17216

13141

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following: **20**

a) If $z_1 = 1 - i$ and $z_2 = -2 + 4i$

Find $z_1 \cdot z_2$ and z_1 / z_2.

b) Find modulus and amplitude of $\dfrac{(2+i)\ (2-i)}{3-i}$

c) Find $\dfrac{dy}{dx}$, if $y = x^x$

d) If $f(x) = x^2 + 5$, find $f(x+2) - f(x-2)$.

P.T.O.

Marks

e) If $f(x) = 16^x + \log_2 x$, find the value of $f\left(\frac{1}{4}\right)$.

f) Evaluate $\lim_{x \to -2} \frac{x^3 + 8}{x^2 + 3x + 2}$

g) Evaluate $\lim_{x \to \infty} \left(\frac{x-1}{x}\right)^x$

h) Evaluate $\lim_{x \to 0} \left(\frac{\sin 5x}{3x}\right)$

i) If $y = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ find $\frac{dy}{dx}$.

j) Find $\frac{dy}{dx}$ if $x = r\cos\theta$, $y = r\sin\theta$.

k) Show that the root of the eqn.

$x \cdot e^x = 1$ lies between 0 & 1.

l) Find the first iteration by using Jacobi's method for the following equation.

$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25.$

Marks

2. Attempt any <u>FOUR</u> of the following: 16

a) Express $\dfrac{2+6\sqrt{3}\,i}{5+\sqrt{3}\,i}$ in the Polar form.

b) Simplify using Demoiver's theorem

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 \cdot (\cos 5\theta - i\sin 5\theta)^{4/5}}{(\cos 3/5\,\theta + i\sin 3/5\,\theta)^5 \cdot (\cos 4/5\,\theta + i\sin 4/5\,\theta)^{10}}$$

c) Using Euler's formula to prove that

$$\cos A + \cos B = 2\cdot\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

d) Prove that

$$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cdot\cos^n\frac{\theta}{2}\cdot\cos\left(\frac{n\theta}{2}\right)$$

e) If $f(x)=\log\left(\dfrac{x-1}{x+1}\right)$ prove that $f\left(\dfrac{x^2+1}{2x}\right) = 2\cdot f(x)$

f) If $f(x)=\dfrac{1}{1-x}$ find $f\{f[f(x)]\}$

Marks

3. Attempt any <u>FOUR</u> of the following: 16

a) If $f(x) = \dfrac{x-4}{4x-1}$ then show that $f[f(x)] = x$.

b) If $f(x) = \log(1+\tan x)$ show that $f\left(\dfrac{\pi}{4} - x\right) = \log 2 - f(x)$.

c) Evaluate $\lim\limits_{x\to\infty} (\sqrt{x^2+1} - x)$

d) Evaluate $\lim\limits_{x\to 0} \left[\dfrac{\sqrt{2} - \sqrt{1+\cos x}}{x^2}\right]$

e) Evaluate $\lim\limits_{x\to 0} \dfrac{1}{x} \left[\log(3+x) - \log(3-x)\right]$

f) Evaluate $\lim\limits_{x\to 0} \dfrac{18^x - 6^x - 3^x + 1}{x \cdot \tan x}$

4. Attempt any <u>FOUR</u> of the following: 16

a) Using first principle find derivative of $f(x) = \cos x$.

b) If u and v are differential function of x and $y = u - v$ then prove that $\dfrac{dy}{dx} = \dfrac{du}{dx} - \dfrac{dv}{dx}$.

c) If $e^x + e^y = e^{x+y}$, find $\dfrac{dy}{dx}$.

d) If $x = 3\sin\theta - 2\sin^3\theta$, $y = 3\cos\theta - 2\cos^3\theta$

find $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{4}$.

Marks

e) Diff. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

f) If $\log\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right)$ find $\frac{dy}{dx}$.

5. Attempt any <u>FOUR</u> of the following: **16**

a) If $y = a\cos(\log x) + b\sin(\log x)$.

Prove that $x^2\frac{d^2y}{dx^2} + x\cdot\frac{dy}{dx} + y = 0$.

b) Find $\frac{dy}{dx}$ if $y = x^{\sin x} + (\tan x)^x$.

c) Find real root of the equation $x^3 - 2x - 5 = 0$ (carry out three iterations only) by Bisection method.

d) Use Regular - Falsi method for finding the root of function $x^2 - 2x - 1 = 0$. (Carry out three iterations only).

e) By using Newton Raphson method find a root of the equation $x^4 - x - 9 = 0$. (Three iterations only).

f) Using Newton Raphson method find approximate value of $\sqrt{10}$ perform three iteration.

Marks

6. Attempt any <u>FOUR</u> of the following: 16

a) Evaluate $\lim_{x \to 3} \frac{x^3 - 7x^2 + 15x - 9}{x^3 - 4x^2 - 3x + 18}$.

b) Evaluate $\lim_{x \to 0} \frac{\tan 3x \cdot \left(5^x - 1\right)}{\sqrt{x^2 + 4} - 2}$.

c) Solve the following equation by Gauss elimination method

$2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16.$

d) Solve the following equation by Gauss-seidal method taking two iterations

$5x + 2y + 7z = 30,\ x + 4y + 2z = 15$ & $x + 2y + 5z = 20.$

e) Solve the following equation by Jacobi's method

$5x + 2y + z = 12,\ x + 4y + 2z = 15$ & $x + 2y + 5z = 20$
(two iterations only)

f) Solve using Gauss elimination method

$x + y + z = 4,\ 2x + y + z = 5$ and $3x + 2y + z = 7$.

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