

**LOGIC DESIGN**

(Common to CSE &amp; ISE)

**Subject Code: 10CS33**  
**Hours/Week: 04**  
**Total Hours: 52**

**I.A. Marks : 25**  
**Exam Hours: 03**  
**Exam Marks: 100**

**PART-A**

**UNIT – 1** **7 Hours**

**Digital Principles, Digital Logic:** Definitions for Digital Signals, Digital Waveforms, Digital Logic, 7400 TTL Series, TTL Parameters The Basic Gates: NOT, OR, AND, Universal Logic Gates: NOR, NAND, Positive and Negative Logic, Introduction to HDL.

**UNIT – 2** **6 Hours**

**Combinational Logic Circuits**

Sum-of-Products Method, Truth Table to Karnaugh Map, Pairs Quads, and Octets, Karnaugh Simplifications, Don't-care Conditions, Product-of-sums Method, Product-of-sums simplifications, Simplification by Quine-McClusky Method, Hazards and Hazard Covers, HDL Implementation Models.

**UNIT – 3** **6 Hours**

**Data-Processing Circuits:** Multiplexers, Demultiplexers, 1-of-16 Decoder, Encoders, Exclusive-or Gates, Parity Generators and Checkers, MagnitudeComparator, Programmable Array Logic, Programmable Logic Arrays, HDL Implementation of Data Processing Circuits

**UNIT – 4** **7 Hours**

**Clocks, Flip-Flops:** Clock Waveforms, TTL Clock, Schmitt Trigger, Clocked D FLIP-FLOP, Edge-triggered D FLIP-FLOP, Edge-triggered JK FLIP-FLOP, FLIP-FLOP Timing, JK Master-slave FLIP-FLOP, Switch Contact Bounce Circuits, Various Representation of FLIP-FLOPs, Analysis of Sequential Circuits, HDL Implementation of FLIP-FLOP

**PART-B**

**UNIT – 5** **6 Hours**

**Registers:** Types of Registers, Serial In - Serial Out, Serial In - Parallel out, Parallel In - Serial Out, Parallel In - Parallel Out, Universal Shift Register, Applications of Shift Registers, Register Implementation in HDL

**UNIT – 6** **7 Hours**

**Counters:** Asynchronous Counters, Decoding Gates, Synchronous Counters, Changing the Counter Modulus, decade Counters, Presetable Counters, Counter Design as a Synthesis problem, A Digital Clock, Counter Design using HDL

**UNIT – 7****7 Hours**

**Design of Synchronous and Asynchronous Sequential Circuits:** Design of Synchronous Sequential Circuit: Model Selection, State Transition Diagram, State Synthesis Table, Design Equations and Circuit Diagram, Implementation using Read Only Memory, Algorithmic State Machine, State Reduction Technique. Asynchronous Sequential Circuit: Analysis of Asynchronous Sequential Circuit, Problems with Asynchronous Sequential Circuits, Design of Asynchronous Sequential Circuit, FSM Implementation in HDL

**UNIT – 8****6 Hours**

**D/A Conversion and A/D Conversion:** Variable, Resistor Networks, Binary Ladders, D/A Converters, D/A Accuracy and Resolution, A/D Converter- Simultaneous Conversion, A/D Converter-Counter Method, Continuous A/D Conversion, A/D Techniques, Dual-slope A/D Conversion, A/D Accuracy and Resolution

Text Book:

1. Donald P Leach, Albert Paul Malvino & Goutam Saha: Digital Principles and Applications, 7th Edition, Tata McGraw Hill, 2010.

**Reference Books:**

1. Stephen Brown, Zvonko Vranesic: Fundamentals of Digital Logic Design with VHDL, 2nd Edition, Tata McGraw Hill, 2005.
2. R D Sudhaker Samuel: Illustrative Approach to Logic Design, Sanguine-Pearson, 2010.
3. Charles H. Roth: Fundamentals of Logic Design, Jr., 5th Edition, Cengage Learning, 2004.
4. Ronald J. Tocci, Neal S. Widmer, Gregory L. Moss: Digital Systems Principles and Applications, 10th Edition, Pearson Education, 2007

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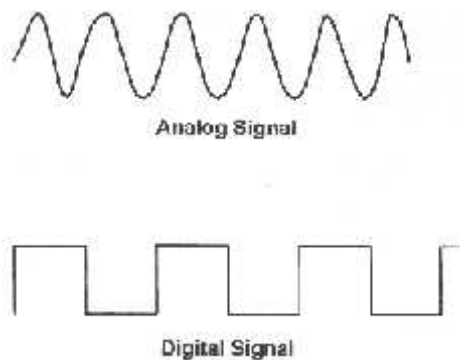
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## Unit-1 : Digital Principles, Digital Logic

### 1.1 Definitions of Analog vs Digital signals

An **Analog signal** is any continuous signal for which the time varying feature (variable) of the signal is a representation of some other time varying quantity, i.e., analogous to another time varying signal. It differs from a digital signal in terms of small fluctuations in the signal which are meaningful.

A **digital signal** uses discrete (discontinuous) values. By contrast, non-digital (or analog) systems use a continuous range of values to represent information. Although digital representations are discrete, the information represented can be either discrete, such as numbers or letters, or continuous, such as sounds, images, and other measurements of continuous systems.



#### Comparison chart

	<b>Analog</b>	<b>Digital</b>
Technology:	Analog technology records waveforms as they are.	Converts analog waveforms into set of numbers and records them. The numbers are converted into voltage stream for representation.
Representation:	Uses continuous range of values to represent information.	Uses discrete or discontinuous values to represent information.
Uses:	Can be used in various computing platforms and under operating systems like Linux, Unix, Mac OS and Windows.	Computing and electronics
Signal:	Analog signal is a continuous signal which transmits information as a response to changes in physical phenomenon.	Digital signals are discrete time signals generated by digital modulation.

	<b>Analog</b>	<b>Digital</b>
Clocks:	Analog clocks indicate time using angles.	Digital clocks use numeric representation to indicate time.
Computer:	Analog computer uses changeable continuous physical phenomena such as electrical, mechanical, hydraulic	Digital computers represent changing quantities incrementally as and when their values change.

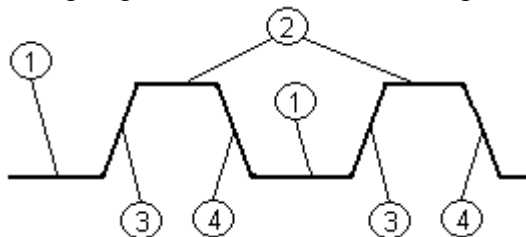
### 1.2 Waveforms in digital systems

In computer architecture and other digital systems, a waveform that switches between two voltage levels representing the two states of a Boolean value (0 and 1) is referred to as a *digital signal*, even though it is an analog voltage waveform, since it is interpreted in terms of only two levels.

The clock signal is a special digital signal that is used to synchronize digital circuits. The image shown can be considered the waveform of a clock signal. Logic changes are triggered either by the rising edge or the falling edge.

The given diagram is an example of the practical pulse and therefore we have introduced two new terms that are:

- Rising edge: the transition from a low voltage (level 1 in the diagram) to a high voltage (level 2).
- Falling edge: the transition from a high voltage to a low one.



### 1.3 TTL Series

Normally Binary Logic Values are defined as either Logic '1' or Logic '0' depending on the level of the output voltage. Another additional (intermediate value) is the 'Undefined value'. Logic levels can either be Positive logic or Negative Logic. For eg:

In TTL Logic Levels (positive logic) logic high or Logic 1 is between  $2.4V \leq V_H \leq 5V$ . Logic '0' or low logic is between  $0V \leq V_L \leq 0.4V$  and the Undefined value is between  $0.4V < \text{undefined} < 2.4V$

Logic families are classified based on either the devices used ,example: diodes ,transistors etc. or the structure of Digital Circuits , example: ECL ,Wired logic etc.

The following are the examples of logic families based on the devices used and their structure,

- DTL :Diode Transistor Logic
- RTL :Resistor Transistor Logic
- TTL :Transistor Transistor Logic
- ECL :Emitter Coupled Logic
- CMOS :Complementary MOSFET Logic

The various logic families differ in the current driving capabilities,Logic Levels, propagation delays and a few other other parameters. The Comparison of TTL and CMOS is clearly illustrated in the following table as an example of differences in the logic families:

TTL	CMOS
<ul style="list-style-type: none"> <li>• Faster drive capability</li> <li>• Stronger drive capability</li> </ul>	<ul style="list-style-type: none"> <li>• Low power consumption</li> <li>• Simpler to make</li> <li>• Greater packing density</li> <li>• Better noise immunity</li> </ul>

Integration Levels:

The devices greatly differ in the density of fabrication ie the levels of integration used.Dependng on the number of transistors/diodes/gates used in the chip they are broadly classified as :

- SSI -small scale integration
- MSI -medium scale integration
- LSI -large scale integration
- VLSI -very large scale integration
- ULSI -ultra large scale integration
- GSI -giant scale integration



Levels of integration	Transistors/package	Gates/chip	Applications
SSI	1-100	<12	Logic gates Op-amps
MSI	100-1000	12-99	Registers Filters
LSI	1000-10000	1000	8 bit processor, A/D converter
VLSI	10k gates/chip		16,32 bit processor 256KB memory DS processor
ULSI	100k gates/chip		64 bit processor 8 MB memory Image processor
GSI	1M gates/chip		64 MB memory multiprocessor

### Speed of Operation:

As signals propagate through the various gates there is a finite time required for the signal change to occur, eg the time required for the input high of a n inverter to change to logic low at the output. This implies that there is a limitation on the no of times the output can change or the speed of operation of the gate. The parameters of importance for the speed of operation are :

- tLH- low to high rise time (tr) : it is defined as the time interval for the signal to rise between 10% to 90% of Vdd
- tHL- high to low time or fall time (tf): it is defined as the time for signal to fall from 90% Vdd to 10% Vdd
- 

The switching is fast with  $t_{min}=t_{hl}+t_{lh}$

Therefore maximum switching freq is achieved when  $f_{max}=1/t_{min}$

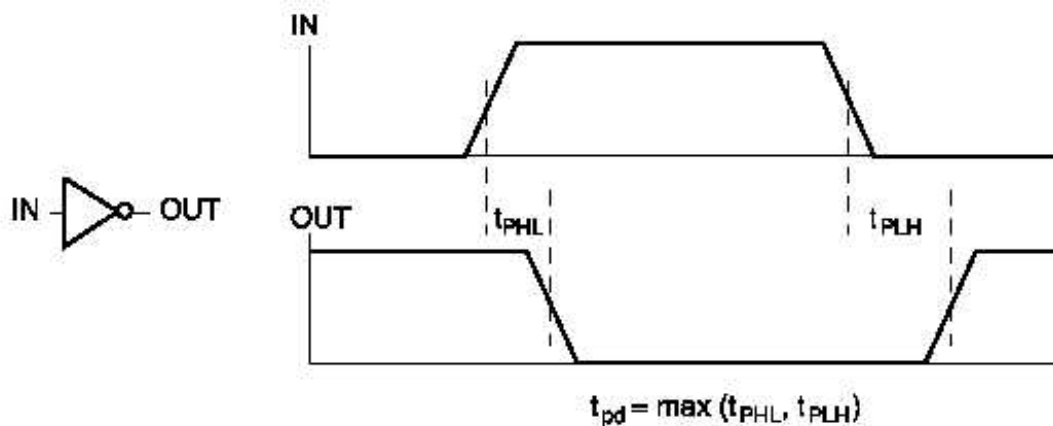
The switching speed is limited due to the effect of capacitance at the base emitter /collector and ground etc.

For eg: if  $t_{hl} = 0.5 \text{ nsec}$ ,  $t_{lh} = 1.0 \text{ nsec}$

Then  $t_{min} = 1.5 \text{ nsec}$  And  $f_{max} = 1/t_{min} = 666.67 \text{ Mhz}$

### Propagation delay:

It is the physical delay as the logical signal propagates through the gates. It differs depending on whether the output transition goes from cutoff to saturation or from saturation to cut-off.



As the loads are connected to gates to realize the necessary logic operations the output signal levels are affected. This is because there is a current flow between the gates due to which there is power consumption. Thus the number of circuits(similar gates) that can be connected to the gates gets limited.

- Fan-out of a gate is the number of gates driven by that gate i.e the maximum number of gates (load ) that can exist without impairing the normal operation of the gate.
- Fan-in of a gate is the number of inputs that can be connected to it without impairing the normal operation of the gate.

### 1.4 Overview of Basic Gates and Universal Logic Gates

A logic gate is an electronic circuit/device which makes the logical decisions. To arrive at this decisions, the most common logic gates used are OR, AND, NOT, NAND, and NOR gates. The NAND and NOR gates are called universal gates. The exclusive-OR gate is another logic gate which can be constructed using AND, OR and NOT gate.

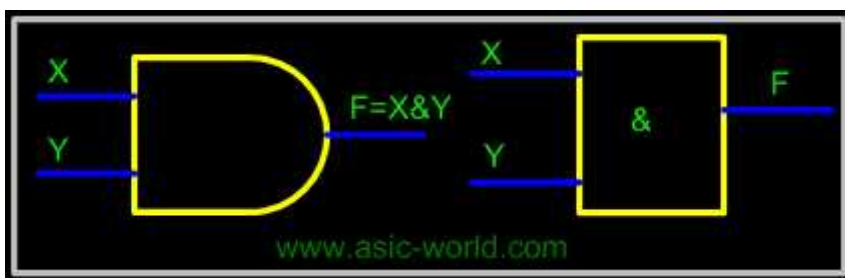
Logic gates have one or more inputs and only one output. The output is active only for certain input combinations. Logic gates are the building blocks of any digital circuit. Logic gates are also called switches. With the advent of integrated circuits, switches have been replaced by

TTL (Transistor Transistor Logic) circuits and CMOS circuits. Here I give example circuits on how to construct simple gates.

### AND Gate

The AND gate performs logical multiplication, commonly known as AND function. The AND gate has two or more inputs and single output. The output of AND gate is HIGH only when all its inputs are HIGH (i.e. even if one input is LOW, Output will be LOW).

If X and Y are two inputs, then output F can be represented mathematically as  $F = X \cdot Y$ . Here dot (.) denotes the AND operation. Truth table and symbol of the AND gate is shown in the figure below.



X	Y	F=(X.Y)
0	0	0
0	1	0
1	0	0
1	1	1

### OR Gate

The OR gate performs logical addition, commonly known as OR function. The OR gate has two or more inputs and single output. The output of OR gate is HIGH only when any one of its inputs are HIGH (i.e. even if one input is HIGH, Output will be HIGH).

If X and Y are two inputs, then output F can be represented mathematically as  $F = X + Y$ . Here plus sign (+) denotes the OR operation. Truth table and symbol of the OR gate is shown in the figure below.

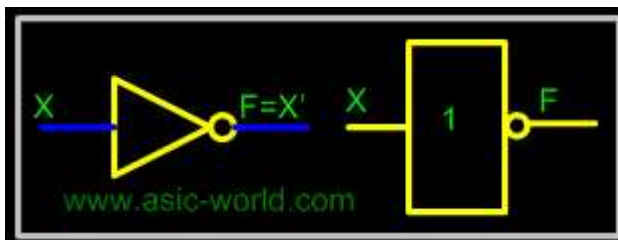


	$YF=(X+Y)$
0	0 0
0	1 1
1	0 1
1	1 1

### NOT Gate

The NOT gate performs the basic logical function called inversion or complementation. NOT gate is also called inverter. The purpose of this gate is to convert one logic level into the opposite logic level. It has one input and one output. When a HIGH level is applied to an inverter, a LOW level appears on its output and vice versa.

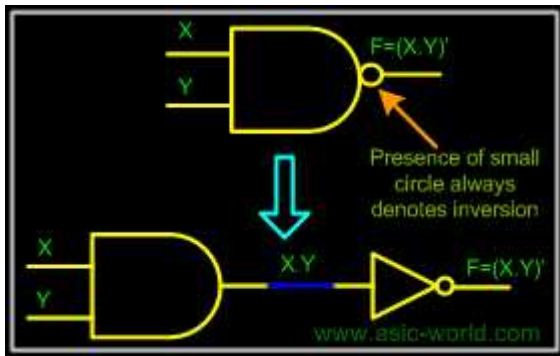
If X is the input, then output F can be represented mathematically as  $F = X'$ , Here apostrophe (') denotes the NOT (inversion) operation. There are a couple of other ways to represent inversion,  $F = !X$ , here ! represents inversion. Truth table and NOT gate symbol is shown in the figure below.



X	$Y=X'$
0	1
1	0

### NAND Gate

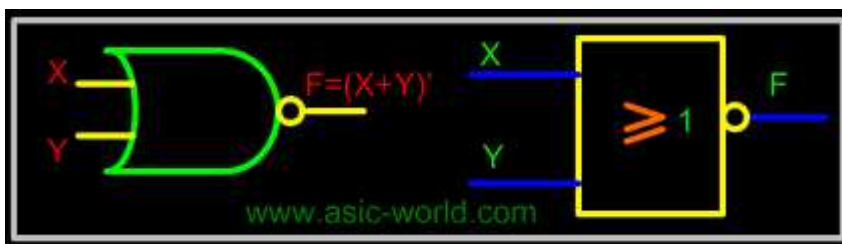
NAND gate is a cascade of AND gate and NOT gate, as shown in the figure below. It has two or more inputs and only one output. The output of NAND gate is HIGH when any one of its input is LOW (i.e. even if one input is LOW, Output will be HIGH).



X	Y	$F=(X.Y)'$
0	0	1
0	1	1
1	0	1
1	1	0

**NOR Gate**

NOR gate is a cascade of OR gate and NOT gate, as shown in the figure below. It has two or more inputs and only one output. The output of NOR gate is HIGH when any all its inputs are LOW (i.e. even if one input is HIGH, output will be LOW).

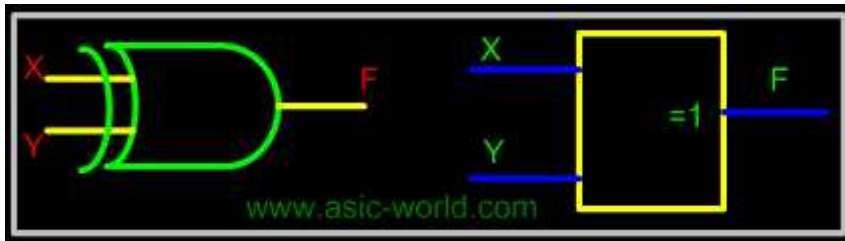


X	Y	$F=(X+Y)'$
0	0	1
0	1	0
1	0	0
1	1	0

**XOR Gate**

An Exclusive-OR (XOR) gate is gate with two or three or more inputs and one output. The output of a two-input XOR gate assumes a HIGH state if one and only one input assumes a HIGH state. This is equivalent to saying that the output is HIGH if either input X or input Y is HIGH exclusively, and LOW when both are 1 or 0 simultaneously.

If  $X$  and  $Y$  are two inputs, then output  $F$  can be represented mathematically as  $F = X \oplus Y$ , Here  $\oplus$  denotes the XOR operation.  $X \oplus Y$  and is equivalent to  $X.Y' + X'.Y$ . Truth table and symbol of the XOR gate is shown in the figure below.

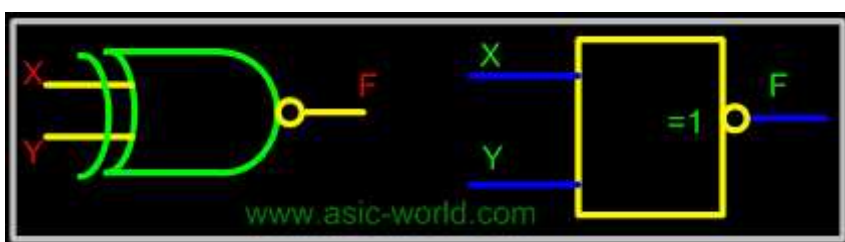


X	Y	$F=(X \oplus Y)$
0	0	0
0	1	1
1	0	1
1	1	0

### XNOR Gate

An Exclusive-NOR (XNOR) gate is gate with two or three or more inputs and one output. The output of a two-input XNOR gate assumes a HIGH state if all the inputs assumes same state. This is equivalent to saying that the output is HIGH if both input  $X$  and input  $Y$  is HIGH exclusively or same as input  $X$  and input  $Y$  is LOW exclusively, and LOW when both are not same.

