{ N-mm}$$

Fig. 2.23

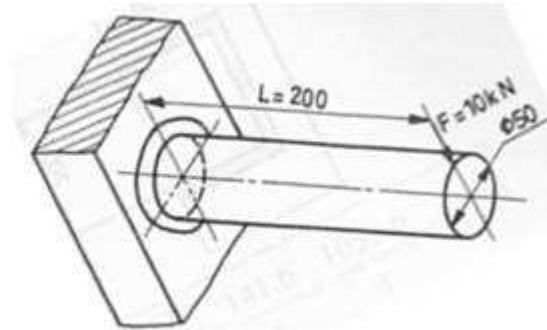
For a circular fillet weld, section modulus, Z is given by,

$$Z = \frac{\pi d^2}{5.66} = \frac{\pi \times 20 \times 50^2}{5.66} = 27752.6 \text{ mm}^3$$

$$\text{Bending stress, } \sigma_b = \frac{M}{Z} = \frac{20 \times 10^5}{27752.6} = 72.1 \text{ N/mm}^2$$

Resultant (maximum) normal stress,

$$\sigma_b = \sqrt{\tau^2 + \sigma_b^2} = \sqrt{4.5^2 + 72.1^2} = 72.24 \text{ N/mm}^2$$



& K. B. Reddy

Example Problem-2: Figure 2.24a shows an eccentrically loaded welded joint.

Determine the fillet weld size. Allowable shear stress in the weld is 80 MPa.

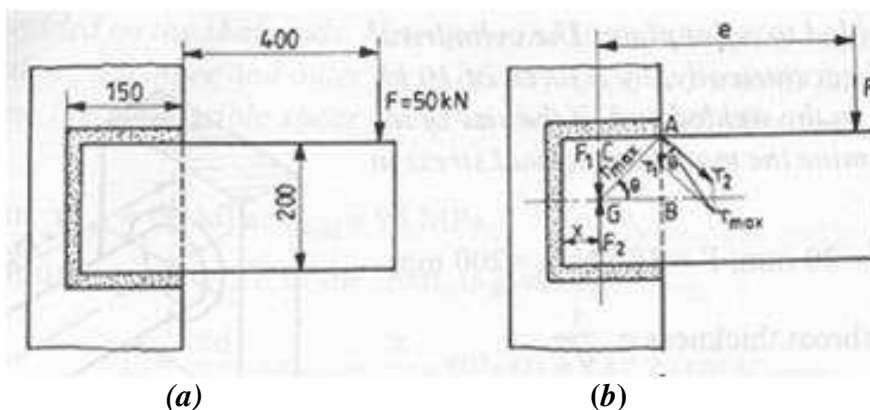


Fig. 2.24

Solution: Given data: $F = 50 \text{ kN}$; $b = 200 \text{ mm}$; $l = 150 \text{ mm}$; $\tau = 80 \text{ MPa}$

Let $t =$ size of the weld; $h =$ throat thickness $= \frac{t}{\sqrt{2}}$

The distance of the centre of gravity, G from the left edge of the plate, x is given by,

$$x = \frac{l^2}{2l + b} = \frac{150^2}{2 \times 150 + 200} = 45 \text{ mm}$$

Eccentrically, $e = 400 + (150 - x) = 400 + (150 - 45) = 505 \text{ mm}$

Polar moment of inertia of the weld throat about G ,

$$J = \frac{t}{\sqrt{2}} \left[\frac{(b + 2l)^3}{12} - \frac{l^2(b + l)^2}{b + 2l} \right]$$

$$= \frac{t}{\sqrt{2}} \left[\frac{(200 + 2 \times 150)^3}{12} - \frac{150^2 \times (200 + 150)^2}{(200 + 2 \times 150)} \right] = 3468.3 \times 10^3 t \text{ mm}^4$$

Maximum radius of the weld, $GA = r_{\max} =$

$$\sqrt{AB^2 + AC^2} = \sqrt{100^2 + 105^2} = 145 \text{ mm}$$

$$\cos \theta = \frac{GB}{GA} = \frac{105}{145} = 0.724$$

Throat area of the weld, $A = (b + 2l) \times \frac{t}{\sqrt{2}} = 353.6t \text{ mm}^2$

Referring Fig. 2.24b, let us introduce two equal and opposite forces, $F_1 - F_2$ through G , and parallel to F such that $F_1 = F_2 = F$.

The effect of $F_1 (= F)$ is to produce primary shear stress.

$$\text{Primary shear stress, } \tau_1 = \frac{F}{A} = \frac{50 \times 1000}{353.6t} = \frac{141.4}{t} \text{ N/mm}^2$$

The effect of $F - F_2 (= F - F)$ is to produce moment, Fe ; inducing secondary shear stress. Maximum secondary shear stress,

$$\tau_2 = \frac{Fe}{J} \times r_{\max} = \frac{50 \times 1000 \times 505}{3468.3 \times 10^3 \times t} \times 145 = \frac{1055.6}{t}$$

Resultant (maximum) shear stress,

$$\tau_{\max} = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos \theta}$$

$$= \sqrt{\left(\frac{141.6}{t}\right)^2 + \left(\frac{1055.6}{t}\right)^2 + 2 \times \frac{141.6}{t} \times \frac{1055.6}{t} \times 0.724}$$

$$= \frac{1162.1}{t}$$

$$80 = \frac{1162.1}{t}$$

$$t = 14.5 \text{ mm}$$

Design of Bolted Joints

Threaded fasteners

Bolts, screws and studs are the most common types of threaded fasteners. They are used in both permanent and removable joints.

Bolts: They are basically threaded fasteners normally used with nuts.

Screws: They engage either with a preformed or a self made internal thread.

Studs: They are externally threaded headless fasteners. One end usually meets a tapped component and the other with a standard nut.

There are different forms of bolt and screw heads for a different usage. These include bolt heads of square, hexagonal or eye shape and screw heads of hexagonal, Fillister, button head, counter sunk or Phillips type. These are shown in Figs. 2.25 and 2.26.

