17216

15116

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any TEN of the following: **20**

a) If $z = 1 + 3i$ evaluate $z^2 + 2z + 4$

b) Express $1 + i$ in modulus and amplitude form.

c) If $f(x) = 16^x + \log_4 x$ find $f\left(\frac{1}{2}\right)$.

d) Define even and odd function.

e) Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 + 2x + 5}{x + 1}$

Marks

f) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{\tan 5x}$

g) Evaluate $\lim_{x \to 0} \frac{3^{2x} - 2^{3x}}{\sin x}$

h) If $y = e^{4x} \cos 3x$ find $\frac{dy}{dx}$

i) If $y = \log [\sin (4x - 3)]$ find $\frac{dy}{dx}$

j) Find $\frac{dy}{dx}$ if $x = 4 \sin 3\theta, y = 4 \cos 6\theta$

k) Show that the root of $x^3 - 9x + 1 = 0$ lies between 2 and 3.

l) Find the first iteration by using Jacobi's method for the following system of equations:

$5x + 2y + z = 12, \quad x + 4y + 2z = 15, \quad x + 2y + 5z = 20$

2. Attempt any <u>FOUR</u> of the following: **16**

a) Find cube roots of unity and show that one root is square the other.

b) Simplify : $\frac{(\cos 2\theta + i \sin 2\theta)(\cos \theta - i \sin \theta)^4}{(\cos 3\theta + i \sin 3\theta)(\cos 5\theta - i \sin 5\theta)^3}$

using De-Moiver's theorem.

c) If $\sin (A + iB) = x + iy$ prove that:

(i) $\frac{x^2}{\cos h^2 B} + \frac{y^2}{\sin h^2 B} = 1$

(ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

Marks

d) Using Euler's exponential formula prove that $\sin^2\theta + \cos^2\theta = 1$

e) If $f(x) = \log\left(\frac{x}{x-1}\right)$ show that $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$

f) If $f(x) = \frac{3x+2}{4x-3}$ show that $f = f^{-1}$

3. Attempt any <u>FOUR</u> of the following: **16**

a) If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$ show that $f(t) = x$.

b) If $f(t) = 50\sin(100\pi t + 0.04)$, then show that $f\left(\frac{2}{100} + t\right) = f(t)$

c) Evaluate $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$

d) Evaluate $\lim_{x \to 0} \frac{\sin 2x - 2\sin x}{x^3}$

e) Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\cos 3x + 3\cos x}{\left(\pi/2 - x\right)^3}$

f) Evaluate $\lim_{x \to 3} \frac{\log(x-2)}{x^2 - 9}$

Marks

4. Attempt any FOUR of the following: **16**

a) If u and v are differentiable functions of x and $y = u/v$ where $v \neq 0$ then prove that $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

b) By using first principle find the derivative of $y = \cos x$.

c) If $y = \sin^{-1}\left[\frac{1}{\sqrt{1+x^2}}\right]$ find $\frac{dy}{dx}$

d) Find $\frac{dy}{dx}$ if $y = \frac{(\cos x)^x}{(1+x^2)}$

e) If $x^p \cdot y^q = (x+y)^{p+q}$ show that $\frac{dy}{dx} = \frac{y}{x}$

f) If $y = 3\sin t - 2\sin^3 t$, $x = 3\cos t - 2\cos^3 t$ find $\frac{dy}{dx}$ at $t = \pi/4$.

5. Attempt any FOUR of the following: **16**

a) Evaluate $\lim_{x \to 0}\left(\frac{6^x - 3^x - 2^x + 1}{x^2}\right)$

b) Evaluate $\lim_{x \to 3}\frac{\log x - \log 3}{x - 3}$

c) Find the approximate roots of the equation $x^3 - x - 4 = 0$ by Bisection method.

Marks

d) Show that root of the equation $x^3 - 4x + 1 = 0$ in (1, 2) and find it by using Newton-Raphson method performing two iterations.

e) Solve the following equations by Gauss elimination method.

$x + 2y + 3z = 14,\ \ 3x + y + 2z = 11,\ \ 2x + 3y + z = 11.$

f) Solve the following equations by Gauss-Seidal method.

$5x - y = 9,\ \ x - 5y + z = -4,\ y - 5z = 6$

6. Attempt any FOUR of the following: **16**

a) If $y = e^{m \sin^{-1} x}$ prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0$

b) If $x = a(\theta + \sin\theta),\ \ y = a(1 + \cos\theta)$ find $\frac{d^2y}{dx^2}$ at $\theta = \pi/2$.

c) Obtain the root of the equation by Regula-Falsi method.

$x^3 - x - 1 = 0$

d) Solve the following equation by Jacobi's method

$20x + y - 2z = 17;\ 3x + 20y - z = -18;\ 2x - 3y + 20z = 25.$

e) Solve the equation by using Gauss - eliminaion method.

$4x + y + 2z = 12,\ -x + 11y + 4z = 33,\ \ 2x - 3y + 8z = 20$

f) Use Newton-Raphson method to evaluate $\sqrt[3]{20}$ correct to three decimal places.