If  $X$  and  $Y$  are two inputs, then output  $F$  can be represented mathematically as  $F = X \oplus Y$ , Here  $\oplus$  denotes the XNOR operation.  $X \oplus Y$  and is equivalent to  $X.Y + X'.Y'$ . Truth table and symbol of the XNOR gate is shown in the figure below.



X	Y	$F=(X \oplus Y)'$
0	0	1
0	1	0
1	0	0
1	1	1

## 1.6 Boolean Laws and Theorems

### A. Axioms

Consider a set  $S = \{0, 1\}$ . Consider two binary operations,  $+$  and  $\cdot$ , and one unary operation,  $'$ , that act on these elements.  $[S, \cdot, +, ', 0, 1]$  is called a switching algebra that satisfies the following axioms  $S$ .

### B. Closure

If  $X \in S$  and  $Y \in S$  then  $X \cdot Y \in S$

If  $X \in S$  and  $Y \in S$  then  $X + Y \in S$

### C. Identity

an identity 0 for  $+$  such that  $X + 0 = X$

an identity 1 for  $\cdot$  such that  $X \cdot 1 = X$

### D. Commutative Laws

$$X + Y = Y + X$$

$$X \cdot Y = Y \cdot X$$

### E. Distributive Laws

$$X \cdot (Y + Z) = X \cdot Y + X \cdot Z$$

$$X + Y \cdot Z = (X + Y) \cdot (X + Z)$$

### Idempotent Law

$$X + X = X$$

$$X \cdot X = X$$

### DeMorgan's Law

$(X + Y)' = X' \cdot Y'$ , These can be proved by the use of truth tables.

Proof of  $(X + Y)' = X' \cdot Y'$

	Y	X+Y	(X+Y)'
0	0	0	1
0	1	1	0
1	0	1	0

1	1	1	0
X	Y	X'	Y'
0	0	1	1
0	1	1	0
1	0	0	1
1	1	0	0

The two truth tables are identical, and so the two expressions are identical.

$(X.Y) = X' + Y'$ , These can be proved by the use of truth tables.

Proof of  $(X.Y) = X' + Y'$

X	Y	X.Y	(X.Y)'
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

X	Y	X'	Y'	X'+Y'
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

### 1.7 Introduction to HDL: Hardware Description Language

HDL is a language that describes the hardware of digital systems in a textual form. It resembles a programming language, but is specifically oriented to describing hardware structures and behaviors.

The main difference with the traditional programming languages is HDL's representation of extensive parallel operations whereas traditional ones represents mostly serial operations. HDL can be used to represent logic diagrams, Boolean expressions, and other more complex digital circuits

There are two standard HDL's that are supported by IEEE.

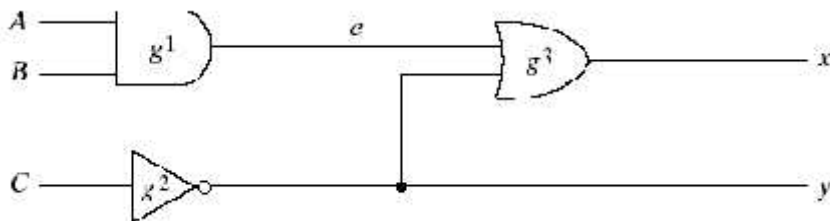
- **VHDL** (*Very-High-Speed Integrated Circuits Hardware Description Language*) - Sometimes referred to as VHSIC HDL, this was developed from an initiative by US. Dept. of Defense.
- **Verilog HDL** – developed by Cadence Data systems and later transferred to a consortium called *Open Verilog International (OVI)*.

Verilog: Verilog HDL has a syntax that describes precisely the legal constructs that can be used in the language.



- ◆ It uses about 100 keywords pre-defined, lowercase, identifiers that define the language constructs.
- ◆ Example of keywords: *module*, *endmodule*, *input*, *output*, *wire*, *and*, *or*, *not*, etc.,
- ◆ Any text between two slashes (//) and the end of line is interpreted as a comment.
- ◆ Blank spaces are ignored and names are case sensitive.

A *module* is the building block in Verilog. It is declared by the keyword *module* and is always terminated by the keyword *endmodule*. Each statement is terminated with a semicolon, but there is no semi-colon after *endmodule*.



HDL Example

```

module smpl_circuit(A,B,C,x,y);
    input A,B,C;
    output x,y;
    wire e;
    and g1(e,A,B);
    not g2(y,C);
    or g3(x,e,y);
endmodule

```

## Unit-2 : Combinational Logic Circuits

DeMorgans Laws are applicable for any number of variables.

### Boundedness Law

$$X + 1 = 1$$

$$X \cdot 0 = 0$$

### Absorption Law

$$X + (X \cdot Y) = X$$

$$X \cdot (X + Y) = X$$

### Elimination Law

$$X + (X' \cdot Y) = X + Y$$

$$X \cdot (X' + Y) = X \cdot Y$$

**Unique Complement theorem**

If  $X + Y = 1$  and  $X \cdot Y = 0$  then  $X = Y'$

**Involution theorem**

$$X'' = X$$

$$0' = 1$$

**Associative Properties**

$$X + (Y + Z) = (X + Y) + Z$$

$$X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$$

**PRINCIPLE OF DUALITY**

One can transform the given expression by interchanging the operation (+) and ( $\bullet$ ) as well as the identity elements 0 and 1. Then the expression will be referred as dual of each other. This is known as the principle of duality.

Example  $x + x = 1$  then the dual expression is

$$x \bullet x = 0$$

A procedure which will be used to write Boolean expressions from truth table is known as canonical formula. The canonical formulas are of two types

**1. Minterm canonical formulas****2. Maxterm canonical formulas****2.1 Sum-of-Products Method****MINTERM CANONICAL FORMULAS**

Minterms are product of terms which represents the functional values of the variables appear either in complemented or uncomplemented form.

Ex:  $f(x,y,z) = x y z + x y z + x y z$

The Boolean expression which is represented above is also known as SOP or disjunctive formula. The truth table is

x y z	f
0 0 0	0

0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	0
1 1 1	0

**m- NOTATION**

To simplify the writing of a minterm in canonical formula for a function is performed using the symbol **mi**. Where **i** stands for the row number for which the function evaluates to 1.

The m-notation for 3- variable an function Boolean function

$$f(x,y,z) = x y z + x y z + x y z \text{ is written as}$$

$$f(x,y,z) = m1 + m3 + m4 \text{ or}$$

$$f(x,y,z) = \sum m(1,3,4)$$

**A three variable m- notation truth variable**

x y z	Decimal designator of row	Minterm	m-notation
0 0 0	0	x y z	m0
0 0 1	1	x y z	m1
0 1 0	2	x y z	m2
0 1 1	3	x y z	m3
1 0 0	4	x y z	m4
1 0 1	5	x y z	m5
1 1 0	6	x y z	m6
1 1 1	7	x y z	m7

**MAXTERM CANONICAL FORM**

Maxterm are sum terms where the variable appear once either in complement or un-complement forms and these terms corresponds to a functional value representing 0.

$$\text{Ex. } f(x,y,z) = (x + y + z) (x + y + z) (x + y + z)$$

$$= \prod M(0, 2, 5)$$

$$= M_0, M_2, M_5$$

## 2.2 KARNAUGH MAPS (K-MAP)

A method for graphically determining implicants and implicates of a Boolean function was developed by Veitch and modified by Karnaugh. The method involves a diagrammatic representation of a Boolean algebra. This graphic representation is called map.

It is seen that the truth table can be used to represent complete function of  $n$ -variables. Since each variable can have value of 0 or 1. The truth table has  $2^n$  rows. Each row of the truth table consists of two parts (1) an  $n$ -tuple which corresponds to an assignment to the  $n$ -variables and (2) a functional value.

A Karnaugh map (K-map) is a geometrical configuration of  $2^n$  cells such that each of the  $n$ -tuples corresponds to a row of a truth table uniquely locates a cell on the map. The functional values assigned to the  $n$ -tuples are placed as entries in the cells, i.e. 0 or 1 are placed in the associated cell.

An important aspect about the construction of K-map is the arrangement of the cells. Two cells are physically adjacent within the configuration if and only if their respective  $n$ -tuples differ in exactly by one element. So that the Boolean law  $x+x=1$  can be applied to adjacent cells. Ex. Two 3-tuples (0,1,1) and (0,1,0) are physically adjacent since these tuples vary by one element.

**One variable** : One variable needs a map of  $2^1 = 2$  cells map as shown below

x f(x)

0 f(0)

1 f(1)

**TWO VARIABLE** : Two variable needs a map of  $2^2 = 4$  cells

x y f(x,y)

0 0 f(0,0)

0 1 f(0,1)

1 0 f(1,0)

1 1 f(1,1)

**THREE VARIABLE** : Three variable needs a map of  $2^3 = 8$  cells. The arrangement of cells are as follows

x	y	z	f(x,y,z)
0	0	0	f(0,0,0)
0	0	1	f(0,0,1)
0	1	0	f(0,1,0)
0	1	1	f(0,1,1)
1	0	0	f(1,0,0)
1	0	1	f(1,0,1)
1	1	0	f(1,1,0)
1	1	1	f(1,1,1)

**FOUR VARIABLE** : Four variable needs a map of  $2^4 = 16$  cells. The arrangement of cells are as follows

w	x	y	z	f(w,x,y,z)	w	x	y	z	f(w,x,y,z)
0	0	0	0	f(0,0,0,0)	1	0	1	0	f(1,0,1,0)
0	0	0	1	f(0,0,0,1)	1	0	1	1	f(1,0,1,1)
0	0	1	0	f(0,0,1,0)	1	1	0	0	f(1,1,0,0)
0	0	1	1	f(0,0,1,1)	1	1	0	1	f(1,1,0,1)
0	1	0	0	f(0,1,0,0)	1	1	1	0	f(1,1,1,0)
0	1	0	1	f(0,1,0,1)	1	1	1	1	f(1,1,1,1)
0	1	1	0	f(0,1,1,0)					
0	1	1	1	f(0,1,1,1)					
1	0	0	0	f(1,0,0,0)					
1	0	0	1	f(1,0,0,1)					

Four variable K-map.

0000	0001	0011	0010
0100	0101	0111	1010

1100	1101	1111	1110
1000	1001	1011	1010

Ex. Obtain the minterm canonical formula of the three variable problem given below

$$f(x, y, z) = x y z + x \bar{y} z + x y \bar{z} + x \bar{y} \bar{z}$$

$$f(x, y, z) = \sum m(0, 2, 4, 5)$$

00                  01                  11                  11

1	0	0	1
1	1	0	0

Ex. Express the minterm canonical formula of the four variable K-map given below

		yz			
		00	01	11	10
wx	1	1	1	0	1
	1	1	1	0	0
	0	0	0	0	0
	1	0	0	0	1

$$f(w, x, y, z) = w x y z + w x y \bar{z} + w x \bar{y} z + w x \bar{y} \bar{z} + w \bar{x} y z + w \bar{x} y \bar{z}$$

$$f(w, x, y, z) = \sum m(0, 1, 2, 4, 5)$$

Ex. Obtain the max term canonical formula

(POS) of the three variable problem stated above

$$f(x,y,z) = (x + y + z)(x + y + z)(x + y + z)$$

$$(x + y + z)$$

$$f(x,y,z) = \Pi M(1,3,6,7)$$

Ex Obtain the max term canonical formula

(POS) of the four variable problem stated above

$$f(w,x,y,z) = (w + x + y + z)(w + x + y + z)(w + x + y + z)$$

$$(w + x + y + z)(w + x + y + z)(w + x + y + z)$$

$$(w + x + y + z)(w + x + y + z)(w + x + y + z)$$

$$f(w,x,y,z) = \Pi M(3,6,7,9,11,12,13,14,15)$$

### 2.3 PRODUCT AND SUM TERM REPRESENTATION OF K –MAP

1.The importance of K-map lies in the fact that it is possible to determine the implicants and implicates of a function from the pattern of 0's and 1's appearing in the map. The cell of a K-map has entry of 1's is referred as 1-cell and that of 0,s is referred as 0-cell.

2. The construction of an n-variable map is such that any set of 1-cells or 0-cells which form a  $2^a \times 2^b$  rectangular grouping describing a product or sum term with n-a-b variables, where a and b are non-negative no.s

3. The rectangular grouping of these dimensions referred as Subcubes. The subcubes must be the power of 2 i.e.  $2^{a+b}$  equals to 1,2,4,8 etc.

4. For three variable and four variable K-map it must be remembered that the edges are also adjacent cells or subcubes hence they will be grouped together.

5. Given an n-variable map with a pair of adjacent 1-cells or 0-cells can result n-1 variable. Where as if a group of four adjacent subcubes are formed than it can result n-2 variables. Finally if we have eight adjacent cells are grouped may result n-3 variable product or sum term.

Typical pair of subcubes

1			
	1	1	
1		1	1
1			

Typical group of four adjacent subcube

1	1		
1	1		

1	1	1	1

Typical group of four adjacent subcubes.

1			1
1			1

1			1
1			1

Typical group of eight

1	1	1	1
1	1	1	1

adjacent subcubes.

	1	1	
	1	1	
	1	1	
	1	1	

1	1	1	1
1	1	1	1

1			1
1			1
1			1
1			1



## 2.4 USING K-MAP TO OBTAIN MINIMAL EXPRESSION FOR COMPLETE BOOLEAN FUNCTIONS :

How to obtain a minimal expression of SOP or POS of given function is discussed.

### PRIME IMPLICANTS and K-MAPS :

#### CONCEPT OF ESSENTIAL PRIME IMPLICANT

<b>00</b>	<b>01</b>	<b>11</b>	<b>10</b>
0	0	0	1
0	0	1	1

$$f(x,y,z) = xy + yz$$

#### ALGORITHM TO FIND ALL PRIME IMPLICANTS

A General procedure is listed below

1. For an n-variable map make  $2^n$  entries of 1's. or 0's.
2. Assign  $I = n$ , so that find out biggest rectangular group with dimension  $2^a \times 2^b = 2^{n-1}$ .
3. If bigger rectangular group is not possible  $I = I-1$  form the subcubes which consist of all the previously obtained subcube repeat the step till all 1-cell or 0's are covered.
4. Remaining is essential prime implicants

1. Essential prime implicants
2. Minimal sums
3. Minimal products

#### MINIMAL EXPRESSIONS OF INCOMPLETE BOOLEAN FUNCTIONS

##### 1. Minimal sums

##### 2. Minimal products.

#### EXAMPLE TO ILLUSTRATE HOW TO OBTAIN ESSENTIAL PRIMES

$$1. f(x,y,z) = \sum m(0,1,5,7)$$

$$\text{Ans } f(x,y,z) = xz + xy$$

$$2. f(w,x,y,z) = \sum m(1,2,3,5,6,7,8,13)$$

$$\text{Ans. } f(w,x,y,z) = w z + w y + x y z + w x y z$$

#### MINIMAL SUMS

$$f(w,x,y,z) = \sum m(0,1,2,3,5,7,11,15)$$

#### MINIMAL PRODUCTS

$$F(w,x,y,z) = \sum m(1,3,4,5,6,7,11,14,15)$$

#### MINIMAL EXPRESSIONS OF INCOMPLETE BOOLEAN FUNCTIONS

$$f(W,X,Y,Z) = \sum m(0,1,3,7,8,12) + dc(5,10,13,14)$$

### 2.5 QUINE – McCLUSKEY METHOD

Using K-maps for simplification of Boolean expressions with more than six variables becomes a tedious and difficult task. Therefore a tabular method illustrate below can be used for the purpose.

#### ALGORITHM FOR GENERATING PRIME IMPLICANTS

The algorithm procedure is listed below

1. Express each minterm of the function in its binary representation.
2. List the minterms by increasing index.
3. Separate the sets of minterms of equal index with lines.
4. Let  $i = 0$ .
5. Compare each term of index  $I$  with each term of index  $I+1$ . For each pair of terms that can combine which has only one bit position difference.
6. Increase  $I$  by 1 and repeat step 5. The increase of  $I$  continued until all terms are compared. The new list containing all implicants of the function that have one less variable than those implicants in the generating list.
7. Each section of the new list formed has terms of equal index. Steps 4,5, and 6 are repeated on this list to form another list. Recall that two terms combine only if they have their dashes in the same relative positions and if they differ in exactly one bit position.
8. The process terminates when no new list is formed.
9. All terms without check marks are prime implicants.

Example: Find all the prime implicants of the function

$$f(w,x,y,z) = \sum m(0,2,3,4,8,10,12,13,14)$$

Step 1: Represent each minterm in its 1-0 notation

no.	minterm	1-0 notation	index
0	w x y z	0 0 0 0	0
2	w x y z	0 0 1 0	1
3	w x y z	0 0 1 1	2
4	w x y z	0 1 0 0	1
8	w x y z	1 0 0 0	1
10	w x y z	1 0 1 0	2
12	w x y z	1 1 0 0	2
13	w x y z	1 1 0 1	3
14	w x y z	1 1 1 0	3

Step 2: List the minterm in increasing order of their index.

No.	w x y z	index
0	0 0 0 0	Index 0
2	0 0 1 0	
4	0 1 0 0	Index 1
8	1 0 0 0	
3	0 0 1 1	
10	1 0 1 0	Index 2
12	1 1 0 0	
13	1 1 0 1	
14	1 1 1 0	Index 3

	W x y z	index

0,2	00-0	
0,4	0-00	
0,8	- 000	
2,3	001-	
2,10	- 010	
4,12	- 100	
8,10	10-0	
8,12	1-00	
10,14	1-10	
12,13	110-	
12,14	11-0	

	w x y z
(0, 2, 8, 10)	__ 0 __ 0
(0, 4, 8,12 )	__ __ 0 0(index 0)
(8,10,12,14)	1__ __ 0 (index 1)

$$F(w,x,y,z)=x z + y z +w z+w x y +w x z$$

PETRICK'S METHOD OF DETERMINING IRREDUNDANT EXPRESSIONS

FIND THE PRIME IMPLICANTS AND IRREDUNDANT EXPRESSION

$$F(W,X,Y,Z)= \sum M(0,1,2,5,7,8,9,10,13,15)$$

$$A=X Y , B= X Z C= Y Z D= X Z$$

$$P = (A+B)(A+C) (B)(C+D)(D)(A+B)(A+C)(B)(C+D)(D)$$

$$P = (A +C)(BD) = ABD +BCD$$

$$F1(W,X,Y,Z)= ABD =X Y +X Z +X Z$$

$$F2(W,X,Y,Z) = BCD = X Z + Y Z +X Z$$

DECIMAL METHOD FOR OBTAINING PRIME IMPLICANTS

The prime implicants can be obtained for decimal number represented minterms. In this procedure binary number are not used to find out prime implicants

$$f(w, x, y, z) = \sum m(0, 5, 6, 7, 9, 10, 13, 14, 15)$$

$$f_{\text{sop}} = xy + xz + xyz + wyz + w x y z$$

### MAP ENTERED VARIABLE (MEV)

It is graphical approach using k-map to have a variable of order n. Where in we are using a K-map of n-1 variable while map is entered with output function and variable.

$$f(w, x, y, z) = \sum m(2, 3, 4, 5, 13, 15) + dc(8, 9, 10, 11)$$

$$\text{Ans. } f_{\text{sop}} = w z + x y + w x y$$

## 2.9 HDL IMPLEMENTATION METHODS

A module can be described in any one (or a combination) of the following modeling techniques.

- **Gate-level modeling** using instantiation of primitive gates and user defined modules.
  - This describes the circuit by specifying the gates and how they are connected with each other.
- **Dataflow modeling** using continuous assignment statements with the keyword **assign**.
  - This is mostly used for describing combinational circuits.
- **Behavioral modeling** using procedural assignment statements with keyword **always**.
  - This is used to describe digital systems at a higher level of abstraction.

Gate-level modeling: Here a circuit is specified by its logic gates and their interconnections.

- ◆ It provides a textual description of a schematic diagram.
- ◆ Verilog recognizes 12 basic gates as predefined primitives.
  - 4 primitive gates of 3-state type.
  - Other 8 are: and, nand, or, nor, xor, xnor, not, buf

```
//Gate-level hierarchical description of 4-bit adder
module halfadder (S,C,x,y);
  input x,y;
  output S,C;
  //Instantiate primitive gates
  xor (S,x,y);
  and (C,x,y);
endmodule
```

Dataflow Modeling: Dataflow modeling uses continuous assignments and the keyword **assign**. A continuous assignment is a statement that assigns a value to a net. The value assigned to the net is specified by an expression that uses operands and operators.