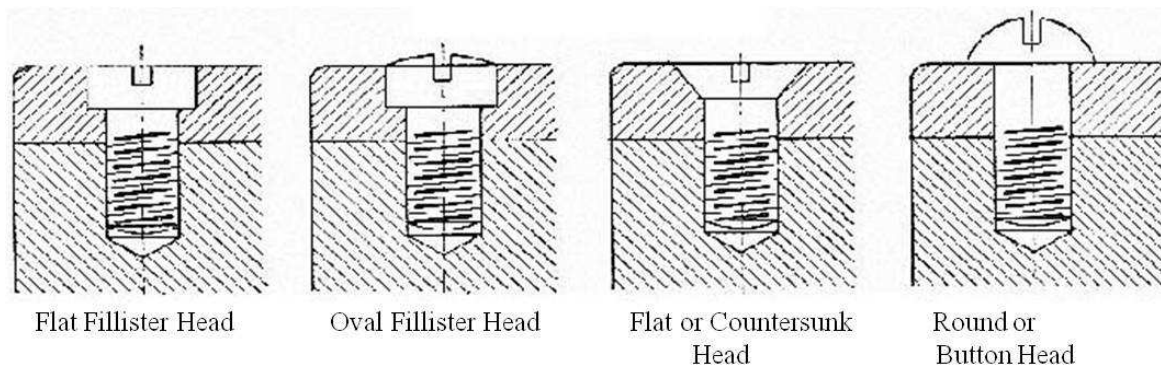


Fig. 2.25 – Types of screw heads

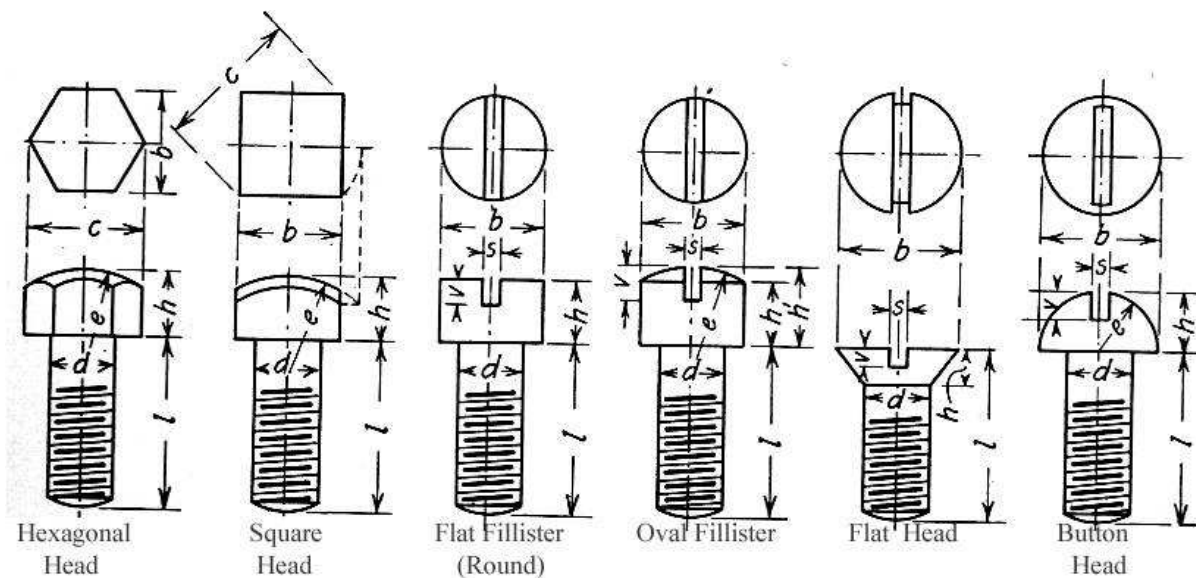


Fig. 2.26 – Types of bolt heads

Tapping screws

These are one piece fasteners which cut or form a mating thread when driven into a preformed hole. These allow rapid installation since nuts are not used.

There are two types of tapping screws. They are known as **thread forming** which displaces or forms the adjacent materials and **thread cutting** which have cutting edges and chip cavities which create a mating thread.

Set Screws

These are semi permanent fasteners which hold collars, pulleys, gears etc on a shaft. Different heads and point styles are available. Some of them are shown in **Fig. 2.27**.

