21415

3 Hours/100 Marks   Seat No. [ ][ ][ ][ ][ ][ ][ ]

Instructions : (1) **All** questions are **compulsory**.
(2) Illustrate your answers with **neat** sketches **wherever** necessary.
(3) Figures to the **right** indicate **full** marks.
(4) **Assume** suitable data, if **necessary**.
(5) **Use** of non-programmable electronic pocket **calculator** is permissible.
(6) Mobile Phone, Pager and any other Electronic Communication devices are not **permissible** in Examination Hall.

MARKS

1. Attempt **any ten** of the following :   **(10×2=20)**

   a) At which point on the curve $y = 3x - x^2$, the slope of tangent is –5.

   b) Divide 80 into two parts such that their product is maximum.

   c) Evaluate $\int \sin^3 x \cdot \cos x \, dx$.

   d) Evaluate $\int x \cdot e^x \, dx$.

   e) Evaluate $\int \dfrac{1}{(x + 3)(x + 2)} \, dx$.

   f) Evaluate $\displaystyle\int_0^{\log_e 2} e^{2x} \, dx$.

   g) Find the area between the line $y = 2x$, X-axis and the ordinates $x = 1$ and $x = 3$.

   h) Find order and degree of following differential equation $\dfrac{d^2 y}{dx^2} + \sqrt{1 + \dfrac{dy}{dx}} = 0$.

   i) Form a differential equation if $y^2 = 4ax$.

P.T.O.

j) From a pack of 52 cards one card is drawn at random. Find the probability of getting a King.

k) An unbiased coin is tossed 5 times. Find the probability of getting three heads.

l) A die is thrown. Find the probability of getting an odd number.

2. Solve **any four** of the following : **(4×4=16)**

   a) Find equation of tangent and normal to the curve $y = x(2 - x)$ at point $(2, 0)$.

   b) Find radius of curvature of the curve $x = a \cos^3\theta$, $y = a \sin^3\theta$ at $\theta = \pi/4$.

   c) Find maximum and minimum value of $y = x^3 - \dfrac{15}{2} x^2 + 18x$.

   d) Evaluate $\int \dfrac{e^x(x + 1)}{\cos^2(xe^x)} \, dx$.

   e) Evaluate $\int \dfrac{\sec^2 x}{3 \tan^2 x - 2 \tan x - 5} \, dx$.

   f) Evaluate $\int \dfrac{x \cdot \sin^{-1} x}{\sqrt{1 - x^2}} \, dx$.

3. Solve **any four** of the following : **(4×4=16)**

   a) Evaluate $\displaystyle\int_0^{\pi/2} \dfrac{dx}{\sqrt{9 - 4x^2}}$.

   b) Evaluate $\displaystyle\int_{\pi/6}^{\pi/3} \dfrac{\sin x}{\sin x + \cos x} \, dx$.

   c) Find area bounded by two curves $y^2 = x$ and $x^2 = y$.

   d) Solve $xy^2 \, dy - (x^3 + y^3) \, dx = 0$ given $y = 0$ when $x = 1$.

   e) Solve the differential equation $(x + y)^2 \dfrac{dy}{dx} = a^2$.

   f) Solve $x \dfrac{dy}{dx} - y = x^2$.

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4. Attempt **any four** of the following : **(4×4=16)**

a) Evaluate $\int_{1}^{5} \dfrac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}\, dx$.

b) Evaluate $\int \dfrac{dx}{4\cos^2 x + 9\sin^2 x}$.

c) Using integration find area of ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

d) Solve $(2x^2 + 6xy - y^2)\, dx + (3x^2 - 2xy + y^2)\, dy = 0$.

e) Solve $(1 + x^2)\, dy - x^2 y\, dx = 0$.

f) Show that $y = \sin(\log x)$ is solution of a differential equation

$$x^2 \dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} + y = 0.$$

5. Attempt **any four** of the following : **(4×4=16)**

a) The probability that 'A' can shoot at a target is 5/7 and B can shoot at same target is 3/5. (A and B) shoot independently. Find probability that

   i) The target is not shot at all

   ii) The target is shot by atleast one of them.

b) If 30% of the bulbs produced are defective. Find probability that out 4 bulbs selected.

   a) One is defective.

   b) At the most two are defective.

c) In a certain examination 500 student appeared, mean score is 68 and S.D. 8. Assuming data are normally distributed find the number of student scoring.

   a) Less than 50

   b) More than 60.

   (Given area between z = 0 to z = 2.25 is 0.4878 and area between z = 0 to z = 1 is 0.3413).

d) Evaluate $\int_0^\pi \dfrac{dx}{5 + 4\cos x}$.

e) Evaluate $\int \dfrac{x}{x^2 + 3x - 4} dx$.

f) Solve $x \cdot \log x \dfrac{dy}{dx} + y = z \log x$.

6. Attempt **any four** of the following : **(4×4=16)**

a) If $P(A) = \dfrac{1}{2}$, $P(B') = \dfrac{2}{3}$, $P(A \cup B) = \dfrac{2}{3}$. Find $P(A' \cap B')$ and $P(A/B)$.

b) If probability that an electric motor is defective is 0.01. What is probability that sample of 300 electric motor will contain exactly 5 defective motor.

   $(e^{-3} = 0.0498)$.

c) Fit a Poisson distribution for following observation :

x_i	20	30	40	50	60	70
f_i	8	12	30	10	6	4

d) A metal wire of 36 m long is bent to form a rectangle. Find its dimensions when its area is maximum.

e) Find equation of tangent to the curve $x = \dfrac{1}{t}$, $y = 1 - \dfrac{1}{t}$ when $t = 2$.

f) Find the area bounded by the parabola $y = 4 - x^2$ and x-axis.

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