//Dataflow description of a 2-to-4-line decoder

```
module decoder_df (A,B,E,D);  
  input A,B,E;  
  output [0:3] D;  
  assign D[0] = ~(~A & ~B & ~E),  
         D[1] = ~(~A & B & ~E),  
         D[2] = ~(A & ~B & ~E),  
         D[3] = ~(A & B & ~E);  
endmodule
```

Behavioral Modeling : Behavioral modeling represents digital circuits at a functional and algorithmic level.

- ♦ It is used mostly to describe sequential circuits, but can also be used to describe combinational circuits.
- ♦ Behavioral descriptions use the keyword **always** followed by a list of procedural assignment statements.
- ♦ The target output of procedural assignment statements must be of the **reg** data type.
- ♦ A **reg** data type retains its value until a new value is assigned.

//Behavioral description of 2-to-1-line multiplexer

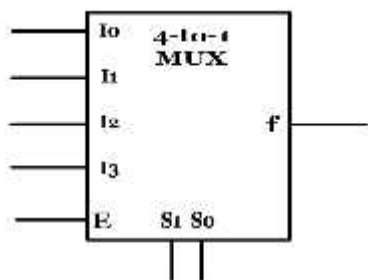
```
module mux2x1_bh(A,B,select,OUT);  
  input A,B,select;  
  output OUT;  
  reg OUT;  
  always @(select or A or B)  
    if (select == 1) OUT = A;  
    else OUT = B;  
endmodule
```

**Unit-3 :Data-Processing Circuits**

3.1 Multiplexer

4-1 line Multiplexer

Multiplexers also called data selectors are another MSI devices with a wide range of applications in microprocessor and their peripherals design. The following diagrams show the symbol and truth table for the 4-to-1 mux.



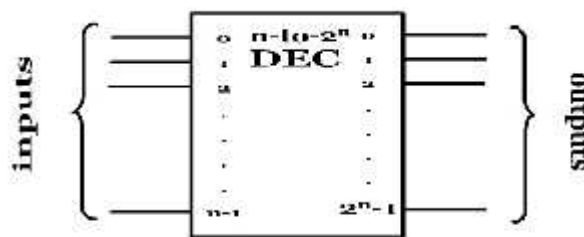
E	S1	S0	I0	I1	I2	I3	f
0	X	X	X	X	X	X	0
1	0	0	0	X	X	X	0
1	0	0	1	X	X	X	1
1	0	1	X	0	X	X	0
1	0	1	X	1	X	X	1
1	1	0	X	X	0	X	0
1	1	0	X	X	1	X	1
1	1	1	X	X	X	0	0
1	1	1	X	X	X	1	1

A 4-to-1 line multiplexer symbol.

Compressed Truth table

3.3 Decoder

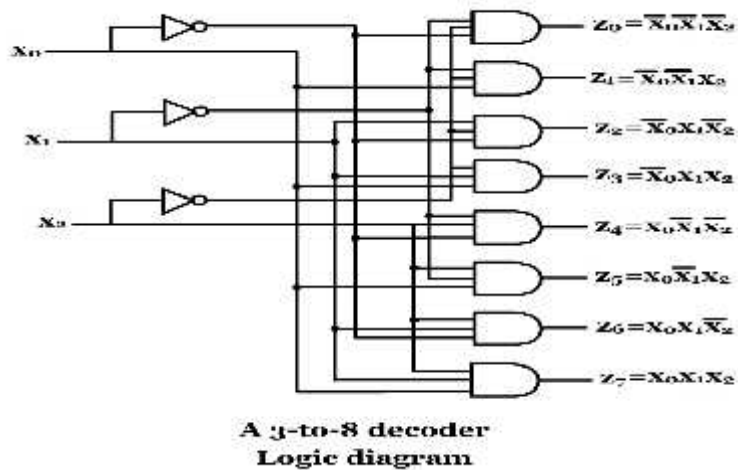
A Decoder is a multiple input, multiple output logic circuit. The block diagram of a decoder is as shown below.



An n-to-2<sup>n</sup> line decoder symbol.

The most commonly used decoder is a n –to 2<sup>n</sup> decoder which ha n inputs and 2<sup>n</sup> Output lines

3-to-8 decoder logic diagram



Inputs			Outputs							
$x_2$	$x_1$	$x_0$	$z_0$	$z_1$	$z_2$	$z_3$	$z_4$	$z_5$	$z_6$	$z_7$
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

**Truth table.**

In this realization shown above the three inputs are assigned  $x_0, x_1,$  and  $x_2,$  and the eight outputs are  $Z_0$  to  $Z_7.$

Function specific decoders also exist which have less than  $2^n$  outputs . examples are 8421 code decoder also called BCD to decimal decoder. Decoders that drive seven segment displays also exist.

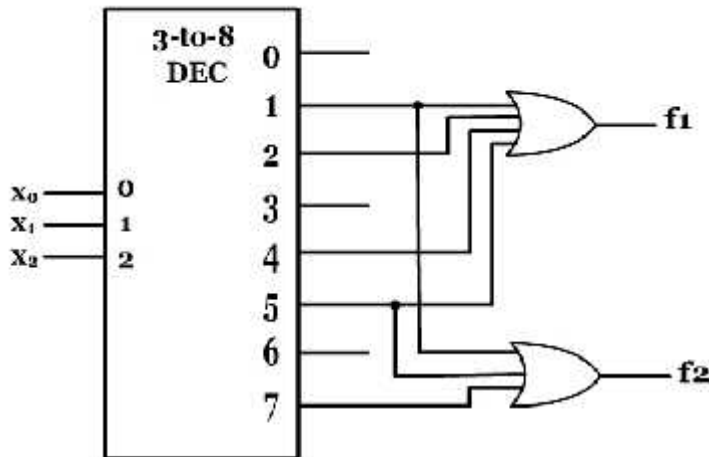
**Realization of boolean expression using Decoder and OR gate**

We see from the above truth table that the output expressions correspond to a single minterm. Hence a  $n$  –to  $2^n$  decoder is a minterm generator. Thus by using OR gates in conjunction with a  $n$  –to  $2^n$  decoder boolean function realization is possible.

Ex: to realize the Boolean functions given below using decoders...

- F1=  $m(1,2,4,5)$
- F2=  $m(1,5,7)$





**Realisation of boolean expressions**

**3.4 Priority encoder**

8-3 line priority encoder

In priority encoder a priority scheme is assigned to the input lines so that whenever more than one input line is asserted at any time, the output is determined by the input line having the highest priority.

The Valid bit is used to indicate that atleast one input line is asserted. This is done to distinguish the situation that no input line is asserted from when the  $X_0$  input line is asserted, since in both cases  $Z_1Z_2Z_3 = 000$ .

Inputs								Outputs			
$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$z_2$	$z_1$	$z_0$	Valid
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	1
x	1	0	0	0	0	0	0	0	0	1	1
x	x	1	0	0	0	0	0	0	1	0	1
x	x	x	1	0	0	0	0	0	1	1	1
x	x	x	x	1	0	0	0	1	0	0	1
x	x	x	x	x	1	0	0	1	0	1	1
x	x	x	x	x	x	1	0	1	1	0	1
x	x	x	x	x	x	x	1	1	1	1	1

**Condensed truth table for an 8-to-3 line priority encoder**

### 3.5 Programmable Logic Devices

Most of the circuits presented so far are available on a TTL IC chip. Circuits can be constructed using these chips and wiring them together. An alternative to this method would be to program all the components into a single chip, saving wiring, space and power. One type of such device is PLA (**P**rogrammable **L**ogic **A**rray) that contains one or more and/or arrays.

Programmable Logic Devices (PLDs)

**PLD's are Standard** logic devices that can be *programmed* to implement any combinational logic circuit. *Programmable* refers to a hardware process used to specify the logic that a PLD implements.

There are various types of PLD devices based on which array is programmable. The Device names and the type of array are listed in the table below.

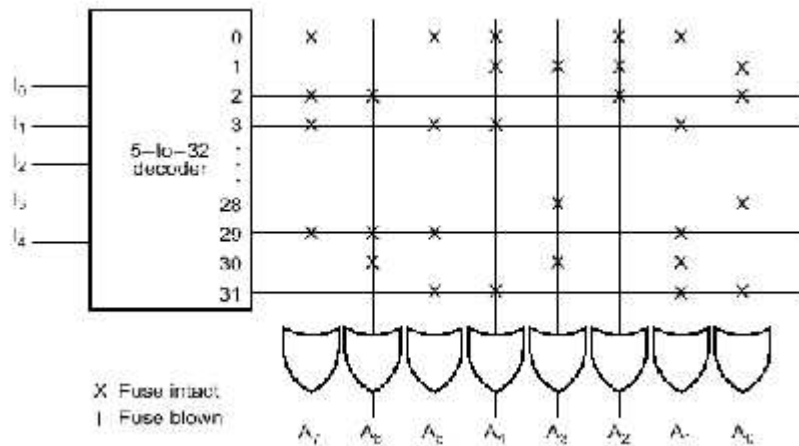
Types of PLDs

DEVICE	AND array	OR array
PROM	Fixed	Programmable
PLA	Programmable	Programmable
PAL	Programmable	Fixed

As an example we will first consider

### 3.3 Programming the ROM

The realization of Boolean expressions using a decoder and or gates was discussed in the earlier chapter on decoders. A similar approach is used in a PROM since a PROM is a device that includes both the decoder and the OR gates within the same network. The programming of the PROM is carried out by blowing the appropriate fuses. Proms are used for Code conversions, generating bit patterns for characters, and as lookup tables for arithmetic functions.



Example: Let  $I_0I_1I_3I_4 = 00010$  (address 2). Then, output 2 of the decoder will be 1, the remaining outputs will be 0, and ROM output becomes  $A_7A_6A_5A_4A_3A_2A_1A_0 = 11000101$ .

### 3.5 Programmable Logic Arrays (PLAs)

Similar concept as in PROM, except that a PLA does not necessarily generate all possible minterms (ie. the decoder is not used). More precisely, in PLAs both the AND and OR arrays can be programmed (in PROM, the AND array is fixed – the decoder – and only the OR array can be programmed).

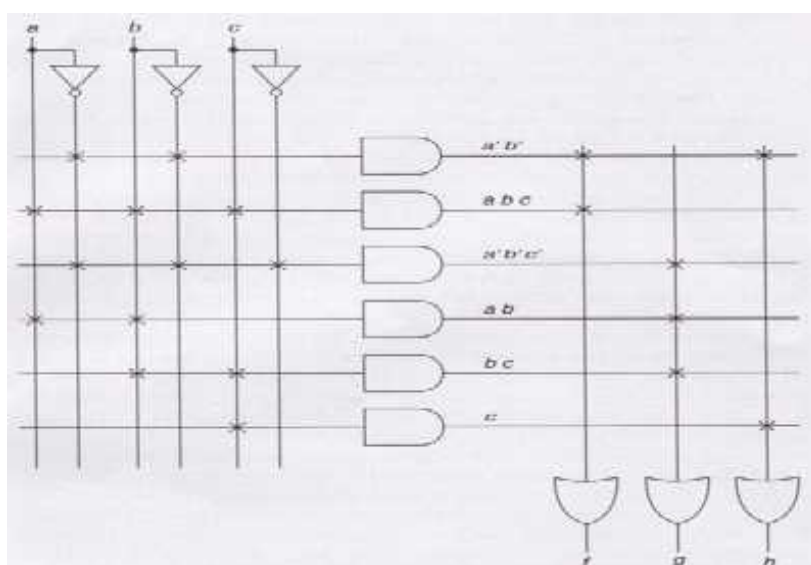
PLA Example

$$f(a,b,c) = a'b' + abc$$

$$g(a,b,c) = a'b'c' + ab + bc$$

$$h(a,b,c) = c$$

PLAs can be more compact implementations than ROMs, since they can benefit from minimizing the number of products required to implement a function.

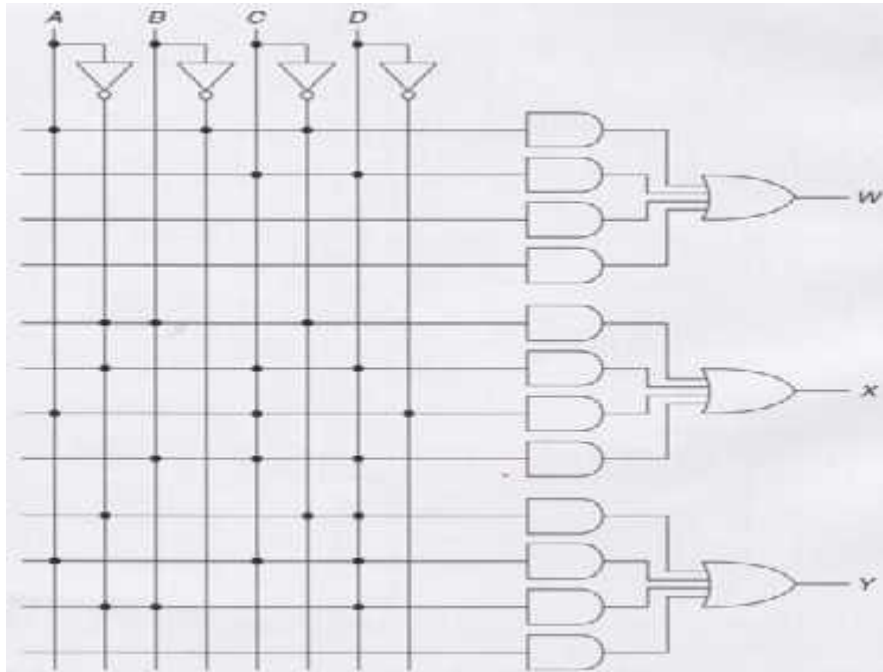


### 3.6 Programmable Array Logic (PAL)

OR plane (array) is fixed, AND plane can be programmed. A PAL is less flexible than PLA

Number of product terms available per function (OR outputs) is limited

PAL-based circuit implementation



$$W = AB'C' + CD$$

$$X = A'BC' + A'CD + ACD' + BCD$$

$$Y = A'C'D' + ACD + A'BD$$

### 3.7 HDL Implementation of Data Processing Circuits

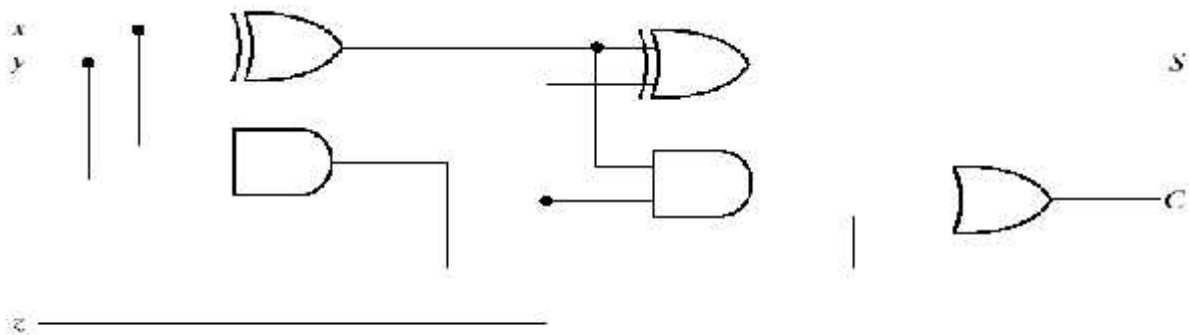


Fig. 4-8 Implementation of Full Adder with Two Half Adders and an OR Gate

//Gate-level hierarchical description of 4-bit adder

```
module halfadder (S,C,x,y);
```

```
  input x,y;
```

```
  output S,C;
```

```
  //Instantiate primitive gates
```

```
  xor (S,x,y);
```

```
  and (C,x,y);
```

```
endmodule
```

```
module fulladder (S,C,x,y,z);
```

```
  input x,y,z;
```

```
  output S,C;
```

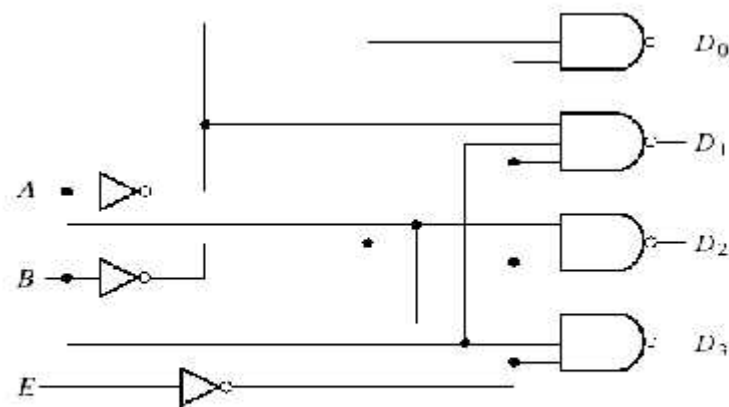
```
  wire S1,D1,D2; //Outputs of first XOR and two AND gates
```

```
  //Instantiate the half adders
```

```
  halfadder HA1(S1,D1,x,y), HA2(S,D2,S1,z);
```

```
  or g1(C,D2,D1);
```

```
endmodule
```



(a) Logic diagram

<i>E</i>	<i>A</i>	<i>B</i>	<i>D</i> <sub>0</sub>	<i>D</i> <sub>1</sub>	<i>D</i> <sub>2</sub>	<i>D</i> <sub>3</sub>
1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

(b) Truth table

Fig. 4.19 2-to-4 Line Decoder with Enable Input

```
module decoder_g1 (A,B,E,D);
```

```
  input A,B,E;
```

```
  output[0:3]D;
```

```
  wire Anot,Bnot,Enot;
```

```
  not
```

```
    n1 (Anot,A),
```

```
    n2 (Bnot,B),
```

```
    n3 (Enot,E);
```

```
  nand
```

```
    n4 (D[0],Anot,Bnot,Enot),
```

```
    n5 (D[1],Anot,B,Enot),
```

```
    n6 (D[2],A,Bnot,Enot),
```

```
    n7 (D[3],A,B,Enot);
```

```
endmodule
```

```
//Dataflow description of 2-to-1-line mux
module mux2x1_df (A,B,select,OUT);
    input A,B,select;
    output OUT;
    assign OUT = select ? A : B;
endmodule

//Behavioral description of 2-to-1-line multiplexer
module mux2x1_bh(A,B,select,OUT);
    input A,B,select;
    output OUT;
    reg OUT;
    always @(select or A or B)
        if (select == 1) OUT = A;
        else OUT = B;
endmodule

//Behavioral description of 4-to-1 line mux
module mux4x1_bh (i0,i1,i2,i3,select,y);
    input i0,i1,i2,i3;
    input [1:0] select;
    output y;
    reg y;
    always @(i0 or i1 or i2 or i3 or select)
        case (select)
            2'b00: y = i0;
            2'b01: y = i1;
            2'b10: y = i2;
            2'b11: y = i3;
        endcase
```

**endmodule**

## Adders

Adders are the basic building blocks of all arithmetic circuits; adders add two binary numbers and give out sum and carry as output. Basically we have two types of adders.

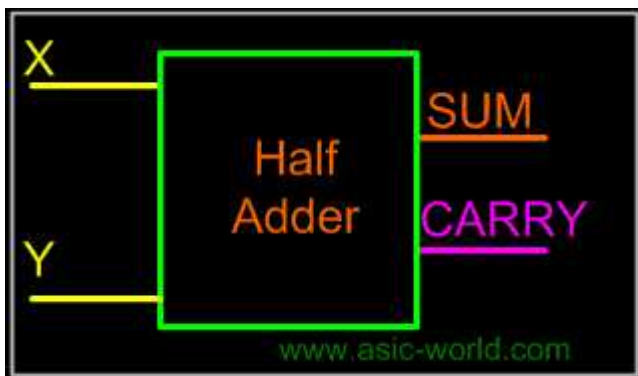
Half Adder.

Full Adder

### Half Adder

Adding two single-bit binary values X, Y produces a sum S bit and a carry out C-out bit. This operation is called half addition and the circuit to realize it is called a half adder.