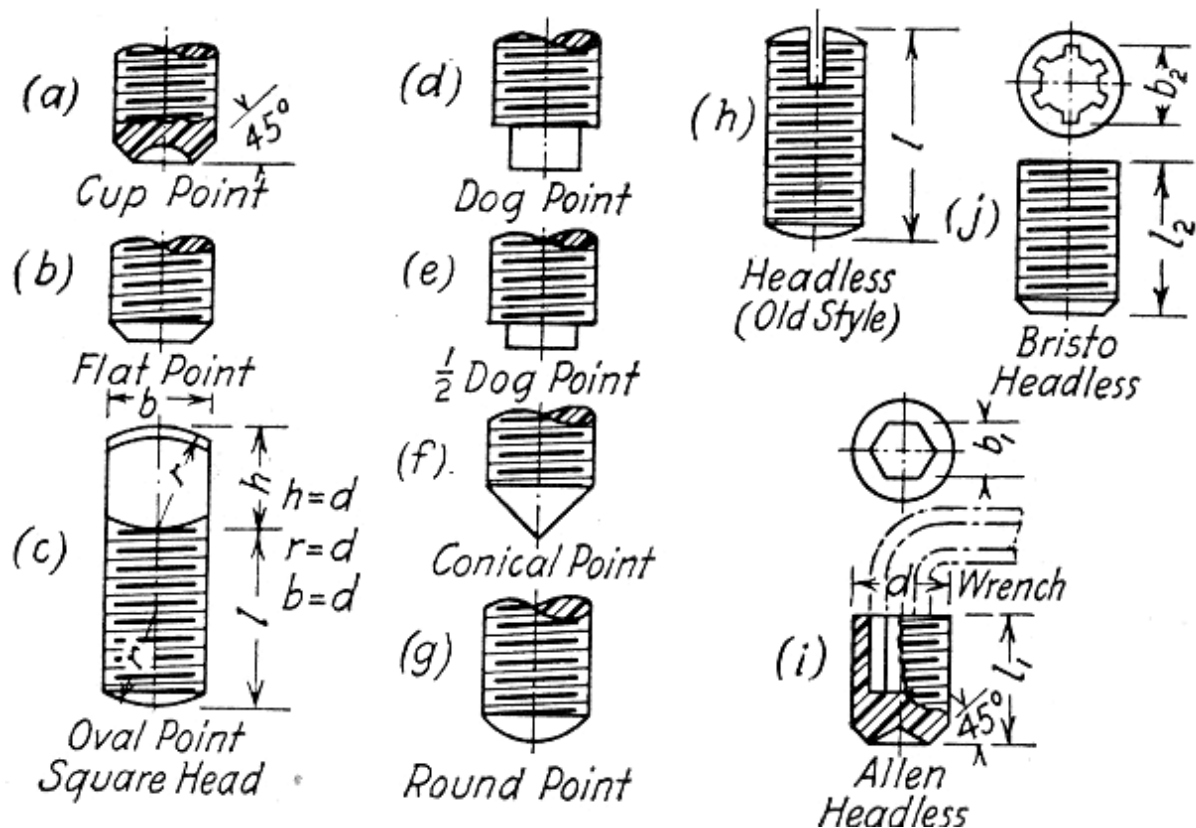


Fig. 2.27 – Different types of set screws

Thread forms

Basically when a helical groove is cut or generated over a cylindrical or conical section, threads are formed. When a point moves parallel to the axis of a rotating cylinder or cone held between centers, a helix is generated. Screw threads formed in this way have two functions to perform in general: (a) to transmit power – Square, ACME, Buttress, Knuckle types of thread forms are useful for this purpose. (b) to secure one member to another- V-threads are most useful for this purpose.

Some standard forms are shown in **Fig. 2.28**.

V-threads are generally used for securing because they do not shake loose due to the wedging action provided by the thread. Square threads give higher efficiency due to a low friction. This is demonstrated in Fig. 2.29.

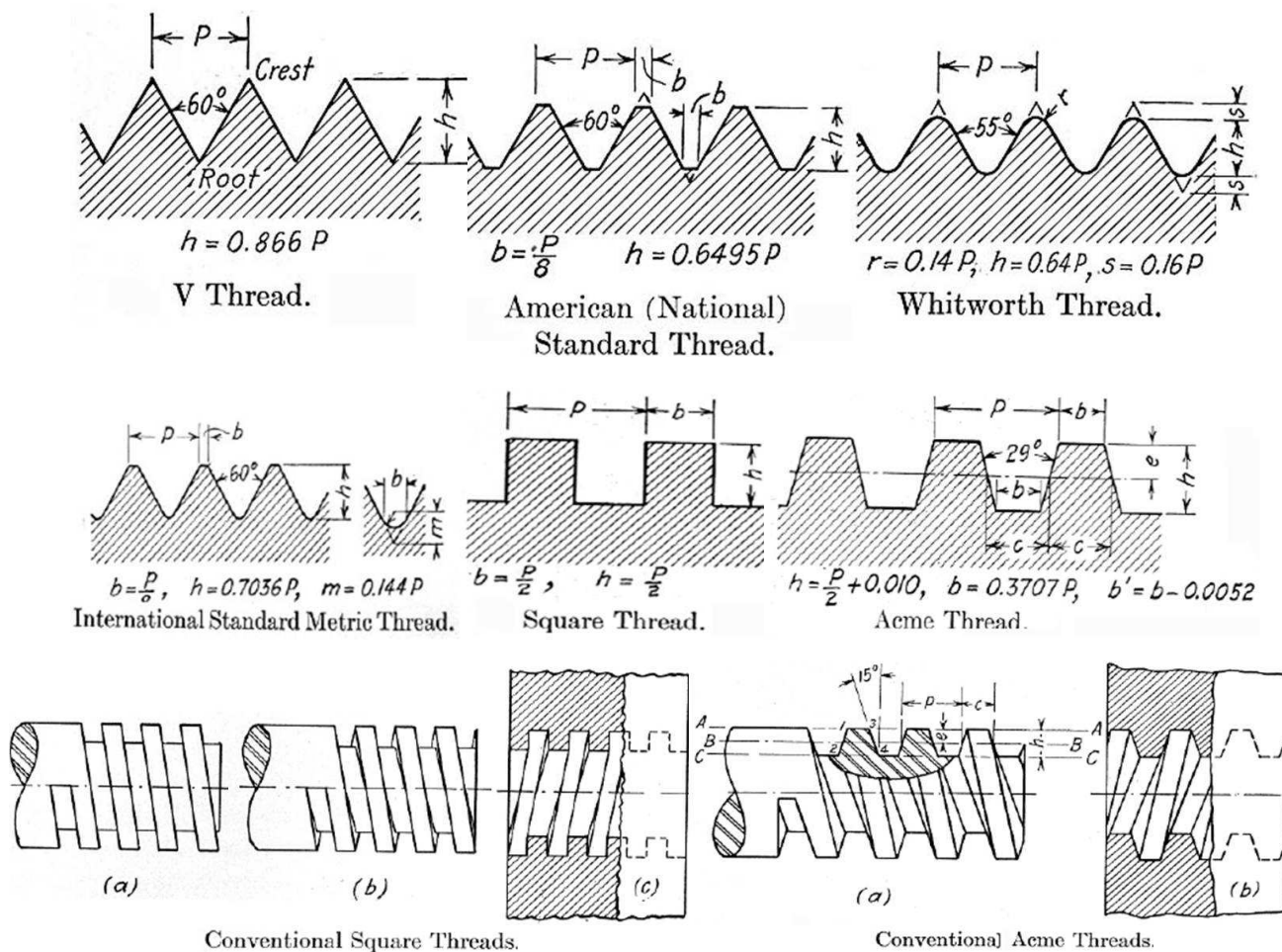


Fig. 2.28 – Different types of thread forms

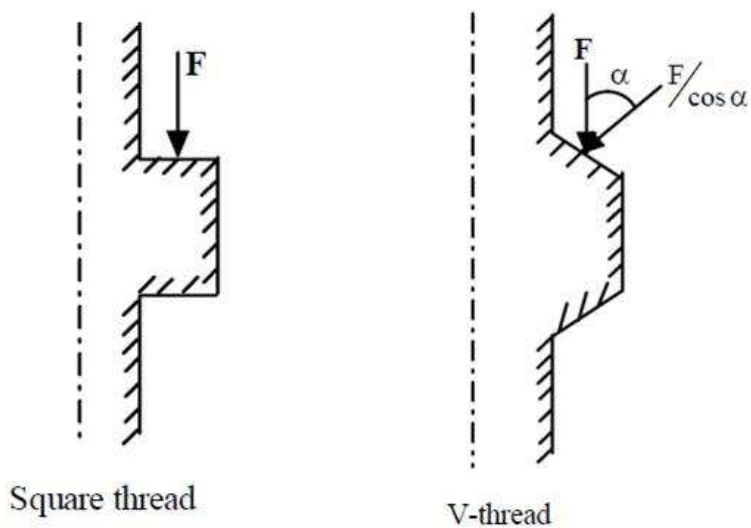


Fig. 2.29 – Loading on square and V-threads

Design of bolted joints

Stresses in screw fastenings

It is necessary to determine the stresses in screw fastening due to both static and dynamic loading in order to determine their dimensions. In order to design for static loading both initial tightening and external loadings need be known.

