## 16117

# 3 Hours / 100 Marks

Seat No.

Instructions:

- (1) All Questions are *compulsory*.
- (2) Figures to the right indicate full marks.
- (3) Assume suitable data, if necessary.
- (4) Use of Non-programmable Electronic Pocket Calculator is permissible.
- (5) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.
- (6) Use of steam tables, logarithmic, Mollier's chart is permitted.

Marks

#### 1. Attempt any TEN of the following:

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- (a) Find 'a', if the tangent to the curve  $y = x^2 + ax$  at the origin is parallel to the line passing through A(-4, -3) and B(-2, 5).
- (b) Find Radius of curvature of  $y = x^3$  at (1, 1).
- (c) Evaluate :  $\int (e^x + x^e + e^e) dx$ .
- (d) Evaluate :  $\int \frac{1}{x + \sqrt{x}} dx.$
- (e) Evaluate :  $\int \sin^2 x \cdot dx$ .
- (f) Evaluate :  $\int \sec^2 x \cdot x \cdot dx$ .

(g) Find 'k', if 
$$\int_{0}^{1} (3x^2 + 2x + k) dx = 0$$
.

- (h) Evaluate  $\int \csc^2(x^\circ) dx$ .
- (i) Find the area under the parabola  $y^2 = 4x$  bounded by the lines x = 0, y = 0, x = 4.
- (j) Find order and degree of the differential equation  $\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx}\right)^{3/2}$ .
- (k) Form a D.E. if  $y = A \sin x + B \cos x$ .
- (1) Form a differential equation, if  $y = ax^2 + b$ .
- (m) Find the probability of getting sum of numbers is 9 with two dice.
- (n) An unbiased coin is tossed 3 times. Find the probability of getting two head.

## 2. Attempt any FOUR of the following:

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- (a) Find equation of tangent to the circle  $x^2 + y^2 + 6x 6y 7 = 0$  at a point it cuts the x-axis.
- (b) Discuss maxima and minima of the function " $\tan x 2x$ ".
- (c) Find radius of curvature of the curve  $\sqrt{x} + \sqrt{y} = 1$  at (1/4, 1/4).
- (d) Evaluate :  $\int \tan^6 x \cdot dx.$
- (e) Evaluate :  $\int \cos(\log x) \cdot dx$ .
- (f) Evaluate:  $\int \frac{\sec^2 x}{(1 + \tan x)(2 + \tan x)} dx.$

#### 3. Attempt any FOUR of the following:

- (a) Evaluate :  $\int_{0}^{4} \frac{1}{\sqrt{4x x^2}} dx.$
- (b) Evaluate :  $\int_{0}^{\pi} \frac{x \cdot \sin x}{1 + \cos^{2} x} dx.$
- (c) Find the area of the loop of the curve  $y^2 = x^2 (1 x)$ .
- (d) Solve:  $\frac{dy}{dx} = e^{(x-y)} \cdot x^2$ .
- (e) Solve:  $(x y) \frac{dy}{dx} = x + y$ .
- (f) Solve:  $(1+x)\frac{dy}{dx} y = e^{3x} (1+x)^2$ .

## 4. Attempt any FOUR of the following:

- (a) Evaluate :  $\int_{1}^{4} \frac{\sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} dx.$
- (b) Evaluate:  $\int_{0}^{\pi/2} \frac{\sin x \cdot \cos x}{\cos^2 x + 3\cos x + 2} dx.$
- (c) Find the area bounded by two parabola  $y^2 = 2x$  and  $x^2 = 2y$ .
- (d) Solve:  $\left[4 \frac{y^2}{x^2}\right] dx + \frac{2y}{x} dy = 0.$
- (e) Solve:  $(y \cdot e^{xy} 2y^3) dx + (x \cdot e^{xy} 6xy^2 2y) dy = 0$
- (f) Verify that  $y^2 = ax^2$  is a solution of  $x \left(\frac{dy}{dx}\right)^2 2y \frac{dy}{dx} + ax = 0$ .

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#### 5. Attempt any FOUR of the following:

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- (a) The probability that a student passes H.S.C. exam is 2/3 and the probability that he passes both H.S.C. and I.I.T. entrance exam is 14/45. The probability that he passes at least one exam is 4/5. What is the probability that he passes the I.I.T. entrance exam?
- (b) In 200 sets of tosses of 5 fair coins in how many ways you can expect
  - (i) at least two heads.
  - (ii) at the most two heads.
- (c) If 5% of the electric bulbs manufacturing by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs.
  - (i) None is defective.
  - (ii) Five bulbs are defective (Given  $e^{-5} = 0.007$ ).
- (d) Evaluate :  $\int \frac{x+1}{(x-1)^2} dx.$

(e) Evaluate:  $\int_{0}^{10/2} \sin 5x \cdot \cos 3x \cdot dx.$ 

(f) Evaluate :  $\int e^x \cdot \sin 4x \, dx.$ 

### 6. Attempt any FOUR of the following:

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- (a) Two six faced unbiased dice are thrown. Find the probability that the sum of the numbers shown is 7 or product is 12.
- (b) In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal. Find
  - (i) How many students score between 12 and 15?
  - (ii) How many students score above 18?

(Given: A(0.8) = 0.2881, A(0.4) = 0.1554, A((1.6) = 0.4452.)

- (c) A metal wire of 40 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.
- (d) Find equation of normal and tangent to the curve  $y = 4 \cdot x \cdot e^x$  at the origin.
- (e) If  $P(A) = \frac{1}{2}$ ,  $P(B') = \frac{2}{3}$ ,  $P(A \cup B) = \frac{2}{3}$ , find  $P(A' \cap B')$  & P(A/B).
- (f) The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that:
  - (i) Exactly two will be defective.
  - (ii) At least two will be defective.
  - (iii) None will be defective.