X	Y	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



$$S(X,Y) = \Sigma(1,2)$$

$$S = X'Y + XY'$$

$$S = X \oplus Y$$

$$\text{CARRY}(X,Y) = \Sigma(3)$$

$$\text{CARRY} = XY$$



**Full Addder**

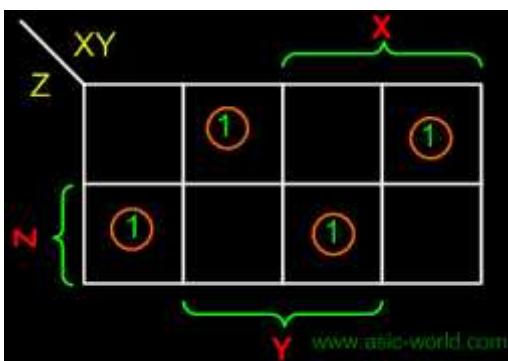
Full addder takes a three-bits input. Adding two single-bit binary values X, Y with a carry input bit C-in produces a sum bit S and a carry out C-out bit.

X	Y	Z	SUM	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\text{SUM}(X,Y,Z) = \Sigma(1,2,4,7)$$

$$\text{CARRY}(X,Y,Z) = \Sigma(3,5,6,7)$$

**Kmap-SUM**

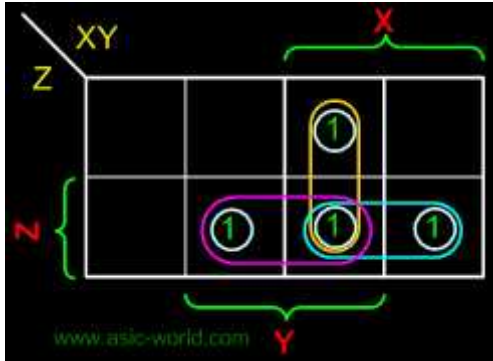


$$\text{SUM} = X'Y'Z + XY'Z' + X'YZ'$$



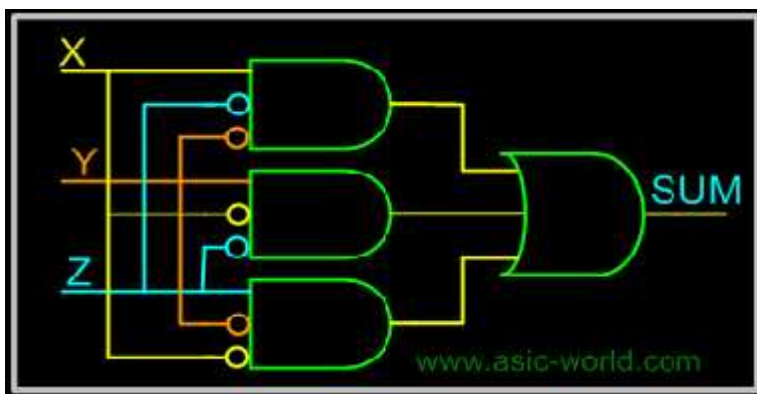
$$\text{SUM} = X \oplus Y \oplus Z$$

**Kmap-CARRY**

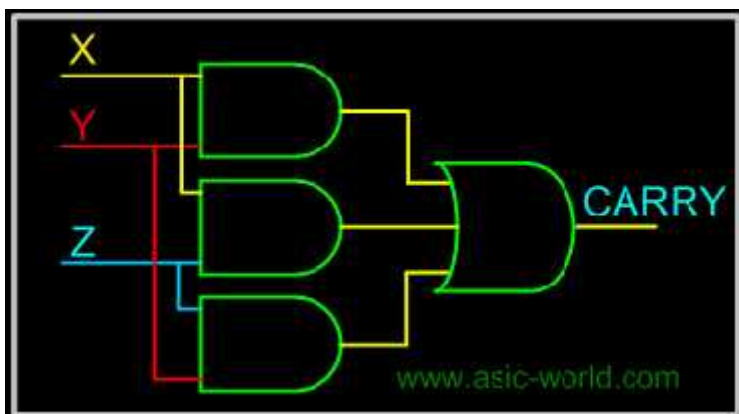


$$\text{CARRY} = XY + XZ + YZ$$

**Circuit-SUM**

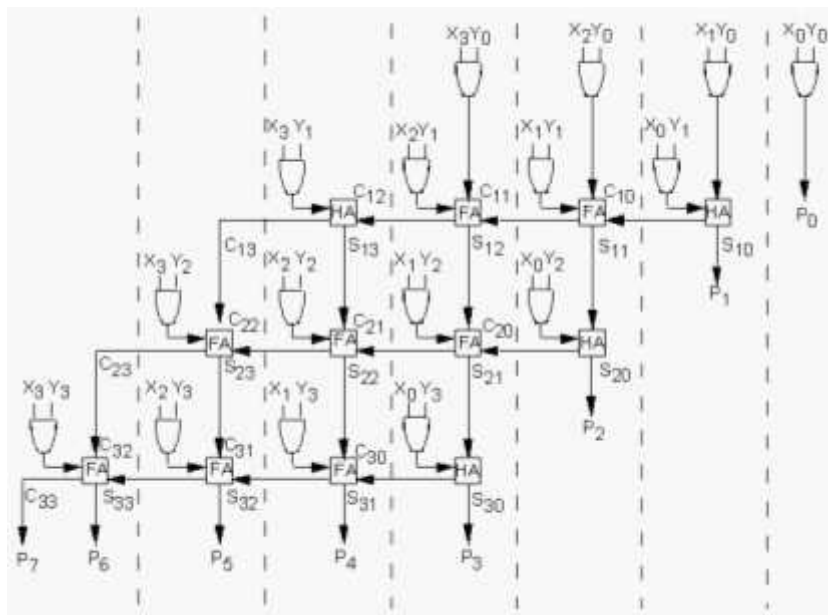
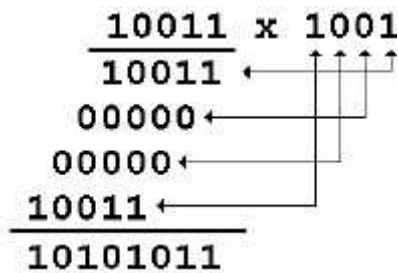


**Circuit-CARRY**



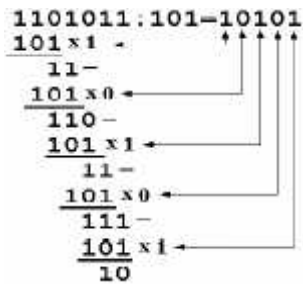
### 3.4 Multipliers

Multiplication is achieved by adding a list of shifted multiplicands according to the digits of the multiplier. An n-bit X n-bit multiplier can be realized in combinational circuitry by using an array of n-1 n-bit adders where each adder is shifted by one position. For each adder one input is the shifted multiplicand multiplied by 0 or 1 (using AND gates) depending on the multiplier bit, the other input is n partial product bits.

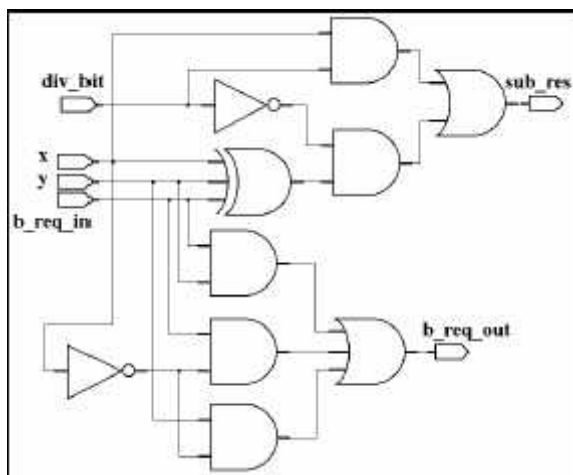


### Dividers

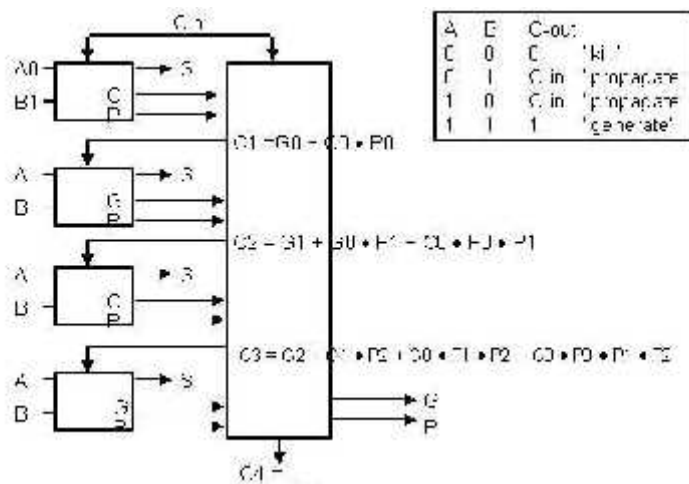
The binary divisions are performed in a very similar manner to the decimal divisions, as shown in the below figure examples. Thus, the second number is repeatedly subtracted from the figures of the first number after being multiplied either with '1' or with '0'. The multiplication bit ('1' or '0') is selected for each subtraction step in such a manner that the subtraction result is not negative. The division result is composed from all the successive multiplication bits while the remainder is the result of the last subtraction step.



This algorithm can be implemented by a series of subtractors composed of modified elementary cells. Each subtractor calculates the difference between two input numbers, but if the result is negative the operation is canceled and replaced with a subtraction by zero. Thus, each divider cell has the normal inputs of a subtractor unit as in the figure below but a supplementary input ('div\_bit') is also present. This input is connected to the b\_req\_out signal generated by the most significant cell of the subtractor. If this signal is '1', the initial subtraction result is negative and it has to be replaced with a subtraction by zero. Inside each divider cell the div\_bit signal controls an equivalent 2:1 multiplexer that selects between bit 'x' and the bit included in the subtraction result X-Y. The complete division can therefore be implemented by a matrix of divider cells connected on rows and columns as shown in figure below. Each row performs one multiplication-and-subtraction cycle where the multiplication bit is supplied by the NOT logic gate at the end of each row. Therefore the NOT logic gates generate the bits of the division result.



Carry Lookahead Adder (CLA)



Since each carry generate function  $G_i$  and carry propagate function  $P_i$  is itself only a function of the operand variables, the output carry and the input carry at each stage can be expressed as a function of the operand variables and the initial carry  $C_0$ . parallel adders whose realizations are based on the above equations are called carry look ahead adders.

## Unit-4 : Clocks , Flip Flops

### Introduction :

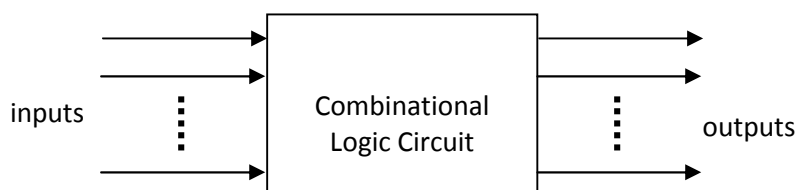
Logic circuit is divided into two types.

1. Combinational Logic Circuit
2. Sequential Logic Circuit

### 4.1 Definition :

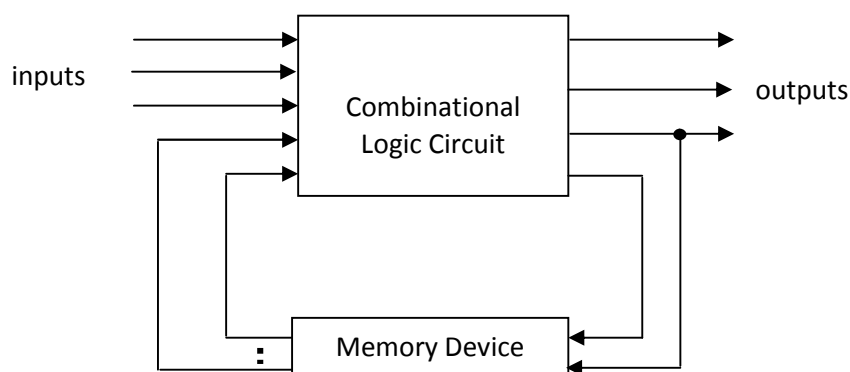
1. Combinational Logic Circuit :

The circuit in which outputs depends on only present value of inputs. So it is possible to describe each output as function of inputs by using Boolean expression. No memory element involved. No clock input. Circuit is implemented by using logic gates. The propagation delay depends on, delay of logic gates. Examples of combinational logic circuits are : full adder, subtractor, decoder, codeconverter, multiplexers etc.



2. Sequential Circuits :

Sequential Circuit is the logic circuit in which output depends on present value of inputs at that instant and past history of circuit i.e. previous output. The past output is stored by using memory device. The internal data stored in circuit is called as state. The clock is required for synchronization. The delay depends on propagation delay of circuit and clock frequency. The examples are flip-flops, registers, counters etc.



- Basic Bistable element.
  - Flip-Flop is Bistable element.
  - It consist of two cross coupled NOT Gates.
  - It has two stable states.
  - Q and  $\bar{Q}$  are two outputs complement of each other.
  - The data stored 1 or 0 in basic bistable element is state of flip-flop.
  - 1 – State is set condition for flip-flop.
  - 0 – State is reset / clear for flip-flop.
  - It stores 1 or 0 state as long power is ON.

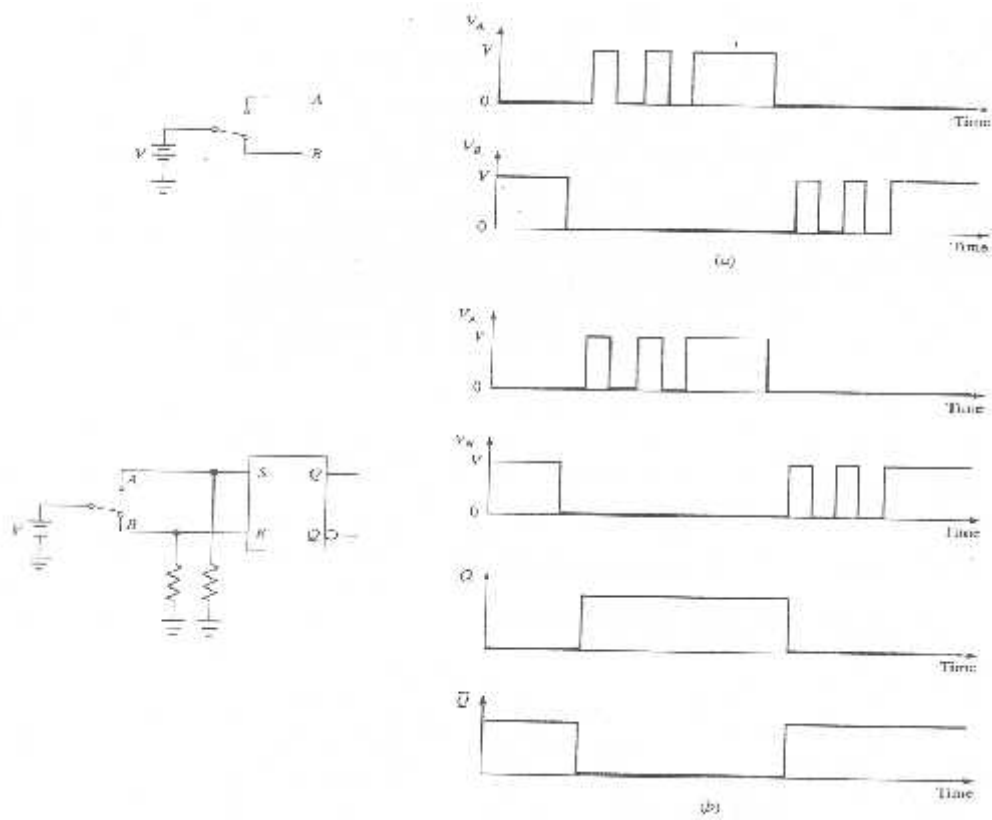
**Latches :**

S-R Latch : Set-reset Flip-Flop

- Latch is a storage device by using Flip-Flop.
- Latch can be controlled by direct inputs.
- Latch outputs can be controlled by clock or enable input.
- Q and  $\bar{Q}$  are present state for output.
- $Q^+$  and  $\bar{Q}^+$  are next states for output.
- The function table / Truth table gives relation between inputs and outputs.
- The S=R=1 condition is not allowed in SR FF as output is unpredictable.

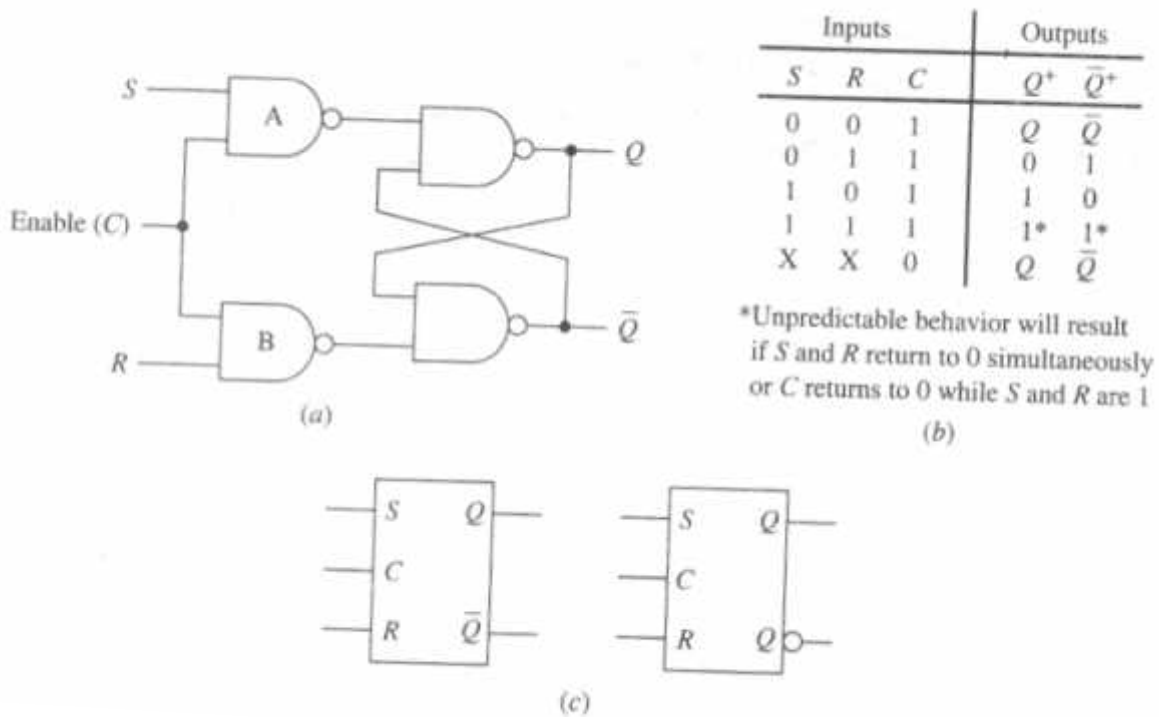
**Application of SR Latch :**

- A switch debouncer



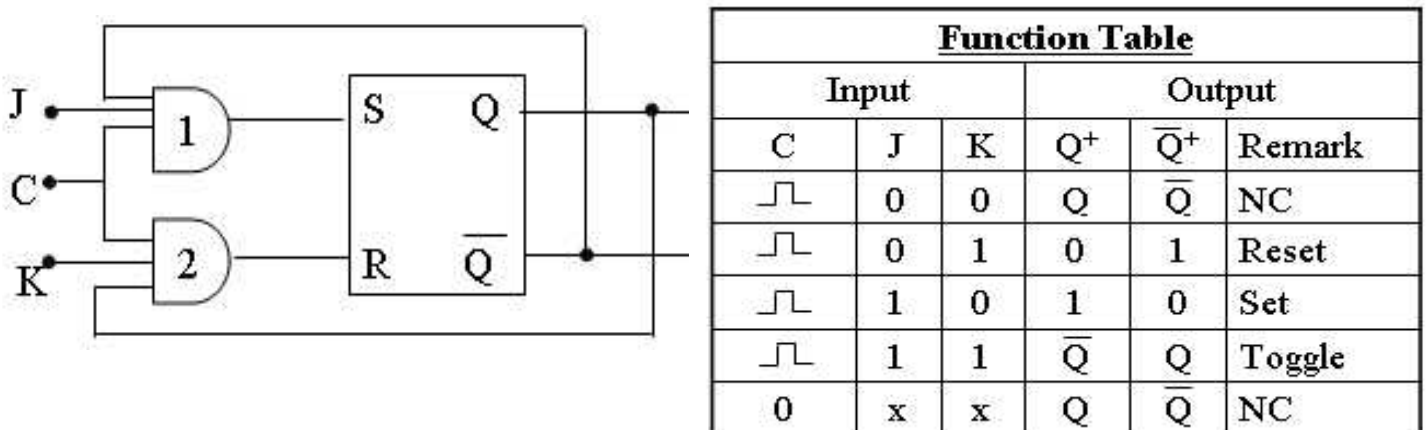
- Bouncing problem with Push button switch.
- Debouncing action.
- SR Flip-Flop as switch debouncer.