4.4.1.1 Initial tightening load

When a nut is tightened over a screw following stresses are induced:

- (a) Tensile stresses due to stretching of the bolt
- (b) Torsional shear stress due to frictional resistance at the threads.
- (c) Shear stress across threads
- (d) Compressive or crushing stress on the threads
- (e) Bending stress if the surfaces under the bolt head or nut are not perfectly normal to the bolt axis.

(a) Tensile stress

Since none of the above mentioned stresses can be accurately determined bolts are usually designed on the basis of direct tensile stress with a large factor of safety. The initial tension in the bolt may be estimated by an empirical relation $P_1 = 284 d$ kN, where the nominal bolt diameter d is given in mm. The relation is used for making the joint leak proof. If leak proofing is not required half of the above estimated load may be used. However, since

initial stress is inversely proportional to square of the diameter $\sigma = \frac{284d}{\frac{\pi}{4}d^2}$, bolts of smaller

diameter such as M16 or M8 may fail during initial tightening. In such cases torque wrenches must be used to apply known load. The torque in wrenches is given by $T = C P_1 d$ where, C is a constant depending on coefficient of friction at the mating surfaces, P is tightening up load and d is the bolt diameter.

(b) Torsional shear stress

This is given by $\tau = \frac{16T}{\pi d_c^3}$ where T is the torque and d_c the core diameter. We may

relate torque T to the tightening load P_1 in a power screw configuration (**figure-2.30**) and taking collar friction into account we may write

$$T = P_1 \frac{d_m}{2} \left(\frac{1 + \mu \pi d_m \sec \alpha}{\pi d_m - \mu L \sec \alpha} \right) + \frac{P_1 \mu_c d_{cm}}{2}$$

where d_m and d_{cm} are the mean thread diameter and mean collar diameter respectively, and μ and μ_c are the coefficients of thread and collar friction respectively and α is the semi thread angle. If we consider that $d_{cm} = \frac{d_m + 1.5d_m}{2}$, then we may write $T = C P_1 d_m$ where C is a constant for a given arrangement. As discussed earlier, similar equations are used to find the torque in a wrench.

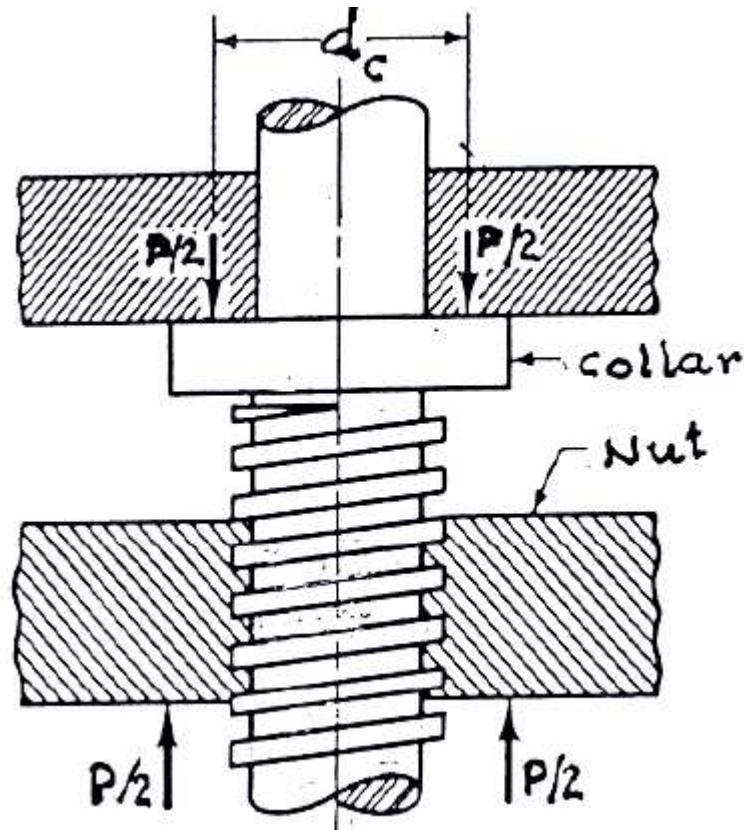


Fig. 2.30 – A typical power screw configuration

(c) Shear stress across the threads

This is given by $\tau = \frac{3P}{\pi d_c b n}$ where d_c is the core diameter and b is the base width of

the thread and n is the number of threads sharing the load.

(d) Crushing stress on threads

This is given by $\sigma_c = \frac{P}{\frac{\pi}{4}(d_o^2 - d_c^2)n}$ where d_o and d_c are the outside and core diameters

as shown in Fig. 2.30.

(e) Bending stress

If the underside of the bolt and the bolted part are not parallel as shown in Fig. 2.31, the bolt may be subjected to bending and the bending stress may be given by $\sigma_B = \frac{xE}{2L}$ where x is the difference in height between the extreme corners of the nut or bolt head, L is length of the bolt head shank and E is the young's modulus.

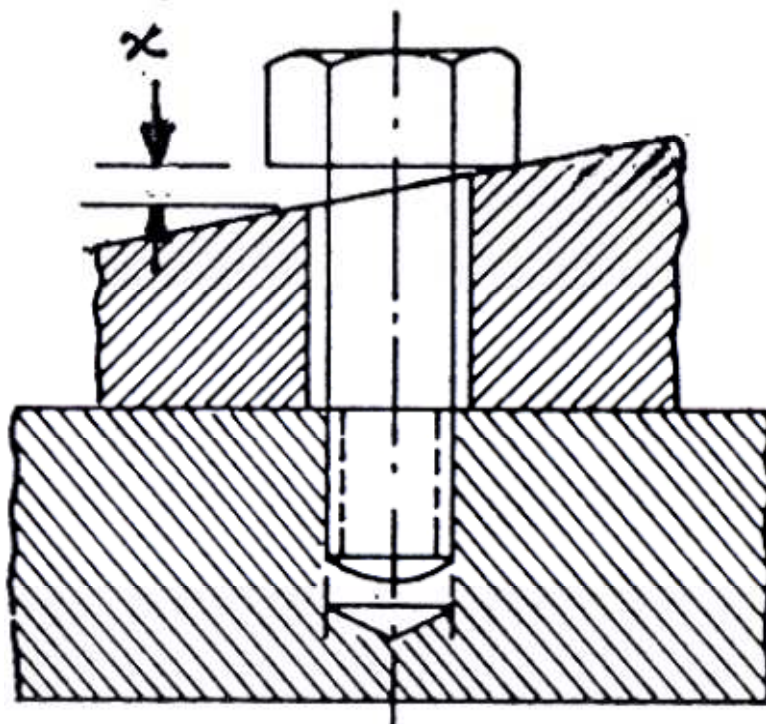


Fig. 2.31 - Development of bending stress in a bolt

Combined effect of initial tightening load and external load

When a bolt is subjected to both initial tightening and external loads i.e. when a preloaded bolt is in tension or compression the resultant load on the bolt will depend on the relative elastic yielding of the bolt and the connected members.

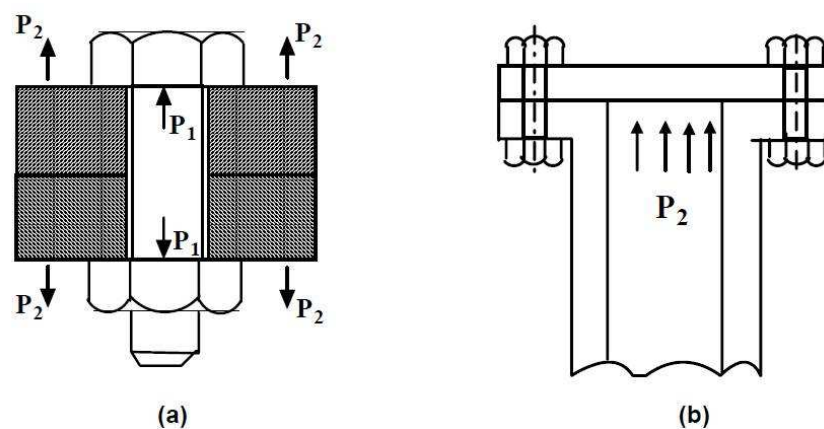


Fig. 2.32 - A bolted joint subjected to both initial tightening and external load

This situation may occur in steam engine cylinder cover joint for example. In this case the bolts are initially tightened and then the steam pressure applies a tensile load on the bolts. This is shown in Fig. 2.32a and b.

Initially due to preloading the bolt is elongated and the connected members are compressed. When the external load P is applied, the bolt deformation increases and the compression of the connected members decrease. Here, P_1 and P_2 in Fig. 2.32a are the tensile loads on the bolt due to initial tightening and external load respectively. The increase in bolt deformation is given by $\delta_b = \frac{P_b}{K_b}$ and decrease in member compression is $\delta_c = \frac{P_c}{K_c}$ where, P_b is the share of P_2 in bolt, and P_c is the share of P_2 in members, K_b and K_c are the stiffnesses of bolt and members. If the parts are not separated then $\delta_b = \delta_c$ and this gives, $\frac{P_b}{K_b} = \frac{P_c}{K_c}$.

Therefore, the resultant load on bolt is $P+KP$. Sometimes connected members may be more yielding than the bolt and this may occurs when a soft gasket is placed between the surfaces.

Under these circumstances $K_b \gg K_c$ or $\frac{K_c}{K_b} \ll 1$ and this gives $K \approx 1$. Therefore the total load

$P = P_1 + P_2$ Normally K has a value around 0.25 or 0.5 for a hard copper gasket with long through bolts. On the other hand if $K_c \gg K_b$, K approaches zero and the total load P equals the initial tightening load. This may occur when there is no soft gasket and metal to metal contact occurs. This is not desirable. Some typical values of the constant K are given in Table 2.1.

Table 2.1

Type of joint	K
Metal to metal contact with through bolt	0-0.1
Hard copper gasket with long through bolt	0.25-0.5
Soft copper gasket with through bolts	0.75
Soft packing with through bolts	0.75-1.00
Soft packing with studs	1.00

Lecture – 15

Cotter Joint

A cotter is a flat wedge-shaped piece of steel as shown in Fig. 2.33. This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods. Examples of cotter joint connections are: connection of piston rod to the crosshead of a steam engine, valve rod and its stem etc.

A typical cotter joint is as shown in Fig. 2.34. One of the rods has a socket end into which the other rod is inserted and the cotter is driven into a slot, made in both the socket and the rod. The cotter tapers in width (usually 1:24) on one side only and when this is driven in, the rod is forced into the socket. However, if the taper is provided on both the edges it must be less than the sum of the friction angles for both the edges to make it self locking i.e.

$\alpha_1 + \alpha_2 < \phi_1 + \phi_2$ where α_1 , α_2 are the angles of taper on the rod edge and socket edge of the cotter respectively and ϕ_1 ,

ϕ_2 are the corresponding angles of friction. This also means that if taper is given on one side only then $\alpha < \phi_1 + \phi_2$ for self locking. Clearances between the cotter and slots in the rod end and socket allows the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the cotter on the rod end.