#### 4.2 Gated SR Latch :



- Enable input C is clock input.
- C=1, Output changes as per input condition.
- C=0, No change of state.
- S=1, R=0 is set condition for Flip-flop.
- S=0, R=1 is reset condition for Flip-flop.
- S=R=1 is ambiguous state, not allowed.

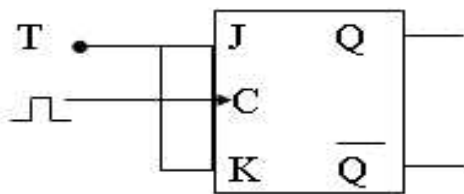
**4.3 JK Flip-Flop by using SR Flip-Flop**





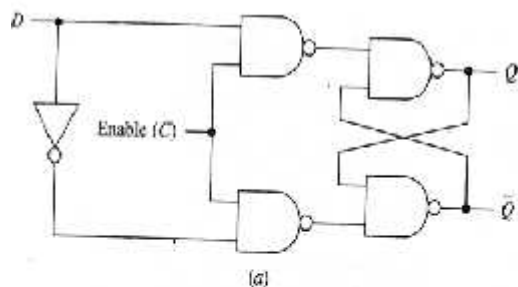
In SR FF, S=R=1 condition is not allowed.

- JK FF is modified version of SR FF.
- Due to feedback from output to input AND Gate J=K=1 is toggle condition for JK FF.
- The output is complement of the previous output.
- This condition is used in counters.
- T-FF is modified version of JK FF in which T=J=K=1.

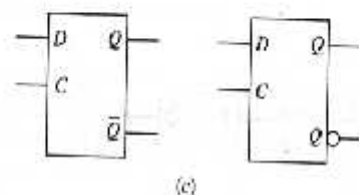


<b>Function Table</b>				
<b>Input</b>		<b>Output</b>		
C	T	Q <sup>+</sup>	$\overline{Q}^+$	Remark
0	0	Q	$\overline{Q}$	NC
0	1	$\overline{Q}$	Q	Toggle
1	x	Q	$\overline{Q}$	NC

**Gated D Latch :**

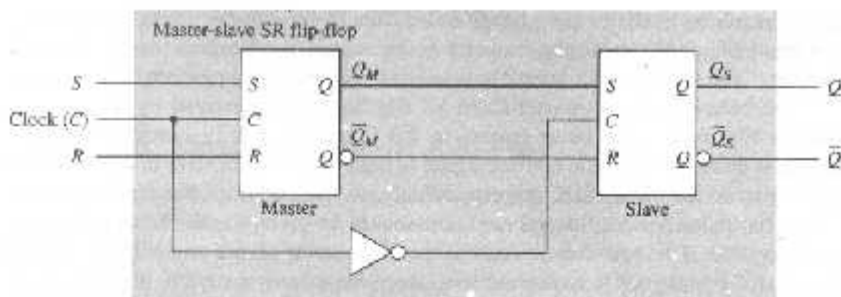


<b>Inputs</b>		<b>Outputs</b>	
D	C	Q	$\overline{Q}$
0	1	0	1
1	1	1	0
X	0	Q	$\overline{Q}$

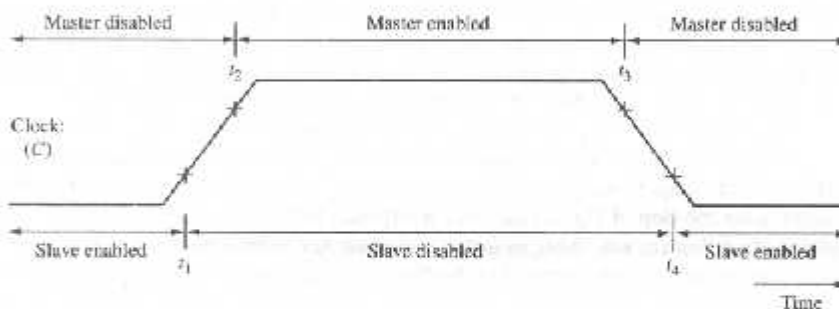


- D Flip-Flop is Data Flip-Flop.
- D Flip-Flop stores 1 or 0.
- R input is complement of S.
- Only one D input is present.
- D Flip-Flop is a storage device used in register.

### 4.4 Master slave SR Flip-Flop



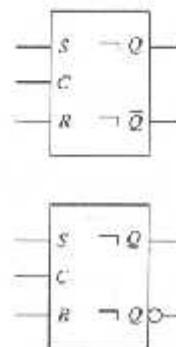
(a)



(b)

Inputs			Outputs	
S	R	C	$Q^+$	$\bar{Q}^+$
0	0		Q	$\bar{Q}$
0	1		0	1
1	0		1	0
1	1		Undefined	Undefined
X	X	0	Q	$\bar{Q}$

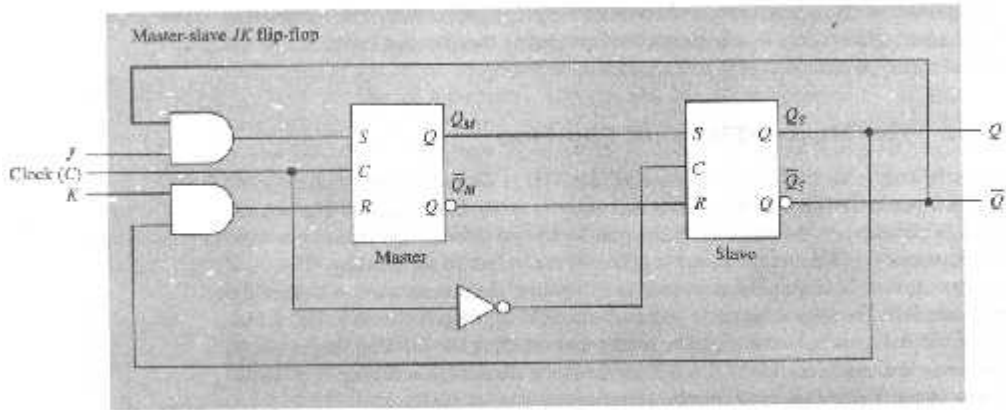
(c)



(d)

- Two SR Flip-Flop, 1<sup>st</sup> is Master and 2<sup>nd</sup> is slave.
- Master Flip-Flop is positive edge triggered.
- Slave Flip-Flop is negative edge triggered.
- Slave follows master output.
- The output is delayed.

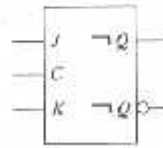
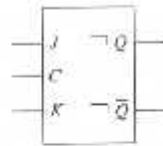
**Master slave JK Flip-Flop**



(a)

Inputs			Outputs	
J	K	C	Q <sup>+</sup>	Q̄ <sup>+</sup>
0	0	⏏	Q	Q̄
0	1	⏏	0	1
1	0	⏏	1	0
1	1	⏏	Q̄	Q
x	x	0	Q	Q̄

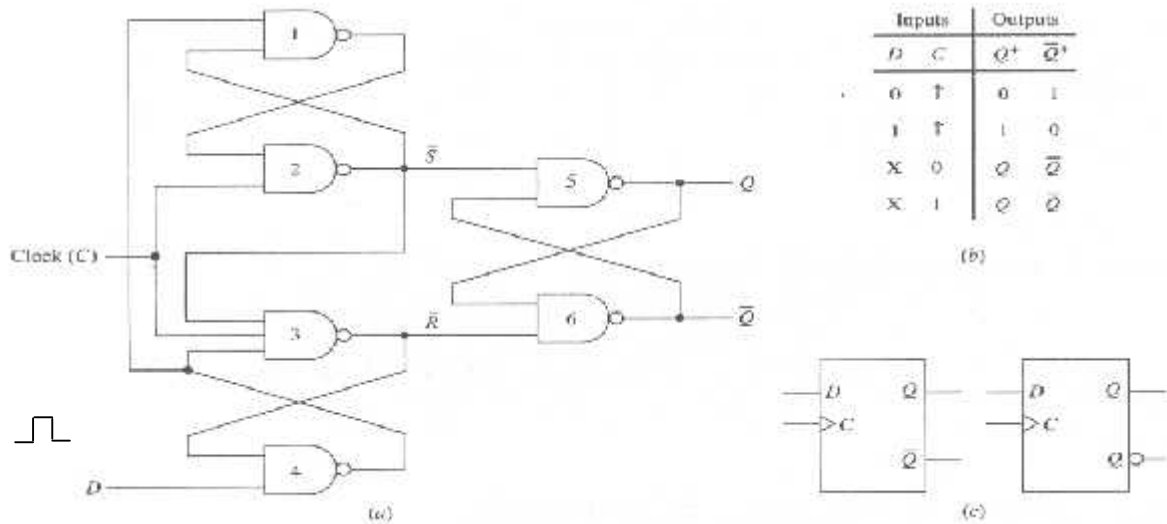
(b)



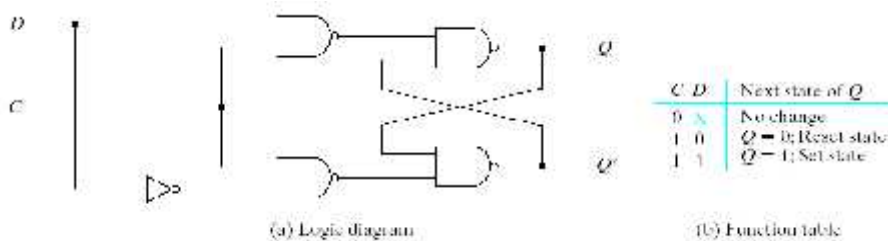
(c)

- In SR Flip-Flop the input combination S=R=1 is not allowed.
- JK FF is modified version of SR FF.
- Due to feedback from slave FF output to master, J=K=1 is allowed.
- J=K=1, toggle, action in FF.
- This finds application in counter.

**4.5 Positive Edge Triggered D Flip-Flop**



4.6 HDL implementation of Flip-flops



```

module D_latch(Q,D,control);
  output Q;
  input D,control;
  reg Q;
  always @(control or D)
    if(control) Q = D; //Same as: if(control=1)
endmodule
    
```

```

//D flip-flop
module D_FF (Q,D,CLK);
  output Q;
  input D,CLK;
  reg Q;
  always @(posedge CLK)
    Q = D;
endmodule
    
```

```

//JK flip-flop from D flip-flop and gates
module JKFF (Q,J,K,CLK,RST);
  output Q;
  input J,K,CLK,RST;
  wire JK;
    
```

```

assign JK = (J & ~Q) | (~K & Q);
//Instantiate D flipflop
DFF JK1 (Q,JK,CLK,RST);
endmodule

```

**// Functional description of JK // flip-flop**

```

module JK_FF (J,K,CLK,Q,Qnot);
output Q,Qnot;
input J,K,CLK;
reg Q;
assign Qnot = ~ Q ;
always @(posedge CLK)
    case({J,K})
        2'b00: Q = Q;
        2'b01: Q = 1'b0;
        2'b10: Q = 1'b1;
        2'b11: Q = ~ Q;
    endcase
endmodule

```

## Unit-5 : Registers

An n-bit register is a collection of n D flip-flops with a common clock used to store n related bits.

### 5.1 Types of Register:

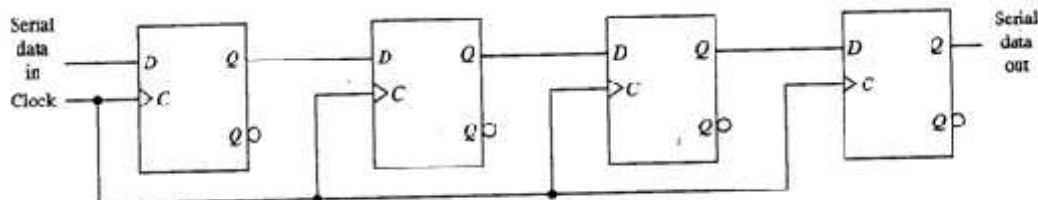


Fig. : Serial-In, Serial-Out Unidirectional Shift Register

- Register is a group of Flip-Flops.
- It stores binary information 0 or 1.
- It is capable of moving data left or right with clock pulse.
- Registers are classified as
  - Serial-in Serial-Out
  - Serial-in parallel Out
  - Parallel-in Serial-Out
  - Parallel-in parallel Out

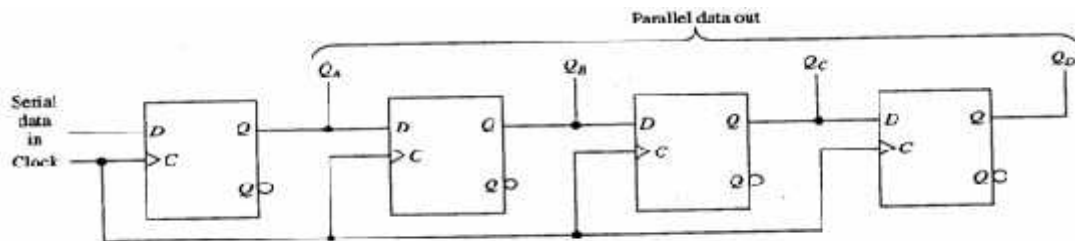


Fig. : Serial-In, Parallel-Out Unidirectional Shift Register

### Parallel-in Unidirectional Shift Register

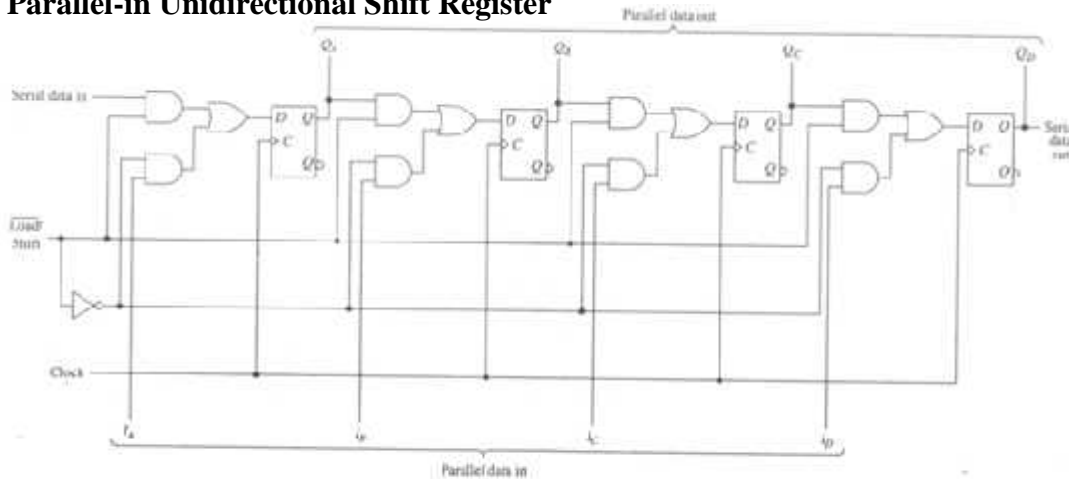
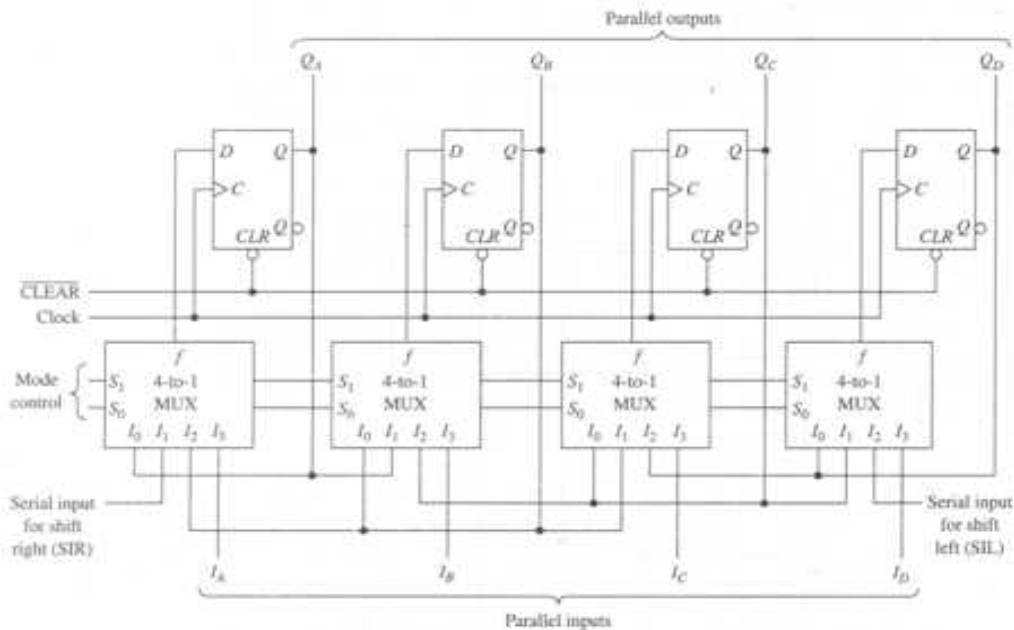


Fig. : Parallel-in Unidirectional Shift Register

- Parallel input data is applied at  $I_A I_B I_C I_D$ .
- Parallel output  $Q_A Q_B Q_C Q_D$ .
- Serial input data is applied to A FF.
- Serial output data is at output of D FF.
- $\bar{L}/\text{Shift}$  is common control input.
- $\bar{L}/S = 0$ , Loads parallel data into register.
- $\bar{L}/S = 1$ , shifts the data in one direction.
- **5.2 Universal Shift Register**



Select lines		Register operation
$S_1$	$S_0$	
0	0	Hold
0	1	Shift right
1	0	Shift left
1	1	Parallel load

- Bidirectional Shifting.
- Parallel Input Loading.
- Serial-Input and Serial-Output.
- Parallel-Input and Serial-Output.
- Common Reset Input.4:1 Multiplexer is used to select register operation.

### 5.3 Shift Register Applications

• **State Registers**

- Shift registers are often used as the state register in a sequential device. Usually, the next state is determined by shifting right and inserting a primary input or output into the next position (i.e. a finite memory machine)
- Very effective for sequence detectors

• **Serial Interconnection of Systems**

- keep interconnection cost low with serial interconnect

• **Bit Serial Operations**

- Bit serial operations can be performed quickly through device iteration
- Iteration (a purely combinational approach) is expensive (in terms of # of transistors, chip area, power, etc).
- A sequential approach allows the reuse of combinational functional units throughout the multi-cycle operation

### Register Implementation in HDL

```
//Behavioral description of Universal shift register
module shftreg (s1,s0,Pin,lfin,rtin,A,CLK,Clr);
  input s1,s0;    //Select inputs
  input lfin, rtin; //Serial inputs
  input CLK,Clr; //Clock and Clear
  input [3:0] Pin; //Parallel input
  output [3:0] A; //Register output
  reg [3:0] A;
  always @ (posedge CLK or negedge Clr)
    if (~Clr) A = 4'b0000;
    else
      case ({s1,s0})
        2'b00: A = A; //No change
        2'b01: A = {rtin,A[3:1]}; //Shift right
        2'b10: A = {A[2:0],lfin}; //Shift left
        //Parallel load input
        2'b11: A = Pin;
      endcase
endmodule
```



## Unit-6 : Counters

### 6.1 Counters

- Counter is a register which counts the sequence in binary form.
- The state of counter changes with application of clock pulse.
- The counter is binary or non-binary.
- The total no. of states in counter is called as modulus.
- If counter is modulus-n, then it has n different states.
- State diagram of counter is a pictorial representation of counter states directed by arrows in graph.