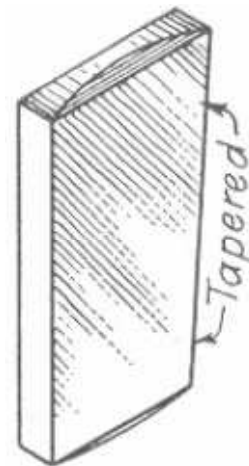


Fig. 2.33

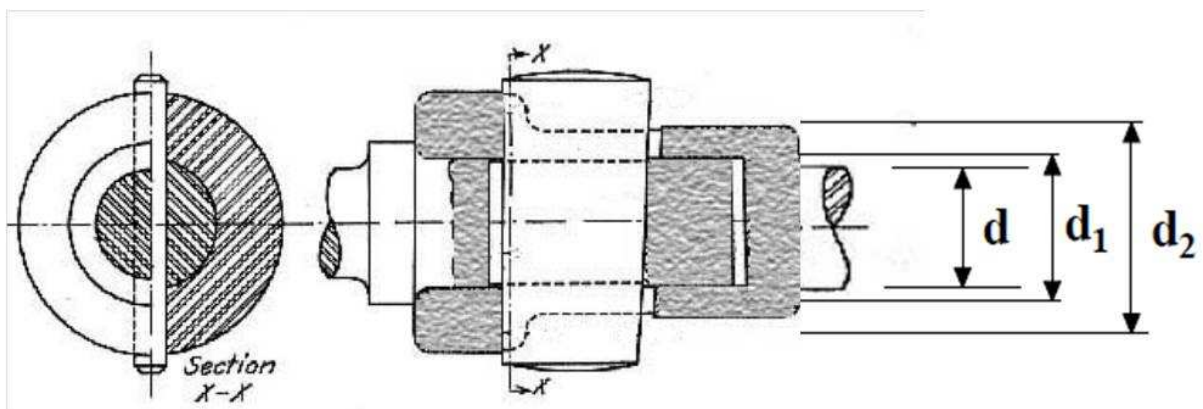


Fig. 2.34 – Cross-sectional views of a typical cotter joint

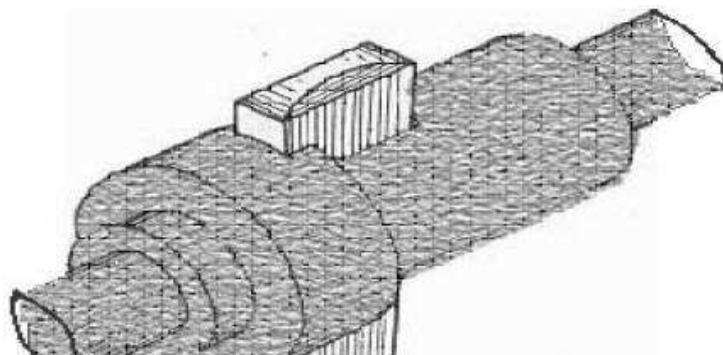


Fig. 2.35 – An isometric view of a typical cotter joint

Design of a cotter joint:

If the allowable stress in tension, compression and shear for the socket, rod and cotter be σ_t , σ_c , and τ respectively, assuming that they are all made of the same material, we may write the following failure criteria:

1. Tension failure of the rod at diameter d (Fig. 2.36)

$$\frac{\pi}{4} d^2 \sigma_t = P$$

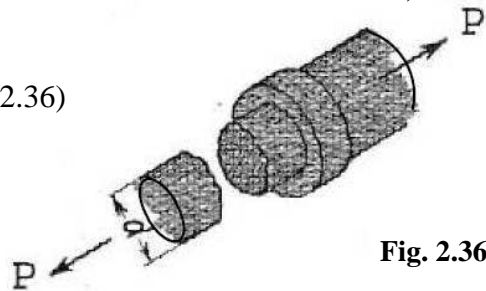


Fig. 2.36

2. Tension failure of the rod across slot (Fig. 2.37)

$$\left(\frac{\pi}{4} d^2 - d_1 t \right) \sigma_t = P$$

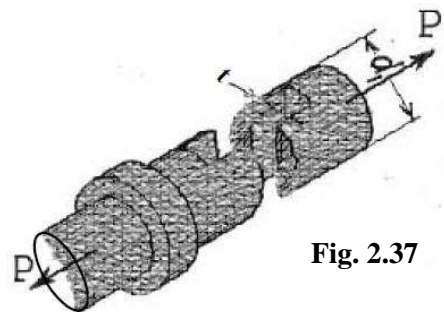


Fig. 2.37

3. Tension failure of the socket across slot (Fig. 2.38)

$$\left(\frac{\pi}{4} (d_2^2 - d_1^2) - (d_2 - d_1) t \right) \sigma_t = P$$

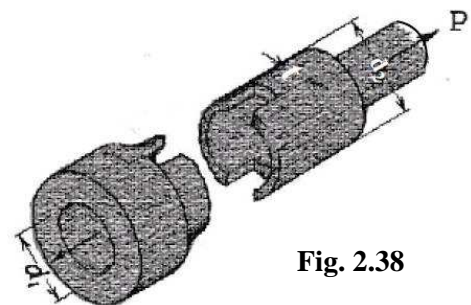


Fig. 2.38

4. Shear failure of cotter (Fig. 2.39)

$$2bt\tau = P$$

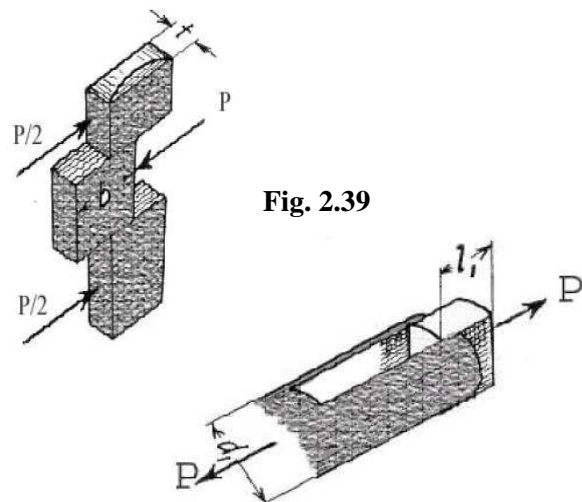
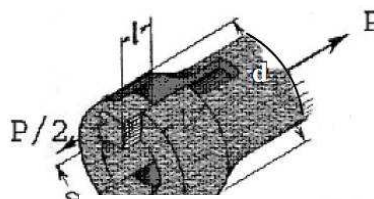