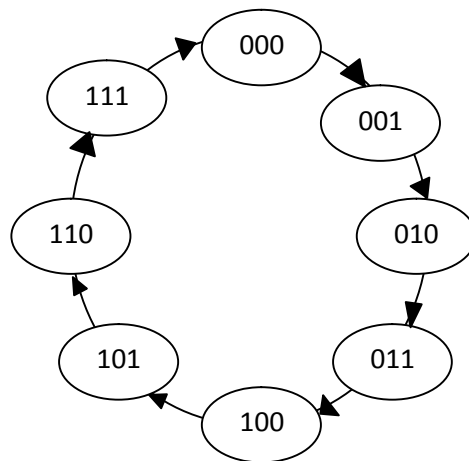
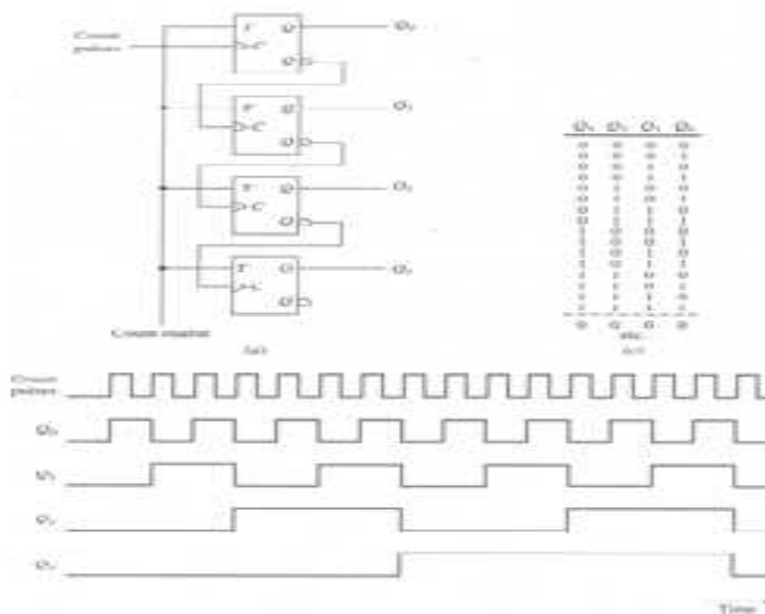


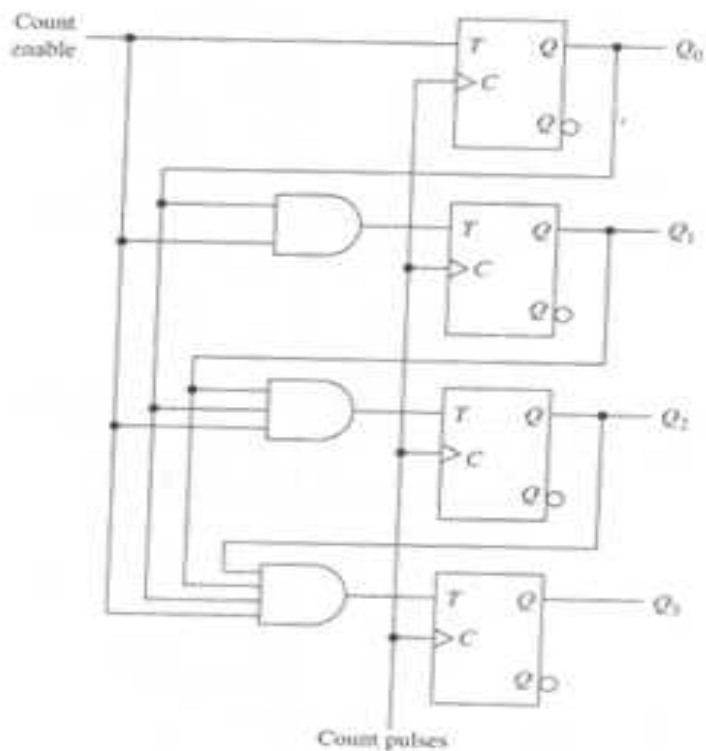
Fig. State diagram of mod-8 counter

### 6.2 4-bit Binary Ripple Counter :



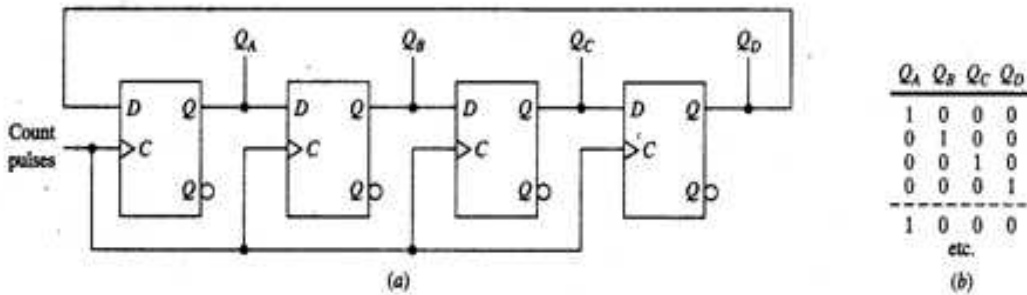
- All Flip-Flops are in toggle mode.
- The clock input is applied.
- Count enable = 1.
- Counter counts from 0000 to 1111.

**6.3 Synchronous Binary Counter :**



- The clock input is common to all Flip-Flops.
- The T input is function of the output of previous flip-flop.
- Extra combination circuit is required for flip-flop input.

**Counters Based on Shift Register**

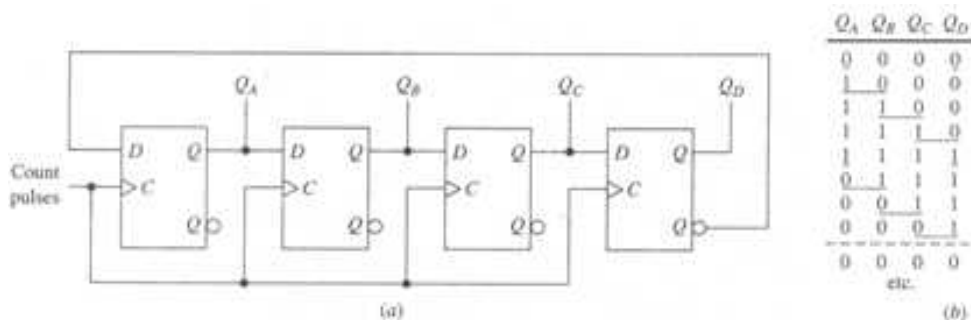


**Mod-4 Ring Counter**

The output of LSB FF is connected as D input to MSB FF.

- This is commonly called as Ring Counter or Circular Counter.
- The data is shifted to right with each clock pulse.
- This counter has four different states.
- This can be extended to any no. of bits.

**Twisted Ring Counter or Johnson Counter**

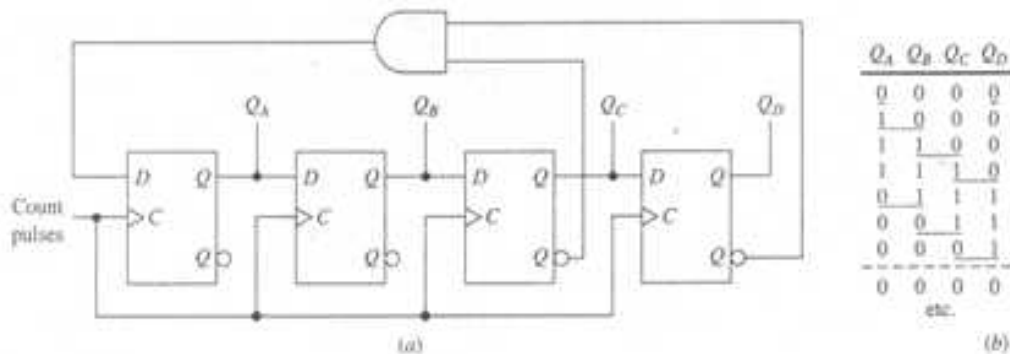


**Mod-8 Johnson Counter**

- The complement output of LSB FF is connected as D input to MSB FF.
- This is commonly called as Johnson Counter.
- The data is shifted to right with each clock pulse.

- This counter has eight different states.
- This can be extended to any no. of bits.

**Mod-7 Twisted Ring Counter**



**Mod-7 Ring Counter**

- The D input to MSB FF is  $\overline{Q_D} \cdot \overline{Q_C}$
- The counter follows seven different states with application of clock input.
- By changing feedback different counters can be obtained.

**Design Procedure for Synchronous Counter**

- The clock input is common to all Flip-Flops.
- Any Flip-Flop can be used.
- For mod-n counter 0 to n-1 are counter states.
- The excitation table is written considering the present state and next state of counter.
- The flip-flop inputs are obtained from characteristic equation.
- By using flip-flops and logic gate the implementation of synchronous counter is obtained.

**Difference between Asynchronous and Synchronous Counter :**

Asynchronous Counter	Synchronous Counter
1. Clock input is applied to LSB FF. The output of first FF is connected as clock to next	1. Clock input is common to all FF.

FF.	
2. All Flip-Flops are toggle FF.	2. Any FF can be used.
3. Speed depends on no. of FF used for n bit . $t_{max} = n \cdot t_p$	3. Speed is independent of no. of FF used. $f_{max} = \frac{1}{t_p}$
4. No extra Logic Gates are required.	4. Logic Gates are required based on design.
5. Cost is less.	5. Cost is more.

### 6.6 Counter Design using HDL

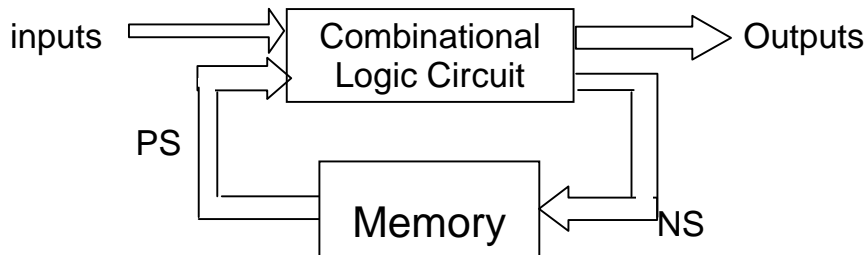
```
//Binary counter with parallel load
module counter (Count,Load,IN,CLK,Clr,A,CO);
input Count,Load,CLK,Clr;
input [3:0] IN; //Data input
output CO; //Output carry
output [3:0] A; //Data output
reg [3:0] A;
assign CO = Count & ~Load & (A == 4'b1111);
always @(posedge CLK or negedge Clr)
  if (~Clr) A = 4'b0000;
  else if (Load) A = IN;
  else if (Count) A = A + 1'b1;
  else A = A; // no change, default condition
endmodule
```

## Unit-7: Design of Synchronous and Asynchronous Sequential Circuits

### 7.1 SYNCHRONOUS SEQUENTIAL NETWORKS

#### Definition :

In sequential networks, the outputs are function of present state and present external inputs. Present state simply called as states or past history of circuit. The existing inputs and present state for sequential circuit determines next state of networks.



Model of Sequential Network

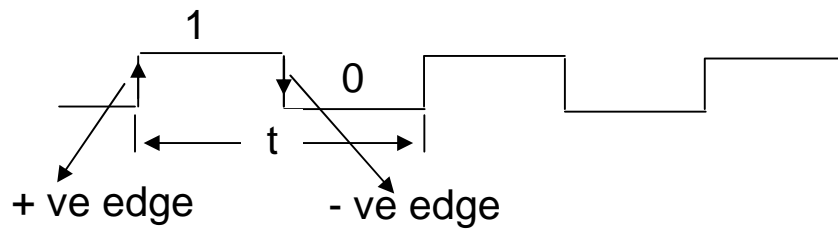
#### Types of Sequential Network :

1. **Asynchronous Sequential Network :** The changes in circuit depends on changes in inputs depending on present state. But the change in memory state is not at given instant of time but depending on input.
2. **Synchronous Sequential Network :** Output depends on present state and present inputs at a given instant of time. So timing sequence is required. So memory is allowed to store the changes at given instant of time.

### 7.2 Structure and Operation of Clocked Synchronous Sequential Circuit :

In synchronous sequential circuit, the network behavior is defined at specific instant of time associated with special timing. There is master clock which is common to all FFs that is used in memory element. Such circuits are called as clocked synchronous sequential circuit.

**Clock :** Clock is periodic waveform with one positive edge and one negative edge during each period.



This clock is used for network synchronization

### 7.3 Basic Operation of Clocked Synchronous Sequential Circuit

Q indicates all present state of FF.

Q+ indicates next state of FF in network.

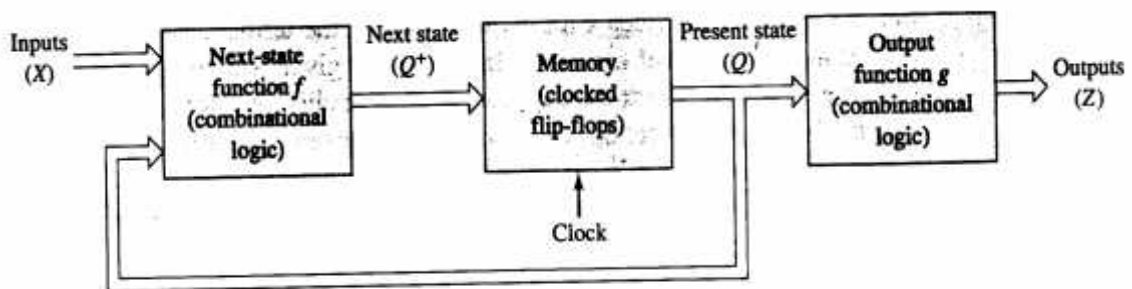
X indicates all external inputs.

$$Q+ = f(x, Q) \quad \text{This is next state of network.}$$

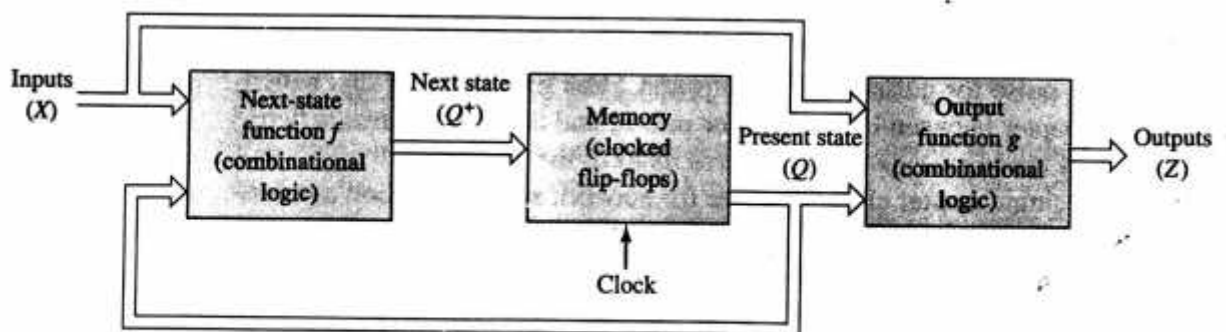
Z indicates output signal of sequential networks.

$$Z = g(X, Q)$$

The structure shown in given figure is called as Mealy Model or Mealy Machine.



**Figure 7.4** Moore model of a clocked synchronous sequential network.



**Figure 7.3** Mealy model of a clocked synchronous sequential network.

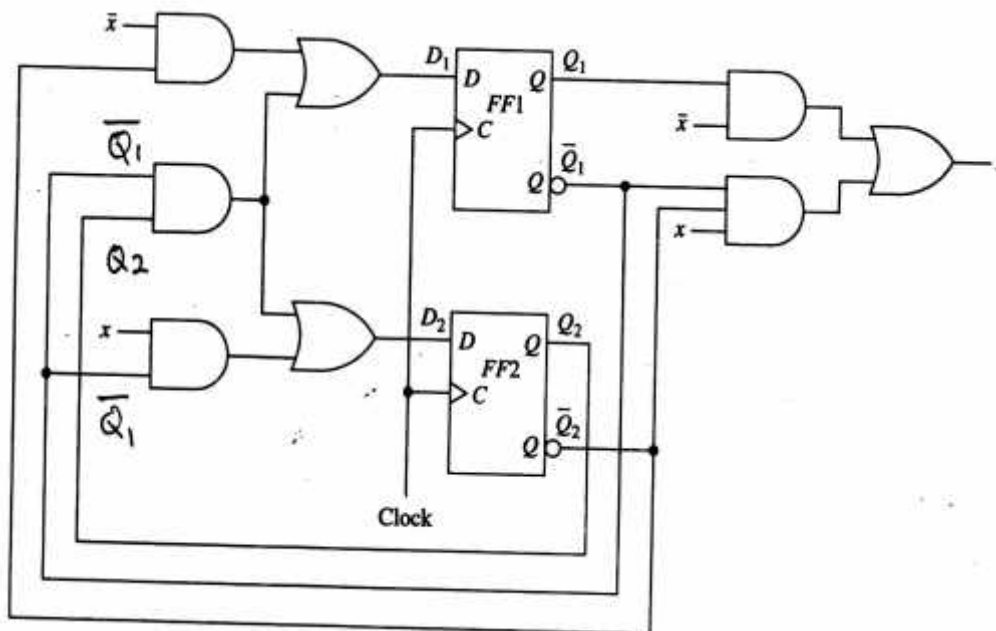
### Difference between Mealy Model and Moore Model of Synchronous Sequential Circuit

**Mealy Model :** In Mealy Model the next state is function of external inputs and present state. The output is also function of external inputs and present state. The memory state changes with master clock.

$$Q^+ = f(X, Q) \qquad Z = g(X, Q)$$

**Moore Model :** In Moore Model the next state is function of external inputs and present state. But the output is function of present state. It is not dependent on external inputs. The no. of FFs required to implement circuit is more compared with Mealy Model,

$$Q^+ = f(X, Q) \qquad Z = g(Q)$$



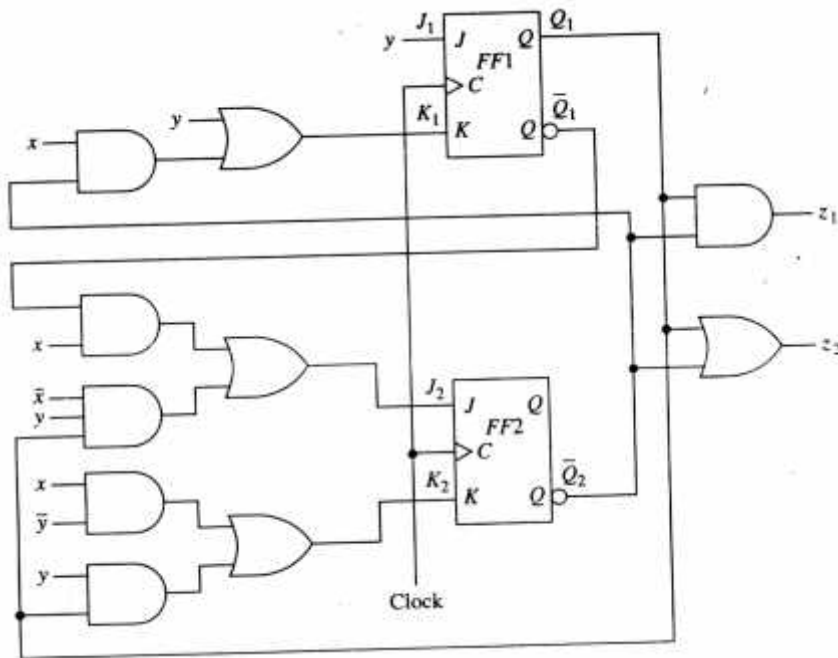
### Logic Diagram for Mealy Network

$$D_1 = \overline{x}Q_2 + \overline{Q_1}Q_2$$

$$D_2 = x\overline{Q_1} + \overline{Q_1}Q_2$$

$$Z = \overline{x}Q_1 + \overline{Q_1}Q_2x$$





**Logic Diagram for Moore Network**

$$\begin{aligned}
 Z_1 &= \overline{Q_2}Q_1 & \text{and} & & Z_2 &= Q_1 + \overline{Q_2} \\
 J_1 &= y & \text{and} & & K_1 &= \overline{Q_2}x + y \\
 J_2 &= \overline{Q_1}x + \overline{xy}Q_1 & \text{and} & & K_2 &= x\overline{y} + y\overline{Q_1}
 \end{aligned}$$

**7.4 Transition Equations :**

To convert excitation expression into next state expression, it is necessary to use the characteristic equations of flip-flops.

The characteristic equations of FF depends on types of FF used.