Fig. 2.39

5. Shear failure of the rod end (Fig. 2.40)

$$2l_1d_1\tau = P$$

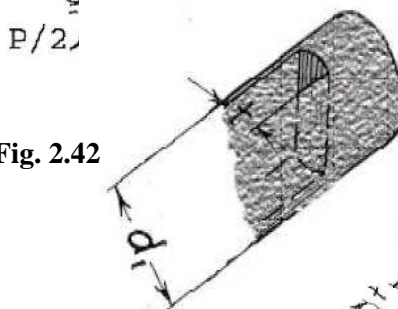
Fig. 2.40



6. Shear failure of socket end (Fig. 2.41)

$$2l(d_2 - d_1)\tau = P$$

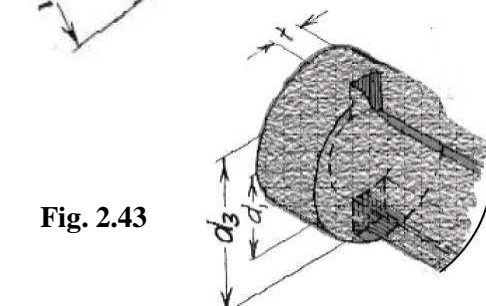
Fig. 2.41



7. Crushing failure of rod or cotter (Fig. 2.42)

$$d_1t\sigma_c = P$$

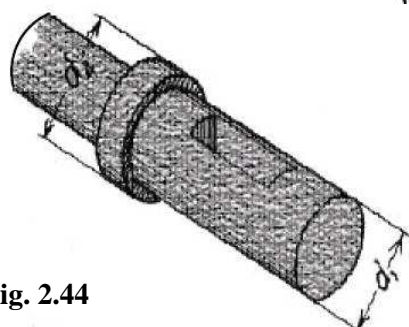
Fig. 2.42



8. Crushing failure of socket or rod (Fig. 2.43)

$$(d_3 - d_1)t\sigma_c = P$$

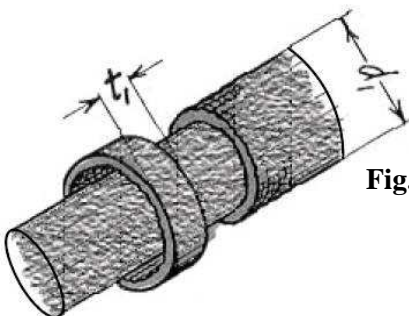
Fig. 2.43



9. Crushing failure of collar (Fig. 2.44)

$$\left(\frac{\pi}{4}(d_4^2 - d_1^2)\right)\sigma_c = P$$

Fig. 2.44



10. Shear failure of collar (Fig. 2.45)

$$\pi d_1 t_1 \tau = P$$

Fig. 2.45

Cotters may bend when driven into position. When this occurs, the bending moment cannot be correctly estimated since the pressure distribution is not known. However, if we assume a triangular pressure distribution over the rod, as shown in Fig. 2.46 (a), we may approximate the loading as shown in Fig. 2.46 (b)

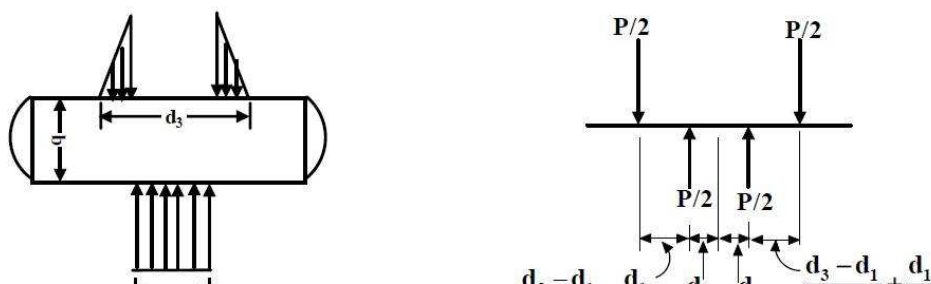


Fig. 2.46

This gives maximum bending moment = $\frac{P}{2} \left(\frac{d_3 - d_1}{6} + \frac{d_1}{4} \right)$ and the bending stress,

$$\sigma_b = \frac{\frac{P}{2} \left(\frac{d_3 - d_1}{6} + \frac{d_1}{4} \right) \frac{b}{2}}{\frac{tb^3}{12}} = \frac{3P \left(\frac{d_3 - d_1}{6} + \frac{d_1}{4} \right)}{tb^2}$$

Tightening of cotter introduces initial stresses which are again difficult to estimate. Sometimes therefore it is necessary to use empirical proportions to design the joint. Some typical proportions are given below:

$$d_1 = 1.21d ; d_2 = 1.75d ; d_3 = 2.4d ; d_4 = 1.5d ; t = 0.31d ; b = 1.6d ; l = l_1 = 0.75d ;$$

$$t_1 = 0.45d ; s = \text{clearance.}$$

Design of a cotter joint:

Knuckle Joint

A knuckle joint (as shown in Fig. 2.47) is used to connect two rods under tensile load. This joint permits angular misalignment of the rods and may take compressive load if it is guided.

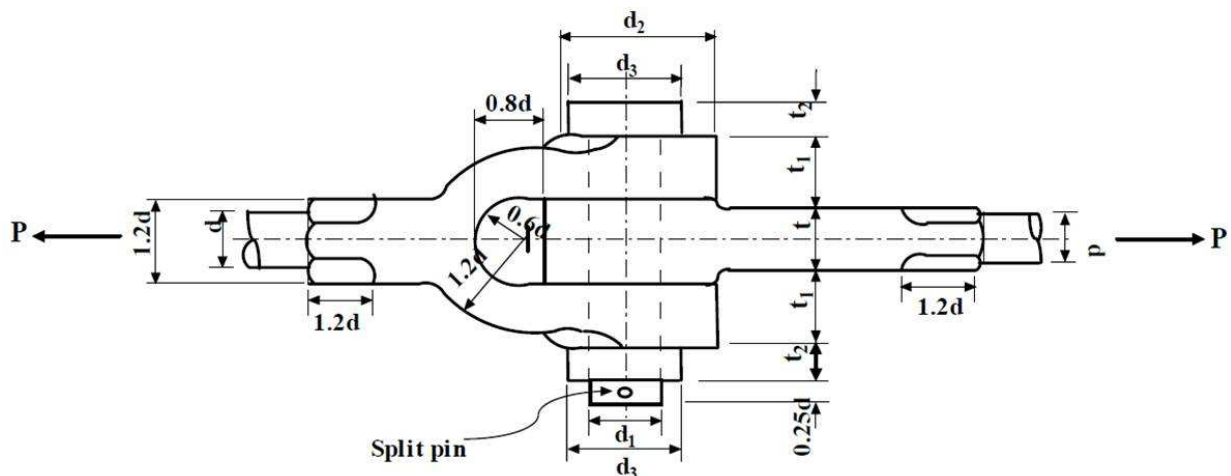


Fig. 2.47 – A typical knuckle joint

These joints are used for different types of connections e.g. tie rods, tension links in bridge structure. In this, one of the rods has an eye at the rod end and the other one is forked with eyes at both the legs. A pin (knuckle pin) is inserted through the rod-end eye and fork-end eyes and is secured by a collar and a split pin. Normally, empirical relations are available to find different dimensions of the joint and they are safe from design point of view. The proportions are given in the Fig. 2.47.

d = diameter of rod

$$d_1 = d \qquad t = 1.25d$$

$$d_2 = 2d \qquad t_1 = 0.75d$$

$$d_3 = 1.5d \qquad t_2 = 0.5d$$

Mean diameter of the split pin = $0.25 d$

However, failures analysis may be carried out for checking. The analyses are shown below assuming the same materials for the rods and pins and the yield stresses in tension, compression and shear are given by σ_t , σ_c and τ .