Ex : For D FF  $Q^+ = D$

For JK FF  $Q^+ = J\overline{Q} + \overline{K}Q$

For T FF  $Q^+ = T \oplus Q$

By substituting the excitation expressions for a FF into characteristic equation, an algebraic description of next state of FF is obtained.

The expression for next state in terms of FF inputs are referred as transition equations.

$$Q1+ = D1 \quad \text{and} \quad Q2+ = D2$$

$$Q_1^+ = x\overline{Q_2} + \overline{Q_1}Q_2$$

$$Q_2^+ = x\overline{Q_1} + \overline{Q_1}Q_2$$

For Moore network

$$Q_1^+ = J_1\overline{Q_1} + \overline{K_1}Q_1$$

$$Q_2^+ = J_2\overline{Q_2} + \overline{K_2}Q_2$$

By substituting the values of J & K inputs we get next state in terms of FF present state and external input.

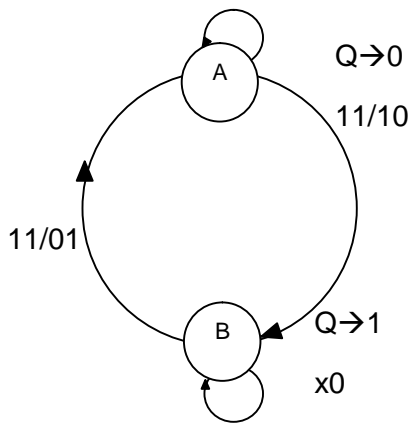
### **Transition Tables :**

Instead of using algebraic equations for next state and outputs of sequential network, it is more convenient and useful to express the information in tabular form.

The Transition Table or State Transition Table or State Table is the tabular representation of the transition and output equations. This table consist of Present State, Next State, external inputs and output variables. If there are n state variables then 2n rows are present in state table.

### **State machine notations :**

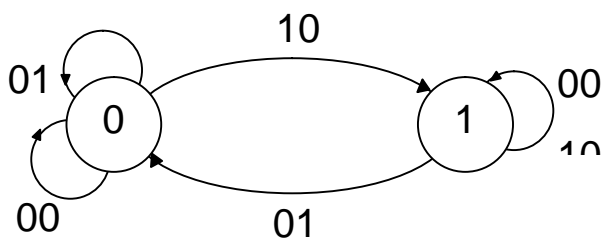
- Input Variables : External input variables to sequential machine as inputs.
- Output Variables : All variables that exit from the sequential machine are output variables.
- State : State of sequential machine is defined by the content of memory, when memory is realized by using FFs.
- Present State : The status of all state variable i.e. content of FF for given instant of time t is called as present state.
- Next State : The state of memory at t+1 is called as Next state.
- State Diagram : State diagram is graphical representation of state variables represented by circle. The connection between two states represented by lines with arrows and also indicates the excitation input and related outputs.
- Output Variables : All variables that exit from the sequential machine are output variables.
-



State diagram of J-K FF

Application Table of JK FF

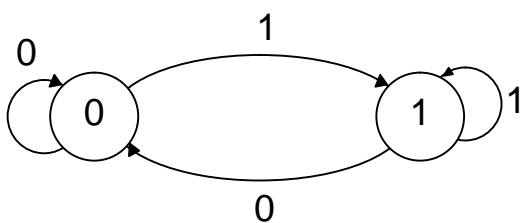
PS	NS	FF input	
Q	Q+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0



State diagram of SR FF

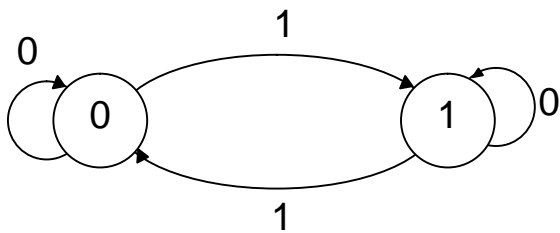
PS	NS	FF i/p	
Q	Q+	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

Application Table of D FF



State diagram of D FF

PS	NS	FF i/p
Q	Q+	D i/p
0	0	0
0	1	1
1	0	0
1	1	1



State diagram of T FF

PS	NS	FF i/p
Q	Q+	T i/p
0	0	0
0	1	1
1	0	1
1	1	0

Transition table for Mealy Network

Present state ( $Q_1Q_2$ )	Next state ( $Q_1^+Q_2^+$ )		Output (z)	
	Input (x)		Input (x)	
	0	1	0	1
00	10	01	0	1
01	11	11	0	0
10	10	00	1	0
11	00	00	1	0

Transition table for Moore Network

PS (Q1Q2)	Next State (Q1+Q2+)				Output (Z1Z2)
	Inputs (xy)				
	00	01	10	11	
00	00	10	01	11	01
01	01	11	00	11	00
10	10	01	00	00	11
11	11	00	10	00	01

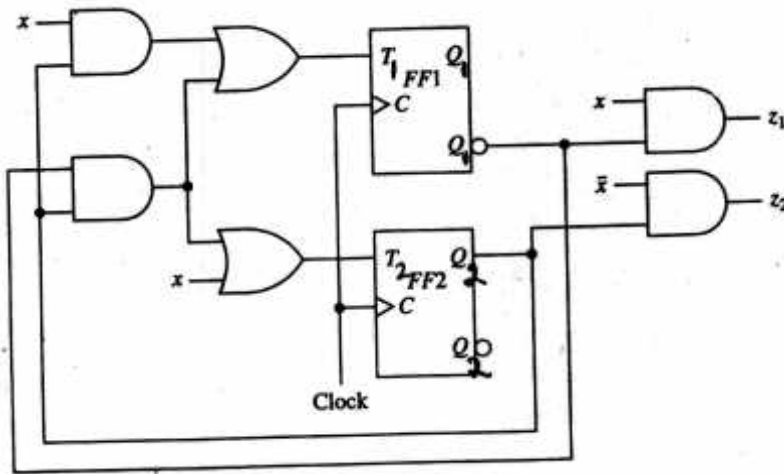
$$Q_1^+ = \overline{x}Q_2 + \overline{Q_1}Q_2, \quad Q_1^+ = D_1$$

$$Q_2^+ = x\overline{Q_1} + \overline{Q_1}Q_2, \quad Q_2^+ = D_2$$

$$Z = \overline{x}Q_1 + \overline{Q_1}Q_2x$$

$$Z_1 = \overline{Q_2}Q_1, Z_2 = Q_1 + \overline{Q_2}, J_1 = y$$

$$K_1 = \overline{Q_2}x + y, J_2 = \overline{Q_1}x + \overline{xy}Q_1, K_2 = xy + y\overline{Q_1}$$



Synchronous Sequential Circuit

$$T_1 = xQ_2 + \overline{Q_1}Q_2, Q_1^+ = T_1 \oplus Q_1$$

$$T_2 = x + \overline{Q_1}Q_2, Q_2^+ = T_2 \oplus Q_2$$

$$Z_1 = x\overline{Q_1}, Z_2 = \overline{x}Q_2$$

**State Tables :**

State table consist of PS, NS and output section. The PS and NS of state tables are obtained by replacing the binary code for each in the transition table by newly defined symbol. The output section is identical to output section of transition table.

Symbols for state can be S1, S2, S3,.....Sn or A, B, C, D, E....

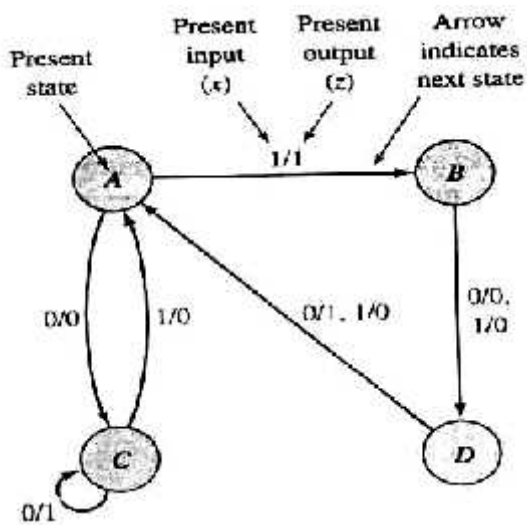
**State table for Mealy Machine**

PS	NS		O/p Z	
	x = 0	x = 1	x = 0	x = 1
00 – A	C	B	0	1
01 – B	D	D	0	0
10 – C	C	A	1	0
11 – D	A	A	1	0

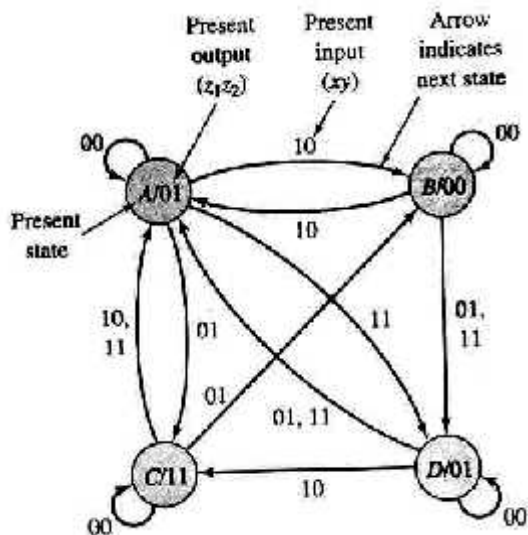
**State Diagram :**

It is graphical representation of state tables. Each state of network is represented by labeled node.

Directed branches connect the nodes to indicate transition between states. The directed branches are labeled according to the values of external input variable that permit transition. The output of sequential network is also entered in state diagram. In case of Moore Network state diagram, the values of input for output is not written.



State diagram for Mealy Network



State diagram for Moore Network

**Network Terminal Behavior**

This is the time response of a network to a sequence of inputs. This can be done from the Logic diagram by tracing signals.

For given example of Mealy Network,

Assume FFs are in 0 state initially, So  $Q_1Q_2 = 00$ .

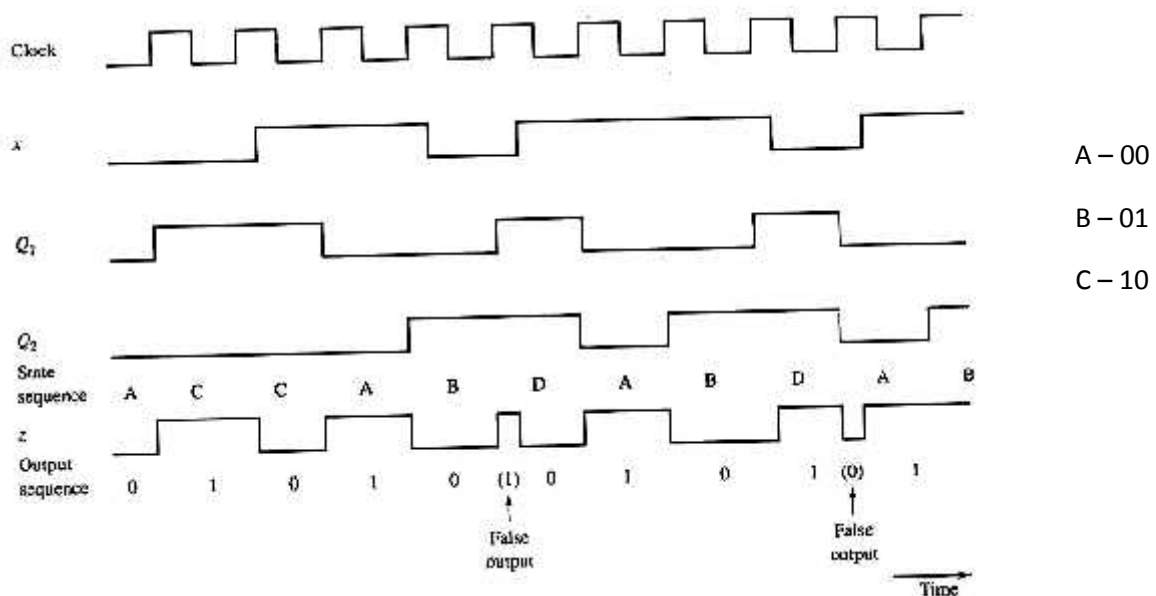
Input Sequence x is 0011011101

Now based on input x, the state of FF output changes and also corresponding network output changes.

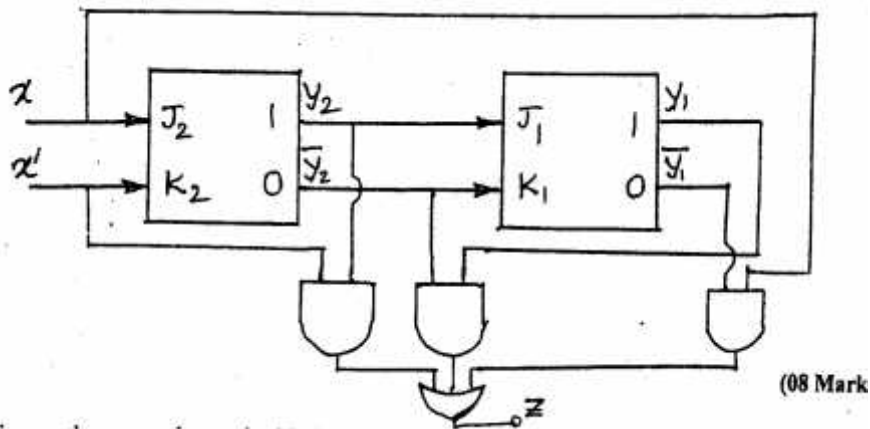
Input sequence x = 0011011101

State sequence = ACCABDABDAB

Output Sequence Z = 0101001011



Timing diagram for Moore Network



**Analysis of Synchronous Circuit**

The given circuit in above figure is Mealy Network and the output is function of input variable and PS of FF. The analysis of above circuit is as follows.

**The Excitation and Output Function**

$$Z = \bar{x}y_2 + y_1\bar{y}_2 + xy_1$$

By substituting the FF inputs in characteristic equation, the next state of FF is obtained in terms of PS of FF and external input.

The characteristic equation of JK FF is  $Q^+ = J\bar{Q} + \bar{K}Q$

$$Q_1^+ = J_1\bar{Q}_1 + \bar{K}_1Q_1 = Q_2$$

$$Q_2^+ = J_2\bar{Q}_2 + \bar{K}_2Q_2 = x$$

**The Excitation Table**

PS Q2 Q1 (y2 y1)	Excitation input			Output Z x=0, x=1
	J2	K2	J1	
		K1		
	x=0, 1		x=0, 1	
0 0	0 1		0 1	1 1

$$J_1 = y_2 = Q_2, \quad K_1 = \bar{y}_2 = \bar{Q}_2$$

$$J_2 = x, \quad K_2 = \bar{x}, \quad Z = \bar{x}y_2 + y_2\bar{y}_1 + xy_1$$

When  $x=0, z = y_2 + \bar{y}_1$  and When  $x=1, z = \bar{y}_1$



0 1	0 1	0 1	0 0
1 0	0 1	1 0	1 1
1 1	0 1	1 0	2 0

**State Table**

PS			NS						O/p Z	
			x = 0			x = 1				
Q2 (y2)	Q1 (y1)	state	Q2+	Q1+	state	Q2+	Q1+	state	X=0	X=1
0	0	A	0	0	A	1	0	C	1	1
0	1	B	0	0	A	1	0	C	0	0
1	0	C	0	1	B	1	1	D	1	1
1	1	D	0	1	B	1	1	D	1	0

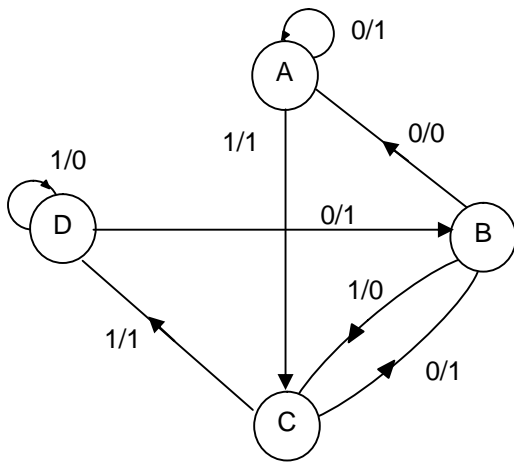
$Q1+ = Q2 = y2$

$Q2+ = x$

*if*  $x = 0, z = y_2 + \overline{y_1}$

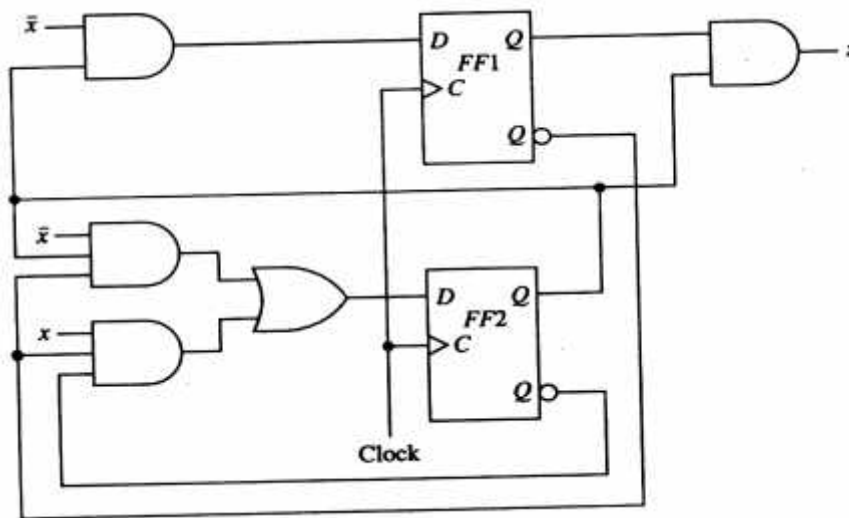
*if*  $x = 1, z = \overline{y_1}$

**State Diagram of Mealy Network**



A, B, C, D are  
Present states.

**Analysis of Moore Network**



$$D_1 = \bar{x}Q_2, \quad D_2 = \bar{x}Q_2\bar{Q}_1 + x\bar{Q}_1Q_2$$

$$Z = Q_1 + Q_2$$

$$\text{if } x = 0, D_1 = Q_2 \ \& \ D_2 = Q_2 \bar{Q}_1$$

$$\text{if } x = 1, D_1 = 0 \ \& \ D_2 = \bar{Q}_1Q_2$$

**State Table / Transition Table**

PS			NS						O/p Z
			x = 0			x = 1			
Q1	Q2	State	Q1+	Q2+	State	Q1+	Q2+	state	
0	0	A	0	0	A	0	1	B	0
0	1	B	1	1	D	0	0	A	1
1	0	C	0	0	A	0	0	A	1
1	1	D	1	0	C	0	0	A	1

$$Q1+ = D1$$

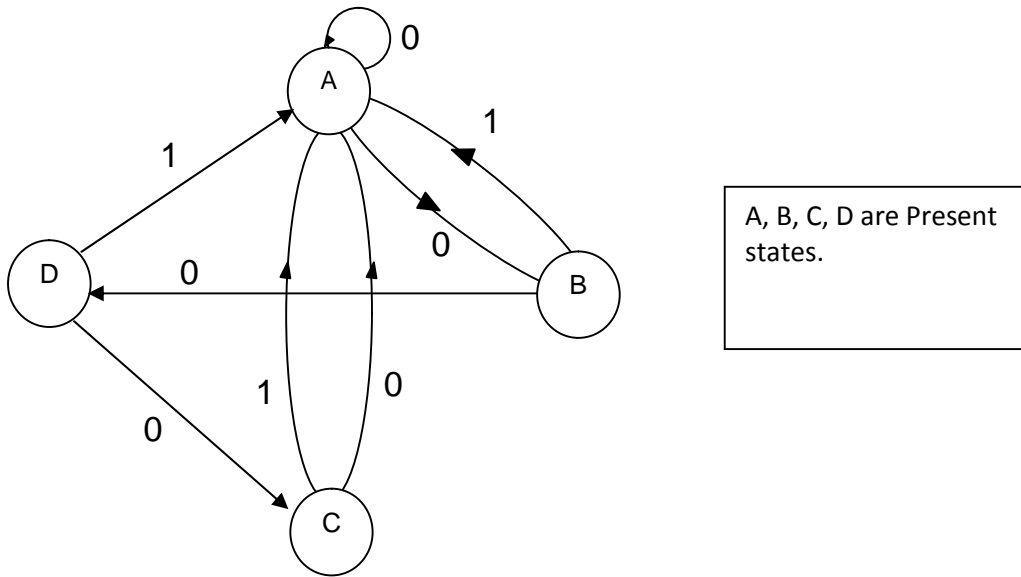
$$Z = Q1 + Q2$$

$$\text{if } x = 0, D_1 = Q_2 \ \& \ D_2 = Q_2 \bar{Q}_1$$

$$Q2+ = D2$$

$$\text{if } x = 1, D_1 = 0 \ \& \ D_2 = \bar{Q}_1Q_2$$

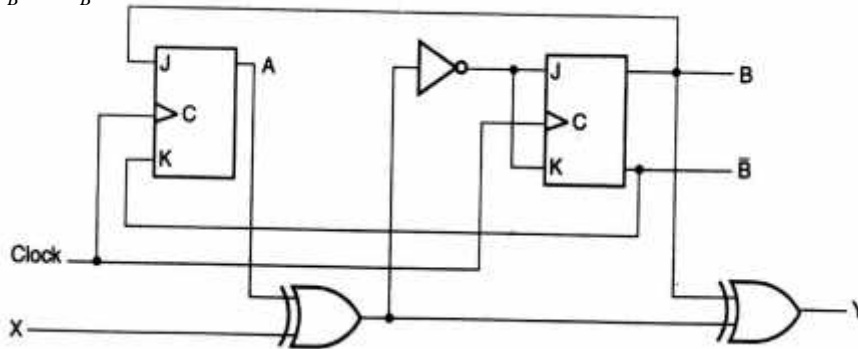
**State Diagram of Moore Network**



**Analysis of Sequential Network**

$$y = x \oplus A \oplus B, \quad J_A = B, K_A = \bar{B}$$

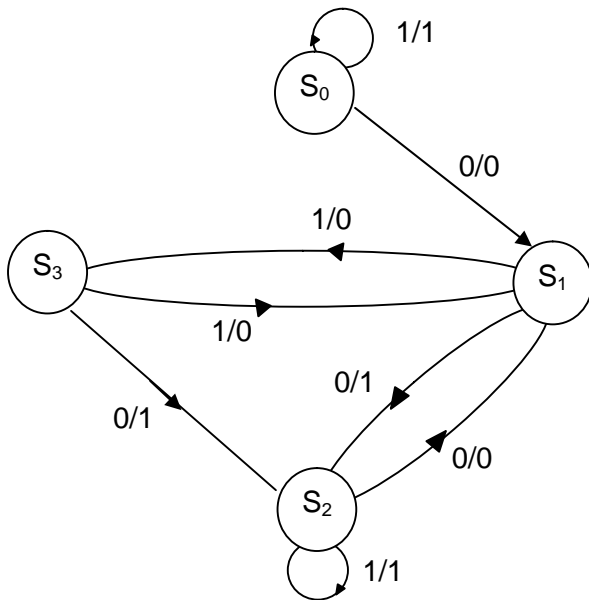
$$J_B = K_B = x \oplus A$$



**State Table**

PS			NS						O/p y	
			x = 0			x = 1				
QA	QB	state	QA+	QB+	state	QA+	QB+	state	x=0	x=1
0	0	S0	0	1	S1	0	0	S0	0	1
0	1	S1	1	0	S2	1	1	S3	1	0
1	0	S2	0	1	S1	1	0	S2	0	1

1	1	S3	1	0	S2	0	1	S1	1	0
---	---	----	---	---	----	---	---	----	---	---



S0, S1, S2, S3 are Present states.

**Unit-8: D/A Conversion and A/D Conversion****Basic Concept:**

- Analog signals are continuous, with infinite values in a given range.
- Digital signals have discrete values such as on/off or 0/1.
- Limitations of analog signals
  - Analog signals pick up noise as they are being amplified.
  - Analog signals are difficult to store.
  - Analog systems are more expensive in relation to digital systems.
- Advantages of digital systems (signals)
  - Noise can be reduced by converting analog signals in 0s and 1s.
  - Binary signals of 0s/1s can be easily stored in memory.
  - Technology for fabricating digital systems has become so advanced that they can be produced at low cost.
- The major limitation of a digital system is how accurately it represents the analog signals after conversion.
- A typical system that converts signals from analog to digital and back to analog includes:
  - A transducer that converts non-electrical signals into electrical signals
  - An A/D converter that converts analog signals into digital signals
  - A digital processor that processes digital data (signals)
  - A D/A converter that converts digital signals into equivalent analog signals
  - A transducer that converts electrical signals into real life non-electrical signals (sound, pressure, and video)

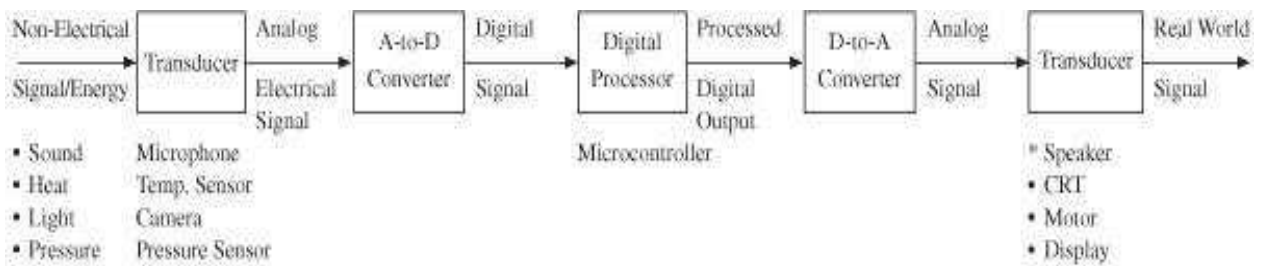
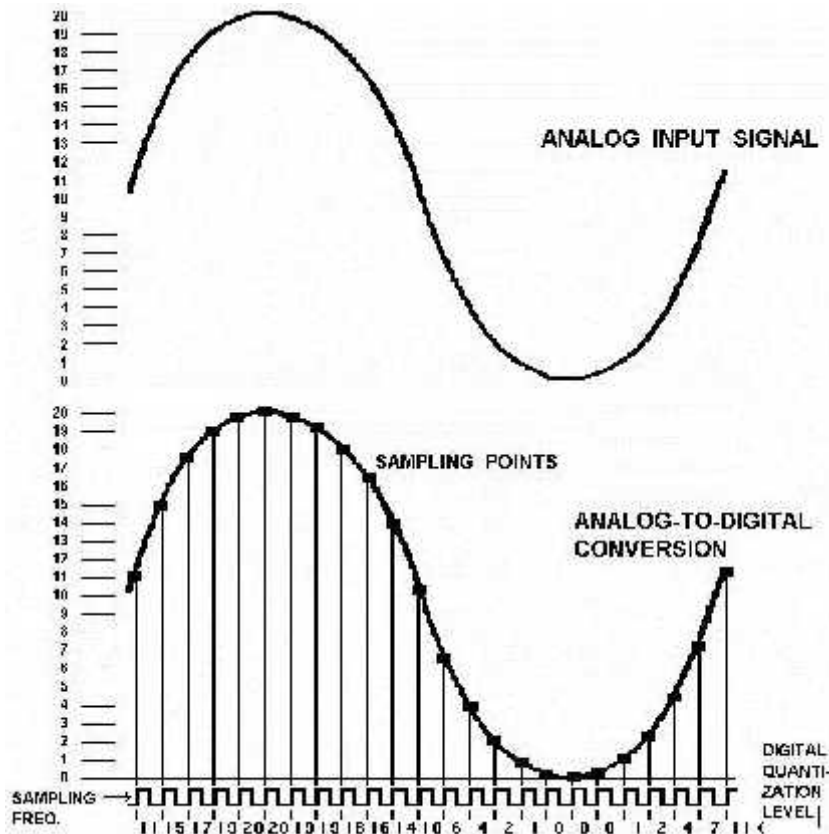


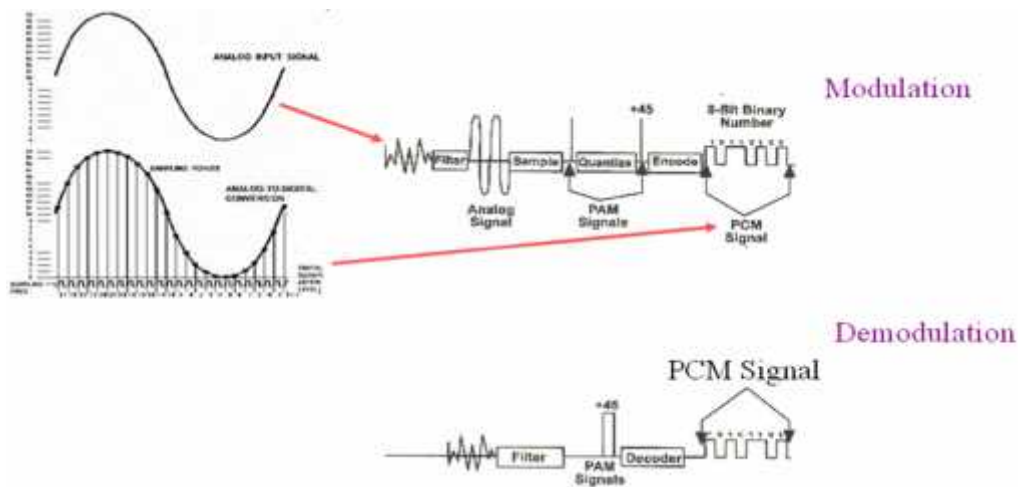
FIGURE 12-1 Embedded Systems: A-to-D and D-to-A Signal Conversion

### A/D Converter

- In order to change an analog signal to digital, the input analog signal is sampled at a high rate of speed.
- The amplitude at each of those sampled moments is converted into a number equivalent – this is called quantization.
- These numbers are simply the combinations of the 0s and 1s used in computer language – this called encoding.



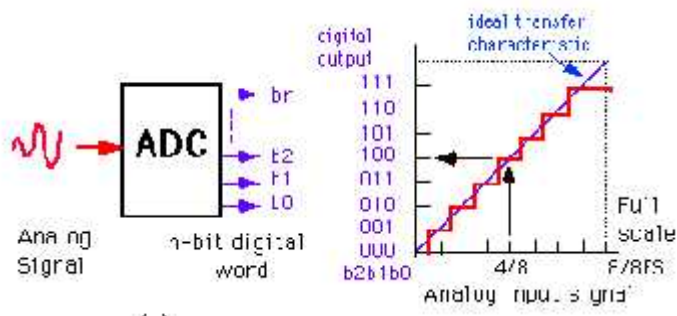
## A/D Conversion – Pulse Code Modulation/Demodulation



### Analog-to-Digital

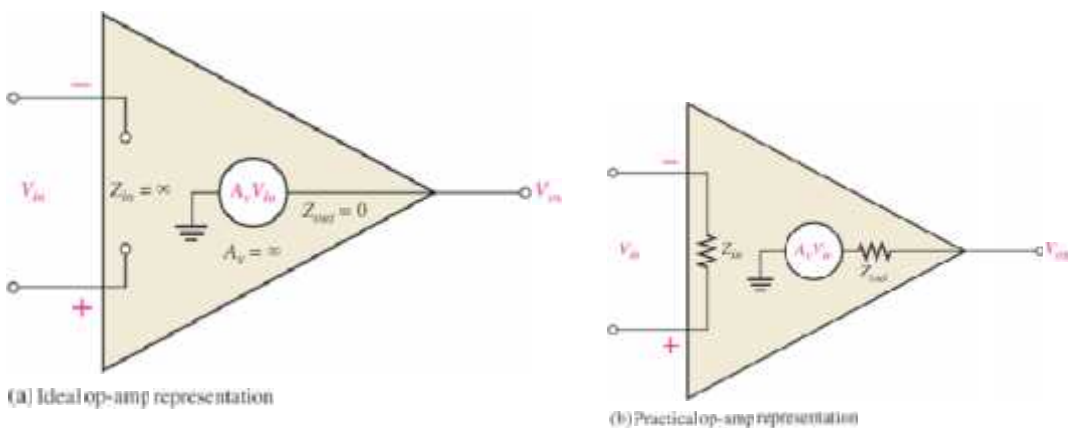
- A simple hypothetical A/D converter circuit with one analog input signal and three digital output lines with eight possible binary combinations: 000 to 111
  - Shows the graph of digital output for FS V analog input
- The following points can be summarized in the above process:
  - Maximum value this quantization process reaches is  $7/8$  V for a 1 V analog signal; includes  $1/8$  V an inherent error
  - $1/8$  V (an inherent error) is also equal to the value of the Least Significant Bit (LSB) = 001.
  - Resolution of a converter is defined in terms of the number of discrete values it can produce; also expressed in the number of bits used for conversion or as  $1/2^n$  where  $n$  = number of bits
  - The value of the most significant bit (MSB) -100- is equal to  $1/2$  the voltage of the full-scale value of 1 V.
  - The value of the largest digital number 111 is equal to full-scale value minus the value of the LSB.
  - The quantization error can be reduced or the resolution can be improved by increasing the number of bits used for the conversion





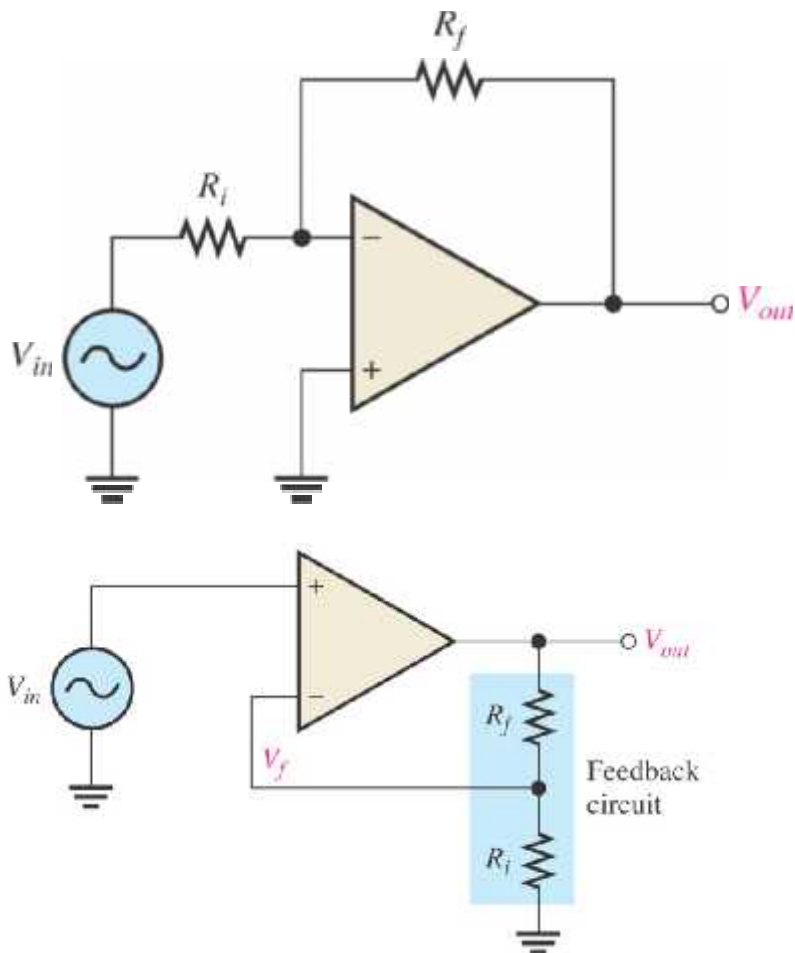
**Opamps**

- Ideal opamps
  - Infinite BW
  - Infinite voltage gain
  - Infinite input impedance
  - Zero output impedance
- Practical opamps
  - wide BW
  - Very high voltage gain
  - Very high input impedance
  - Very low output impedance



### Closed Loop Frequency Response

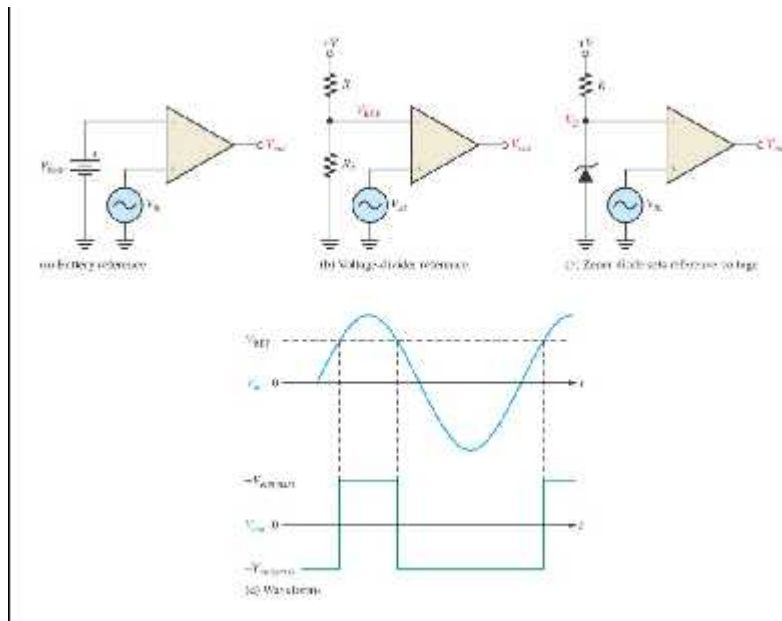
- Non-inverting
  - Source is connected to the non-inverting input
  - Feedback is connected to the inverting input
  - If  $R_f$  and  $R_i$  are zero, then unity feedback used for buffering
  - $A_v = 1 + R_f/R_i$
- Inverting
  - Feedback and source are connected to the inverting input
$$A_v = -R_f/R_i$$



### Comparators

- Determines which input is larger
- A small difference between inputs results maximum output voltage (high gain)

- Zero-level detection
- Non-zero-level detection



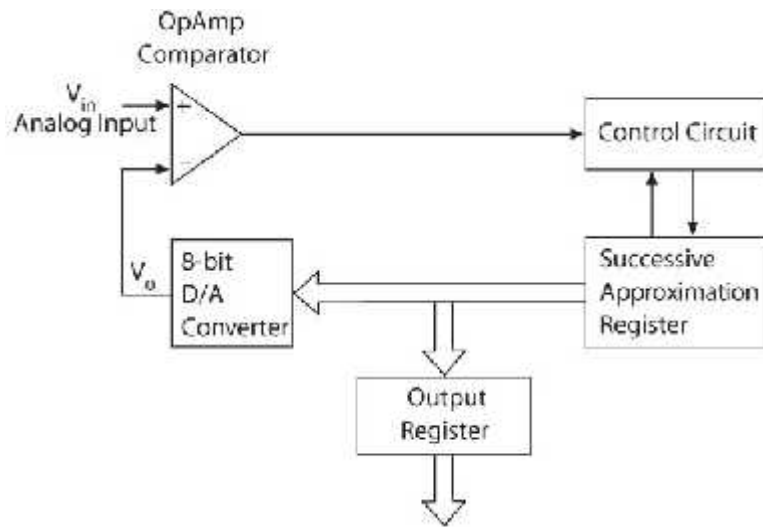
### A/D Conversion – Types

- Can be classified in four groups:
  - Integrator:
    - Charges a capacitor for a given amount of time using the analog signal.
    - It discharges back to zero with a known voltage and the counter provides the value of the unknown signal.
    - Provides slow conversion but low noise.
    - Often used in monitoring devices (e.g., voltmeters)
  - Flash: uses multiple comparators in parallel.
    - The known signal is connected to one side of the comparator and the analog signal to be converted to the other side of the comparator.
    - The output of the comparators provides the digital value.
    - This is a high-speed, high cost converter.

### A/D Conversion

Successive approximation: Includes a D/A (digital to analog) converter and a comparator. An internal analog signal is generated by turning on successive bits in the D/A converter.

Counter: Similar to a successive approximation converter except that the internal analog signal is generated by a counter starting at zero and feeding it to the D/A converter.



### Sample and Hold Circuit

- If the input voltage to an A/D converter is variable, the digital output is likely to be unreliable and unstable. Therefore, the varying voltage source is connected to the ADC through a sample and hold circuit.
- Basic Operation:
  - When the switch is connected, it samples the input voltage.
  - When the switch is open, it holds the sampled voltage by charging the capacitor.
  - Acquisition time: time to charge the capacitor after the switch is open and settle the output.
  - Conversion time: total time needed from the start of a conversion (turning on the MSB in the SAR) until the end of the conversion (turning on/off Bit0 in the SAR) - TAD: conversion time per bit.