1. Failure of rod in tension:

$$\frac{\pi}{4} d^2 \sigma_t = P$$

2. Failure of knuckle pin in double shear:

$$2 \frac{\pi}{4} d_1^2 \tau = P$$

3. Failure of knuckle pin in bending (if the pin is loose in the fork):

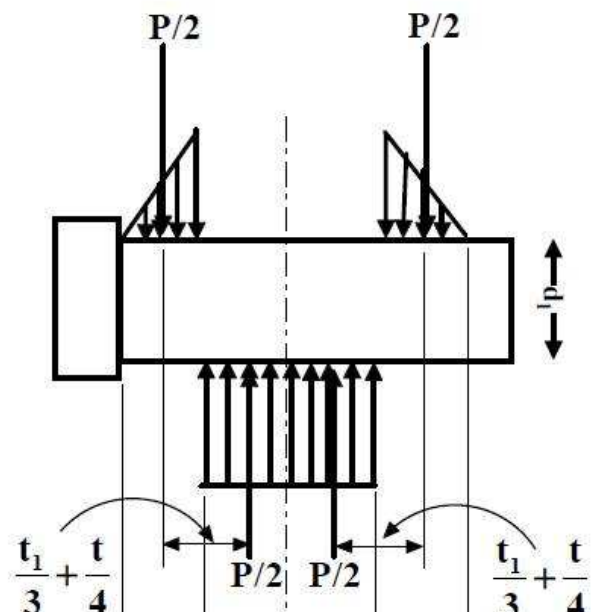
Assuming a triangular pressure distribution on the pin, the loading on the pin is shown in Fig. 2.48.

Equating the maximum bending stress to tensile or compressive yield stress we have,

$$\sigma_t = \frac{16P \left(\frac{t_1}{3} + \frac{t}{4} \right)}{\pi d_1^3}$$

4. Failure of rod eye in shear:

$$(d_2 - d_1) t \tau = P$$



5. Failure of rod eye in crushing:

$$d_1 t \sigma_c = P$$

6. Failure of rod eye in tension:

$$(d_2 - d_1) t \sigma_t = P$$

Fig. 2.48

7. Failure of forked end in shear:

$$2(d_2 - d_1) t_1 \tau = P$$

8. Failure of forked end in tension:

$$2(d_2 - d_1) t_1 \sigma_t = P$$

9. Failure of forked end in crushing:

$$2d_1 t_1 \sigma_c = P$$

The design may be carried out using the empirical proportions and then the analytical relations may be used as checks. For example using the 2nd equation we have, $\tau = \frac{2P}{\pi d_1^2}$. We

may now put value of d_1 from empirical relation and then find $F.S. = \frac{\tau_y}{\tau}$ which should be more than one.

<p>Example Problem-1: Design a typical cotter joint to transmit a load of 50 kN in tension or compression. Consider that the rod, socket and cotter are all made of a material with the following allowable stresses: Allowable tensile stress $\sigma_y = 150$ MPa; Allowable crushing stress $\sigma_c = 110$ MPa; Allowable shear stress $\tau_y = 110$ MPa.</p> <p>Solution:</p> <p>Axial load, $P = \frac{\pi}{4} d^2 \sigma_y$. On substitution this gives $d=20$ mm. In general standard shaft size in mm are:</p> <table data-bbox="177 1904 1235 2018"> <tr> <td>6 mm to 22 mm diameter</td> <td>2 mm in increment</td> </tr> <tr> <td>25 mm to 60 mm diameter</td> <td>5 mm in increment</td> </tr> </table>	6 mm to 22 mm diameter	2 mm in increment	25 mm to 60 mm diameter	5 mm in increment	<p>Reference</p> <p>Fig. 2.34 and 2.36</p>
6 mm to 22 mm diameter	2 mm in increment				
25 mm to 60 mm diameter	5 mm in increment				

60 mm to 110 mm diameter	10 mm in increment	
110 mm to 140 mm diameter	15 mm in increment	
140 mm to 160 mm diameter	20 mm in increment	
500 mm to 600 mm diameter	30 mm in increment	
We therefore choose a suitable rod size to be 25 mm.		
For tension failure across slot $\left(\frac{\pi}{4}d^2 - d_1t\right)\sigma_t = P$. This gives $d_1t = 1.58 \times 10^{-4}$ m ² . From empirical relations we may take $t = 0.4d$ i.e. 10 mm and this gives $d_1 = 15.8$ mm. Maintaining the proportion let $d_1 = 1.2d = 30$ mm.		Fig. 2.37
The tensile failure of socket across slot, $\left(\frac{\pi}{4}(d_2^2 - d_1^2) - (d_2 - d_1)t\right)\sigma_t = P$. This gives $d_2 = 37$ mm. Let $d_2 = 40$ mm.		Fig. 2.38
For shear failure of cotter $2bt\tau = P$. On substitution this gives $b = 22.72$ mm. Let $b = 25$ mm.		Fig. 2.39
For shear failure of rod end $2l_1d_1\tau = P$ and this gives $l_1 = 7.57$ mm. Let $l_1 = 10$ mm.		Fig. 2.40
For shear failure of socket end $2l(d_2 - d_1)\tau = P$. This gives $l = 22.72$ mm. Let $L = 25$ mm.		Fig. 2.41
For crushing failure of socket or rod $(d_3 - d_1)t\sigma_c = P$. This gives $d_3 = 75.5$ mm. Let $d_3 = 77$ mm.		Fig. 2.43
For crushing failure of collar $\left(\frac{\pi}{4}(d_4^2 - d_1^2)\right)\sigma_c = P$. On substitution this gives $d_4 = 38.4$ mm. Let $d_4 = 40$ mm.		Fig. 2.44
For shear failure of collar $\pi d_1 t_1 \tau = P$ which gives $t_1 = 4.8$ mm. Let $t_1 = 5$ mm.		Fig. 2.45
Therefore the final chosen values of dimensions are: $d = 25$ mm; $d_1 = 30$ mm; $d_2 = 40$ mm; $d_3 = 77$ mm; $d_4 = 40$ mm; $t = 10$ mm; $t_1 = 5$ mm; $l = 25$ mm; $l_1 = 10$ mm; $b = 27$ mm.		